Implementing a Group- and Project/Problem-Based Learning in a College Algebra Course

Abdramane Serme[a],*

[a]Ph.D., Associate Professor, Mathematics Department, the City University of New York (BMCC/CUNY), New York, USA.
* Corresponding author.

Address: Mathematics Department, The City University of New York (BMCC/CUNY), New York, 199 Chambers Street, Room N-599, New York, NY 10007-1097, USA; E-mail: aserme@bmcc.cuny.edu

Abstract: The idea of this paper originated from reading the interesting article written by Mohammad A. Alseweed in Studies in Literature and Language (2013). In the article, the author defined and analyzed traditional learning, blended/hybrid learning and virtual learning. The result favored blended/hybrid learning in test scores and students' attitudes suggests that students are more receptive when instructors use different teaching approaches. In this paper we describe an innovative approach to project-based learning in a group setting environment. Traditional science instruction has tended to exclude students who need to learn from contexts that are real-world, graspable, and self-evidence meaningful (Kolodner et al., 2003). As emphasized by Blumenfeld, one way of encouraging student engagement and addressing the contextualization of students' inquiry is through project-based instruction (Bumenfeld et al., 1991; Petrosino, 2004). The learning sciences community agrees that deep and effective learning is best promoted by situating learning in purposeful and engaging activity (Bransford et al., 1999; Collins et al., 1989; Kolodner et al., 2003). Our goal for developing this collaborative project/problem-based learning technique is to engage the students in deep learning by encouraging them to write and explain all the steps of their reasoning when yielding to the answers.

1. INTRODUCTION

According to Blumenfelfd et al. (1991) and Helle (2006), the essence of project-based learning is that a question or problem serves to organize and drive activities; and these activities culminate in a final product that addresses the driving question. The benefits of learning in a group setting environment is well known as it provides opportunities for students to share ideas, extend their thinking and learn by talking and collaborating with others (Marx et al., 1997). The collaborative project-based learning that we developed involved both vertical learning (i.e., accumulation of subject matter knowledge) and horizontal learning (i.e., general skills such as writing, management skills). We achieved those two goals by asking the students to write all the steps of the proofs, justifications or calculations that they are performing. The collaborative part consists of putting the students in groups of four or five students based on a questionnaire that the students filled out the first day of class. The questionnaire asks them their major, their GPA, their last mathematics class and the grade earned, where they live, the days that they are on campus attending classes, and how confident they feel about their mathematical ability. The projects/problems that students will be working on are designed base on the chapters covered in the syllabus or curriculum. Every important theme or component of the syllabus/curriculum can be derived/taught by breaking it into a series of small questions. One of the students in the group of four or five will be called "The Prover", "The Solver" or "The Brain", one other student in the same group will be called "The Explainer" or "The Calculator" and the last student of the first trio will be called "The Checker", "The Recorder" or "The Caller". Many other variations of this kind of collaborative project/ problem-based learning exist. This article is divided into two main parts. In the first part we will explain how the collaborative project/problem-based learning is implemented. The second part of the article will be devoted in giving in detail an example of a project that was done with the students in class.

2. IMPLEMENTATION OF THE COLLABORATIVE PROJECT/PROBLEM-BASED LEARNING

The first day of class is devoted to the explanation of how the class will be conducted and the way the project-based learning roles will be rotated. The

students will be provided an instructional sheet that will be explained in class and given to the students to keep in their portfolio. Details of the contents of the instructional sheet are provided in section 2.1 below. The students should be reminded to bring the instructional sheet to every class. In the implementation we recommend that class participation occupies an important part of the grades distribution. We also recommend that instructors get some type of training before the beginning of the semester in order to familiarize themselves with the projects, their concepts and their implementation.

2.1 Instructions and Roles Definition

Instructions: In this collaborative project-based learning project you will be in groups of 4 or 5 students. Each student is required to participate and participation is graded. One student will play the role of The Prover, The Solver or The Brain. The student sitting on his right (or left) hand will play the role of The Explainer or The Calculator and the student sitting on the right of The Explainer plays the role of The Checker, The Recorder or 'The Caller". Each role is explained below.

