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The Innovation and Martingale Pricing of Mortgage Insurance Under O-U Process

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Abstract: Give an innovative design for housing mortgage insurance in the basis of the guarantee insurance, then obtain the pricing formula of the mortgage insurance by using the method of martingale pricing, when the unpaid part is a constant and the real estate price is driven by the process of O-U index.

Key words: Mortgage; Insurance; Option; Martingale pricing

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1. INTRODUCTION

Housing mortgage is a new kind of real estate's financial business which improving commercialization of China housing and supporting residents buy house by their own. It is practiced several years and have achieved results, but there is many risks existed, such as borrowers can not reimbursed because of some reasons, mortgage property is insufficient to reimburse according to law disposal; mortgaged goods is existing the risks of lost or break; owing to did not register mortgage causing mortgage risk, interest exchange risk and inflation risk and so on. In order to escape these risks, The People's Bank of China has issued "The Measures of Personal Housing Mortgage Management". That is ordained that borrowers who applying housing mortgage to commercial bank can through insurance to assisting bank to solving risks of housing mortgage, but now our country is the beginning of housing mortgage. At present, China's housing mortgage loans insurance mainly has three kinds of forms, first, housing property insurance; second, mortgage guarantee insurancethird, life insurance of housing mortgage.The kind of insurance is single, high premium, the means of paying is inflexible, few cities implement the insurance, which are far from practical demand. Furthermore, the cities are doing the business of housing mortgage insurance which premium is usually calculated by a proportion of amount of loan. The calculation of premium is lack of theoretical foundation, it is unfair for insurer and restrained the development of insurance's business. Different housing buyers have different states of credit and economic power. Diversity of insurance is good of loan institutions and housing buyers to make flexible decisions. Thus, it is necessary to develop new kind of mortgage insurances or innovate on the basis of primitive insurances, in order to vastly adapt to actual situation of housing buyers. Meanwhile, deepen discussing the pricing theory of housing mortgage insurance in order to offer theory support for premium [1–3].

2. THE CONSTRUCTION OF HOUSING MORTGAGE INSURANCE INNOVATION

Housing mortgage guarantee insurance is the insurance that lending institutions requires borrowers to cover. Housing buyers should pay a certain amount of premium to insurance company when they lending, and the insurance company makes the guarantee of repayment in turn, then the lending institutions will give buyers the corresponding loans and offer certain preference in the down payment, interest and loan terms and so on, but this insurance has shortcomings. Due to the risks have totally transferred to insurance company, the lending institutions usually neglect the stringent review of housing buyers' credit. Therefore, the insurance company shall charging high premium because of enormous risk. It is not only restrained the development of housing mortgage and increased economic costs of housing buyers, but also can be causing system risks. Thus, it is necessary to encourage lending institutions to conduct rigorous credit review of the borrower to avoid systemic risk, but also take into account the interests of the lending institutions, insurance companies and buyers. It is a proper choice for loan institutions and insurance companies that both are responsible for risks.

This paper have learned the experiences of home and abroad, and constructed a certain innovation on the basis of part guarantee insurance. If the loss was within primitive principal A_0 of a certain proportion of k_1, it is entirely borne by insurance company; if it is beyond k_1, the extra proportion of the quota k_1A_0 should be distributed to insurance companies and loan institutions by a certain proportion of k_2, thus, we called it common insurance.

If borrowers have broken the contract at the time of $t = T$, then the insurance institutions can adopt the following two means to performing compensation:

- If the amount of loss is below the insurance companies' quota of k_1A_0, the insurance companies should reimburse the unpaid loan to the lending institutions balance and take the right of property in order to get the mortgage right. Let $M(T)$ be the amount of unpaid at time T, let $H(T)$ be property price at time T, let α be the proportion of housing value after realization of mortgage right, and α is a constant, then the amount of compensation is $\max(M(T) - \alpha H(T),\ 0)$.

- If the amount of loss is beyond the insurance companies' quota of $k_1 A_0$, then the lending institutions shall retain the right of property, the amount of compensation that the insurance companies should pay to the lending institutions is $k_1 A_0 + k_2 [M(T) - \alpha H(T) - k_1 A_0]$.

