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Proof of the Jie Bove Conjecture

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Abstract: In 1855, the mathematician Jie Bove proposed a primes distribution conjecture of small regional called Jie Bove conjecture. This article proposed and demonstrated the regional distribution theorem of primes and proved Jie Bove conjecture.

Key words: Primes; Theorem; Region; Calculation

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1. INTRODUCTION

In 1855, the mathematician Jie Bove proposed integer x^2 and $(x+1)^2$ between at least two primes [1-3, 5, 6]. In 1905, Mai Lunte proved $x < 9{,}000{,}000$ the Jie Bove conjecture is Established. Benpian proved:

$$\pi(x^2+2x+1)-\pi(x^2)>1 \tag{1}$$

where the $\pi\,(x^2)$ is not greater than x^2 number of prime numbers. If the (1) is proved established, the Jie Bove conjecture is established.

2. THEOREM INTERVAL DISTRIBUTION OF PRIMES

Set $x > y$, interval (x, y) is small, primes p, ignore decimals, we have [7-8]:

$$\pi(x)-\pi(y)=s(x)-s(y), \quad (x\to\infty) \tag{2}$$

$$s(x)-s(y)=\sum_{y\le p\le x}\frac{x-y}{p\ln\lambda}, \quad \lambda=\frac{x}{y}$$

Here (2) is known as *theorem interval distribution of primes*. For example, Let $y = 9$, $x = 16$, by (2) calculation:

$$\begin{aligned} s(16)-s(9) &= \sum_{y\le p\le x}\frac{x-y}{p\ln\frac{x}{y}} \\ &= \sum_{9\le p\le 16}\frac{16-9}{p\ln\frac{16}{9}} \\ &= \frac{7}{11\ln\frac{16}{9}}+\frac{7}{13\ln\frac{16}{9}} \\ &= 2+0.041881103 \end{aligned}$$

If ignore decimals 0.041881103, we get:

$$s(16)-s(9)=2$$

$$\pi(16)-\pi(9)=2$$

Let $y = x^2$, and x into $(x+1)^2$, calculation:

x,	$\pi(x)-\pi(y)$,	$s(x)-s(y)$,
2,	2,	2 + 0.11,
4,	3,	3 + 0.12,
8,	4,	4 - 0.004,
16,	7,	7 + 0.023,
32,	9,	9 + 0.0238,
64,	14,	14 + 0.0037,
128,	24,	24 - 0.0025,
256,	53,	53 + 0.02,

By (2) if x approaching infinity, the decimals are infinitely small.

3. PROVE THEOREMS INTERVAL

Proof. Set x approaching infinity, interval (x, y) is small, by (2) we get:

$$\begin{aligned} \ln\lambda &= \ln\frac{x}{y} \\ &= \ln\left(1+\frac{x-y}{y}\right) \end{aligned}$$

$$= \frac{x-y}{y} - \frac{(x-y)^2}{2y^2} + \frac{(x-y)^3}{3y^3} - \cdots$$
$$= \frac{x-y}{y}$$

and:

$$\ln\lambda = \ln\frac{x}{y}$$
$$= \ln\left(1 - \frac{x-y}{x}\right)^{-1}$$
$$= \frac{x-y}{x} + \frac{(x-y)^2}{2x^2} + \frac{(x-y)^3}{3x^3} + \cdots$$
$$= \frac{x-y}{x}$$

We can get:

$$\ln\lambda\ln\lambda = \frac{x-y}{y}\frac{x-y}{x}$$
$$\ln^2\lambda = \frac{(x-y)^2}{xy}$$

and

$$\ln\lambda = \frac{x-y}{\sqrt{xy}} \tag{3}$$

By (2) we have:

$$y \le p \le x \tag{4}$$

Interval (x, y) is small, by (4) we can get:

$$p = \sqrt{xy} \tag{5}$$

By (3) and (5) we get:

$$\ln\lambda = \frac{x-y}{p}, \quad (x \to \infty)$$

Into (2) we can get:

$$s(x) - s(y) = \sum_{y \le p \le x} 1 = \pi(x) - \pi(y) \;, \quad (x \to \infty)$$

Then (2) is proved confirmed.

4 CONVERT THE THEOREMS INTERVAL

By (2) we get:

$$\frac{1}{2}s(x) - \frac{1}{2}s(y) < \pi(x) - \pi(y) < 2s(x) - 2s(y) \tag{6}$$
$$s(x) - s(y) = \sum_{y \le p \le x} \frac{x-y}{p\ln\lambda}$$

By (6) we prove the Jie Bove conjecture.

5. MERTENS THEOREM

In 1874, mathematicians Mertens proved [4]:

$$\sum_{p\le x}\frac{1}{p}=\ln\ln x+A_1+o\left(\frac{1}{\ln x}\right) \tag{7}$$

Here (7) is known as *Mertens theorem*. Wherein A_1 is a constant.
Set $x\to\infty$ by (7) we get:

$$\sum_{p\le x}\frac{1}{p}=\ln\ln x+A_1$$

Set $x > y$, we get:

$$\sum_{p\le x}\frac{1}{p}-\sum_{p\le y}\frac{1}{p}=\ln\ln x+A_1-\ln\ln y-A_1$$

$$=\ln\frac{\ln x}{\ln y}$$

$$=\ln\frac{\ln y+\ln x-\ln y}{\ln y}$$

$$=\ln\left(1+\frac{\ln\lambda}{\ln y}\right),\qquad \lambda=\frac{x}{y}$$

Interval (x, y) is small, we have:

$$\ln\left(1+\frac{\ln\lambda}{\ln y}\right)=\frac{\ln\lambda}{\ln y}-\frac{\ln^2\lambda}{2\ln^2 y}+\frac{\ln^3\lambda}{3\ln^3 y}-\cdots\cdots=\frac{\ln\lambda}{\ln y}$$

We can get:

$$\sum_{y\le p\le x}\frac{1}{p}=\frac{\ln\lambda}{\ln y}$$

Set $y = x^2$, and x into $(x+1)^2$, we can get:

$$\sum_{x^2\le p\le(x+1)^2}\frac{1}{p}=\frac{\ln\frac{x+1}{x}}{\ln x},\quad (x\to\infty) \tag{8}$$

Here (8) is known as *theorem interval distribution of Mertens*.

6. PROVE THE JIE BOVE CONJECTURE

By (2) we get:

$$s(x^2+2x+1)-s(x^2)=\sum_{x^2\le p\le(x+1)^2}\frac{2x+1}{p\ln\frac{(x+1)^2}{x^2}} \tag{9}$$

By (9) we have:

$$s(x^2+2x+1)-s(x^2)=\frac{2x+1}{2\ln\frac{x+1}{x}}\sum_{x^2\le p\le(x+1)^2}\frac{1}{p} \tag{10}$$

Set (10) into (8) we can get:

$$s(x^2+2x+1)-s(x^2)=\frac{2x+1}{2\ln x}, \quad (x\to\infty) \tag{11}$$

Set (11) into (6) we get:

$$\frac{x}{2\ln x}<\pi(x^2+2x+1)-\pi(x^2)<\frac{2x+1}{\ln x} \tag{12}$$

By (12), (2) is proved confirmed.

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