   • ***The Prover*:** The role of The Prover is to write out the mathematical proof, explanation, or justification of the given equation by starting with the expression to the left of the equals sign and doing one operation to it in each step. Each of the steps will be written on a new line. The Prover will verbally explain out loud his thinking of each step to the group as he goes. The Prover can be helped by other members of the group. Below is an example of how The Prover will operate.

column of The Prover	column of The Explainer
$a^r a^s = (a \cdots a) \cdot (a \cdots a)$ *r gimes s times*	"a^r means that we should multiply r– many a^s together and a^s means that we should multiply s– many a^s together."
$= a \cdots a$ $(r+s) times$	"We can remove the parentheses because of the property of the multiplication, when multiplying, the way we group variables does not change the outcome of the multiplication. Furthermore, r– many a^s and s– many a^s being multiplied together means that we are multiplying a total of $(r+s)$– many a^s all together."
$= a^{r+s}$	"*A* product of $(r+s)$– many a^s is just a raised to the power $(r+s)$– which is a^{r+s}."

   • ***The Explainer*:** "The Explainer" not only checks the work of The Prover out loud but also writes the explanation, or justification in the column of The Explainer. Each step of the calculation or proof written by the The Prover should be explained, or justified in writing by The Explainer on the same line to the right of the calculation or proof. The Explainer while explaining The Prover's work may detect some mistakes. In that case The Explainer, The Prover and the other members of the group will discuss the issue and come up with the correct answer or formulation. When the disagreement or mistake cannot be resolved,

the instructor is called for clarification. It is normal that The Explainer does not understand some of the (correct) steps of The Prover and therefore asks for clarification to better understand to process. Each step and its correctness are agreed upon as a group before moving to the next step or equation. The explanation of the reasoning behind each step is written in a short sentence or two on the right of The Prover's work and if possible on the same line.

• **_The Checker_:** The Checker role is to check the work of The Explainerand The Prover. Any mistakes found will be the subject of a group discussion until the group can decide on the correct way to clearly explain that step. Every correction in which the group agrees on is written on the original paper next to the calculation or proof.

It is worth noticing that the discussions that arise when The Prover, The Explaner or The Checker are doing their part of the work is what this collaborative project/problem-based learning is about. The students' learning is enriched by engaging each other and they end up learning from each other. The instructor should monitor the group activity by walking around and should intervene as soon as there is a disagreement, mistake or if the group is not working at an acceptable pace. Time is of essence and the instructor should make sure that everything is going smoothly in each group at all time. After the first group session the rotation begins, The Explainer becomes The Prover, The Checker becomes The Explainer and the fourth student becomes The Checker and so forth until all the questions in the project are solved. The instructor should make sure that everybody, especially the weak students participate and volunteer to be The Prover or The Explainer. Throughout the project and for each question, each student will write is name next to the work that he actually did. Each question on the sheet that the instructor collects will start as follows:

The Prover: Yusuf,
The Explainer: Bryant,
The Checker: Abdul.

The instructor can collect the students work done after each session and give the sheet back to the students the following session. If the instructor trusts the students, then the unfinished work can be done by the group outside of the classroom during a mutually agreed upon time and day.

2.2 Class Preparation for the Project

The instructors are encouraged before handing out the project to give a very short overview of the project on the chalk or smart board, or with a computer (power point is preferable). This is the instructor's opportunity to provide some important rules, equations, definitions, propositions and/or theorems to the students. He will define a clear goal and outline of the project. It is also worth adding that for this group project/problem-based learning to be efficient that the students be provided some podcasts to view online. The podcasts may be followed by a quiz of couple questions to motivate the students and make sure that they view the podcasts. The contents of the podcasts will be a summary of a traditional lecture. In the

podcasts the students will learn some definitions, rules, formulae, equations that will be helpful for the projects.