Thus, the income at the expiration of the mortgage loan insurance hold by lending institutions can be expressed as:

$$V_T = \begin{cases} \max(M(T) - \alpha H(T),\ 0), & \max(M(T) - \alpha H(T),\ 0) < k_1 A_0, \\ k_1 A_0 + k_2 [M(T) - \alpha H(T) - k_1 A_0], & M(T) - \alpha H(T) \geq k_1 A_0. \end{cases} \tag{1}$$

3. THE CONSTRUCTION OF MATHEMATICAL MODELS

Given the financial market in continuous time, taking 0 as now and T as the due date; Given a complete probability space $(\Omega,\ F,\ \mathrm{P})$, assume that the unpaid amount $M(T)$ is a constant at moment T (can be obtained by credit evaluation of risk and suppose $M(T) > k_1 A_0$), and the risk-free rate $r(t)$ is the function of time t, property values $H(t)$ meet stochastic differential equation as follows:

$$\frac{\mathrm{d}H(t)}{H(t)} = [\mu(t) - a\ln(H(t))]\,\mathrm{d}t + \sigma(t)\mathrm{d}B(t), \qquad H(0) = H \tag{2}$$

Where, $\sigma(t)$ are continuous functions of the time t, $\sigma(t) > 0$, $\{B(t)\}_{0 \leq t \leq T}$ is one-dimensional standard Brownian Motion of $(\Omega,\ F,\ \mathrm{P})$, $(F_t)_{0 \leq t \leq T}$ is the corresponding natural information flow, $F_t = F$. The role of the constant $a (> 0)$ is that when prices rise to a certain height, it makes a downward trend in $H(t)$, and the expected rate of return in this model depends on the property values.

Lemma 3.1 Assume property values meet (2), then we have

$$H(t) = H^{e^{-at}} \exp\left\{ \int_0^t \left[\mu(s) - \frac{1}{2}\sigma^2(s)\right] e^{-as}\mathrm{d}s + e^{-at} \int_0^t e^{as}\sigma(s)\mathrm{d}B(s) \right\} \tag{3}$$

$$\mathrm{E}[H(t)] = H^{e^{-\alpha t}} \exp\left\{ \int_0^t \left[\mu(s) - \frac{1}{2}\sigma^2(s)\right] e^{-as}\mathrm{d}s + \frac{1}{2} \int_0^t \sigma^2(t) e^{-2as}\mathrm{d}s \right\} \tag{4}$$

4. PRICING MORTGAGE COMMON INSURANCE BY THE METHOD OF MARTINGALE PRICING

The traditional martingale pricing technique has often supposed that the financial market is no arbitrage and complete. On the circumstances of martingale pricing technique, a stock (or derivative securities) present price can be get from the discounted future expected cash flow, and expected value discount can be carried out under the risk neutral. It is assumed that the financial market is complete and no arbitrage. The traditional martingale pricing methods is used to obtain the insurance pricing.

Let $\theta(t) = \dfrac{\mu(t) - a\ln(H(t)) - r(t)}{\sigma(t)}$, $0 \leqslant t \leqslant T$ be a process adapted to $(F_t)_{0\leq t\leq T}$. Define a new probability $\tilde{\mathrm{P}}$ by

$$\frac{\mathrm{d}\tilde{\mathrm{P}}}{\mathrm{dP}} = \exp\{-\int_0^T \theta(u)\mathrm{d}B(u) - \frac{1}{2}\int_0^T (\theta(u))^2\mathrm{du}\},$$

and define a process $\{\tilde{B}(t)\}_{0\leqslant t\leqslant T}$ by

$$\mathrm{d}\tilde{B}(t) \triangleq \theta(t)\mathrm{d}t \ + \mathrm{d}B(t).$$

According to the Girsanov Theorem, the probability P and $\tilde{\mathrm{P}}$ are equivalent; Under $\tilde{\mathrm{P}}$, the process $\tilde{\mathrm{B}}(t)$, $0 \leq t \leq T$ is a Brownian motion, and we have

$$\frac{\mathrm{d}H(t)}{H(t)} = r(t)\mathrm{d}t + \sigma(t)\mathrm{d}\tilde{B}(t).$$

Furthermore

$$H(t) = H\exp\left\{\int_0^t \left(r(s) - \frac{1}{2}\sigma^2(s)\right)\mathrm{d}s + \int_0^t \sigma(s)\mathrm{d}\tilde{B}(s)\right\}.$$

Theorem 4.1 Assume the financial market is complete and no arbitrage. Let $M(T)$, a constant, be the unpaid amount. Let $r(t)$, a function of t, be the risk-free rate. Let the income of co-insurance policy satisfies Equation (1), and property values $H(t)$ meets Equation (2), then we obtain the martingale pricing formula of mortgage common insurance as follows:

$$\begin{aligned} V_0 =& \mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\left[M\Phi(d_2) + (k_1A_0 - k_1k_2A_0 + k_2M - M)\Phi(d_1)\right] \\ &- \alpha H\left[\Phi(\sqrt{\kappa} - d_1) - \Phi(\sqrt{\kappa} - d_2)\right] - \alpha Hk_2\Phi(d_1 - \sqrt{\kappa}) \end{aligned}$$

Where $d_1 = \dfrac{\ln\left(\dfrac{M - k_1A_0}{\alpha H}\right) - \int_0^T r(t)dt + \dfrac{1}{2}\kappa}{\sqrt{\kappa}}$, $d_2 = \dfrac{\ln\left(\dfrac{M}{\alpha H}\right) - \int_0^T r(t)dt + \dfrac{1}{2}\kappa}{\sqrt{\kappa}}$, $\kappa = \int_0^T \sigma^2(t)\mathrm{d}t$ and $\Phi(x)$ represents normal distribution function.