The example of a project, provided below, in the second part of this paper is on "Exponential and logarithmic functions". At the Borough of Manhattan Community College (BMCC) which is a two-year college within the City University of New York (CUNY), "Exponential and logarithmic functions" is the title of the chapter seven of Intermediate Algebra and Trigonometry course. The class is a remedial class that meets for six hours a week in two sessions of three hours each or three sessions of two hours each. The following rules and definitions with examples can be given to the students.

a)   The five basic exponent rules (we assume that a, $b \neq 0$, and r, s are positive integers):

1) $a^r a^s = a^{r+s}$,
2) $(a^r)^s = a^{rs}$,
3) $(ab)^r = a^r b^r$,
4) $\left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r}$,

5) $\dfrac{a^r}{b^s} = a^{r-s}$

b)   The statement $y = \log_b x$ is read y "is the logarithm to the base b of x " and is equivalent to the statement $x = by$. In words, we say "y is the number we raise to in order to get x". b is a positive number other than 1.

c)   *Special identities*. If b is a positive number other than 1 then: $b^{\log_b x} = x$ and $\log_b b^x = x$.

d)   Goal: The goal of this assignment is to define a logarithm form as exponent in a way that is consistent with the five basic exponent rules above; to understand the connection between exponential and logarithmic forms; to understand how to simplify or expand certain logarithmic forms, and to understand and use the properties of logarithms to write a logarithm expression as a single logarithm. Another easy goal to attain is to solve equations that involve logarithms.

Here is the Outline of this project:

1)   We will find a way to define logarithm forms as exponents, using the basic rules of exponents.

2)   Then we will restate some of the rules of exponents in the context of logarithms.

3)   We will see how to use the properties of logarithms to simplify, condense and expand expressions involving logarithms.

4)   Finally, we will explore the methods used for solving equations that involve logarithms, as well as considering the reasonable values that might be in the solution set of the equations.

3. EXAMPLE OF A PROJECT DONE IN CLASS BY THE STUDENTS

Project On Exponential and Logarithmic Functions

3.1 What Does $y=\log_b x$ Mean, Where b is a Positive Real Number Other Than 1?

We will assume throughout this project that x is a positive number in the expression $\log_b x$. The statement $y=\log_b x$ is equivalent to the statement $x=b^y$

We will rewrite the logarithmic form $y=\log_b$ in exponential form $x=b^y$ and use what we know about exponents and their properties to simplify, expand expressions with logarithms, and solve equations involving logarithms.

1) ***Specific Example***: We begin by rewriting the statement $3=\log_2 8$ in exponential form. We assume that all of the rules of exponents apply.

$b=2$ The base b is 2.
$y=3$ The left-hand side y is 3.
$x=8$ x on the right-hand side is 8.
$8=2^3$ We write then the exponential form of $3=\log_2 8$.

2) ***Specific Example***: We will write the exponential form of $\log_{10} 0.001=-3$**.** We assume that all of the rules of exponents apply.

$b=10$ The base b is 10.
$y=-3$ The left-hand side is -3.
$x=0.001$ *x on the right-hand side is 0.001.*
$10^{-3}=0.001$ *We write then the exponential form of* $\log_{10} 0.001=-3$**.**

3) ***Specific Example***: We want to solve the equation $\log_4 \frac{1}{8}=x$ *for x*. We assume that all of the rules of exponents apply.

$b=4$ The base b is *4*.
$y=x$ The left-hand side y in this case is *x*.
$x=\frac{1}{8}$ x on the right-hand side, in this case is $x=\frac{1}{8}$.

$\frac{1}{8}=4^x$ We write then the exponential form of $\log_4 \frac{1}{8}=x$**.**

$2^{-3}=(2^2)^x$ *T*he exponential form of $\frac{1}{8}$ is 2^{-3} and the exponential form of *4 is* 2^2.
$2^{-3}=2^{2x}$ We rewrite $(2^2)^x$ as 2^{2x} using the exponential rule *b)* above.
$-3=2x$ The only way the left and right side are equal is if the exponents are equal.
$\frac{-3}{2}=x$ We then solve for x

4) <u>**Your turn.**</u> Use the example in number 1) as a model. All you have to do is fill in the empty blanks. Student 1: copy the steps below onto your own paper filling in the missing steps. As you write, explain aloud to the group, one step at a time, why each step above makes sense, just as you would in the role of the The Prover. Student 2 should play the role of The Explainer and fill in the blanks of the explanation*,* and Student 3 should play the role of The Checker.