Proof. For convenience, let $X = \dfrac{\int_0^T \sigma(t)\mathrm{d}\tilde{B}(t)}{\sqrt{\kappa}}$, $\kappa = \int_0^T \sigma^2(t)\mathrm{d}t$, then $X \sim \mathrm{N}(0,1)$; let

$$A = \{M(T) > \alpha H(T),\ M(T) - \alpha H(T) < k_1A_0\},$$
$$B = \{M(T) - \alpha H(T) \geq k_1A_0\}.$$

According to martingale pricing method, the value of housing mortgage loan co-insurance V_0 satisfies

$$\begin{aligned} V_0 =& \mathrm{E}^{\tilde{P}}\left[\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}(M - \alpha H(T))\mathrm{I}_A\right] \\ &+ \mathrm{E}^{\tilde{P}}\left[\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}(k_1A_0 + k_2(M - \alpha H(T) - k_1A_0))\mathrm{I}_B\right] \\ =& M\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{E}^{\tilde{P}}\left[\mathrm{I}_A\right] - \alpha\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{E}^{\tilde{P}}\left[H(T)\mathrm{I}_A\right] \\ &+ (k_1A_0 - k_1k_2A_0 + k_2M)\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{E}^{\tilde{P}}\left[\mathrm{I}_B\right] - \alpha k_2\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{E}^{\tilde{P}}\left[H(T)\mathrm{I}_B\right] \end{aligned} \tag{5}$$

To compute V_0, we first compute set A.

$$\begin{aligned}A =&\{M(T) > \alpha H(T), M(T) - \alpha H(T) < k_1 A_0\}\\ =&\left\{\ln(\frac{M - k_1 A_0}{\alpha H}) - \int_0^T r(t)dt + \frac{1}{2}\kappa < \int_0^T \sigma(t)\mathrm{d}\tilde{B}(t) < \ln(\frac{M}{\alpha H}) - \int_0^T r(t)dt + \frac{1}{2}\kappa\right\}\\ =&\left\{d_1 < \frac{\int_0^T \sigma(t)\mathrm{d}\tilde{B}(t)}{\sqrt{\kappa}} < d_2\right\}\\ =&\{d_1 < X < d_2\}\end{aligned}$$

Similarly,

$$B = \{M(T) - \alpha H(T) \geqslant k_1 A_0\} = \{X \leqslant d_1\}.$$

Then, we have

$$\mathrm{E}^{\tilde{P}}[\mathrm{I}_A] = \Phi(d_2) - \Phi(d_1), \tag{6}$$

$$\mathrm{E}^{\tilde{P}}[\mathrm{I}_B] = \Phi(d_1) \tag{7}$$

$$\begin{aligned}\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{E}^{\tilde{P}}[H(T)\mathrm{I}_A] =&H\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{e}^{\int_0^T r(t)-\frac{1}{2}\sigma^2(\mathrm{t})\mathrm{d}t}\mathrm{E}^{\tilde{P}}[\mathrm{e}^{\int_0^T \sigma(t)\mathrm{d}\tilde{B}(t)}\mathrm{I}_{\{d_1<X<d_2\}}]\\ =&H\mathrm{e}^{-\frac{1}{2}\kappa}\mathrm{E}^{\tilde{P}}[\mathrm{e}^{X\sqrt{\kappa}}\mathrm{I}_{\{d_3<X<d_1\}}]\\ =&H[\Phi(\sqrt{\kappa} - d_1) - \Phi(\sqrt{\kappa} - d_2)]\end{aligned} \tag{8}$$

$$\begin{aligned}\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{E}^{\tilde{P}}[H(T)\mathrm{I}_B] =&H\mathrm{e}^{-\int_0^T r(t)\mathrm{d}t}\mathrm{e}^{\int_0^T r(t)-\frac{1}{2}\sigma^2(\mathrm{t})\mathrm{d}t}\mathrm{E}^{\tilde{P}}[\mathrm{e}^{\int_0^T \sigma(t)\mathrm{d}\tilde{B}(t)}\mathrm{I}_{\{X\leqslant d_1\}}]\\ =&H\mathrm{e}^{-\frac{1}{2}\kappa}\mathrm{E}^{\tilde{P}}[\mathrm{e}^{X\sqrt{\kappa}}\mathrm{I}_{\{-X\geqslant -d_1\}}]\\ =&H\Phi(d_1 - \sqrt{\kappa})\end{aligned} \tag{9}$$

Inserting (6–9) into (5), the theorem is proved.

□

5. CONCLUSION

According to the reality of China, we give an innovative design for housing mortgage insurance in the basis of the guarantee insurance. The innovative design is conducive to the healthy development of the banking and the insurance industry, and also conducive to avoiding systemic risk.

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