Solve for x: $\log_3 x = -2$	We are trying to find the value of x
$b = 3$	The _____ b is *3*.
$x = ?$	x is the unknown.
$y = -2$	y is found on the _____ of the _____ symbol.
$x = 3^{-2}$	Is the _____ form of _____ .
$x = \dfrac{1}{3^2}$	We use the _____ exponent rule.
$x = \dfrac{1}{9}$	

5) **Your turn.** Solve the following equations a), b), c) for x and simplify d) and e). First rewrite in exponential form. Rotate roles/tasks at each example. Work out each one step-by-step, with **a different step on each line**, just like in the models above.

  a. $\log_x 16 = 4$

  b. $\log_{16} \dfrac{1}{2} = x$

  c. $\log_{27} x = \dfrac{1}{3}$,

  d. $\log_{10} 100,000 =$

  e. $\log_3(\log_2 8) =$

3.2 Properties of Logarithms

6) **Examples:** The next task is to show that $\log_b(xy) = \log_b x + \log_b y$. Assume that b is a positive real number other than 1. Using the steps you followed in the previous tasks as a model, try to show that $\log_b(xy) = \log_b x + \log_b y$.

$b^{A+B} = b^A \cdot b^B$	First of the five basic exponent rules.
If we denote $x = b^A$ then $A = \log_b x$	We rewrite the exponential form in logarithmic form.
If we denote $y = b^B$ then $B = \log_b y$	We rewrite the exponential form in logarithmic form.
$xy = b^A \cdot b^B$	We multiply $x = b^A$ by $y = b^B$.
$xy = b A + B$	From step *1 above* $b^{A+B} = b^A \cdot b^B$
If $xy = b^{A+B}$ then $A + B = \log_b xy$	We rewrite the exponential form in logarithmic form.
$A + B = \log_b x + \log_b y$	From step *2* and *3* above since $A = \log_b x$ and $B = \log_b y$
$\log_b xy = \log_b x + \log_b y$	From last two equations.

7) **Your turn.** Use 6) above as a model to show that $\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$. Rotate the roles. As before, assume that b is a positive real number other than 1. Start by writing $b^{A-B} = \dfrac{b^A}{b^B}$ and using the same notation follow 6) above.

8) **Your turn.** Use 6) above as a model to show that $\log_b x^r = r\log_b x$. As before, assume that b is a positive real number other than 1. From the product rule of real numbers we know that $rA = Ar$ All you have to do is fill in the empty

blanks. Rotate the roles. Student 4 or 5: Copy the steps below onto your own paper, filling in the missing steps. As you write, explain aloud to the group, one step at a time, why each step above makes sense, just as you would in the role of The Prover. Student 5 or 1 should play the role of The Explainer and fill in the blanks of the explanation, and Student 1 or 2 should play the role of The Checker.

$(b^A)r=b^A r$	_____ of the five basic exponent rules.
If we denote $x=b^A$ then $A=\log_b x$	By rewriting the __in _____ .
$rA=r\log_b x$	We ___the left and right side of $A=\log_b x$ by r.
$x^r=b^{Ar}$	We ____both side of $x=b^A$ to ____ power.
If $x^r=b^{Ar}$ then $Ar=\log_b x^r$	We rewrite the _____ in _____.
$rA=r\log_b x$	From step number _____ above.
$\log_b x^r=r\log_b x$	From _____equations.

3.3 How to Use the Properties of Logarithms

9) **Specific Examples:**

We would like to use the properties of logarithms to expand $\log_2 \dfrac{5x^4}{\sqrt{y}\,z^3}$.

$\log_2\dfrac{5x^4}{\sqrt{y}\,z^3}=\log_2\dfrac{5x^4}{y^{\frac{1}{2}}z^3}$ We know that $\sqrt{y}=y^{\frac{1}{2}}$ from the rational exponents rule.

$\log_2\dfrac{5x^4}{\sqrt{y}\,z^3}=\log_2 5x^4-\log_2 y^{\frac{1}{2}}z^3$ We use the distributive property to the left side

of the equation We use the property of logarithms in 7) to rewrite the logarithm of a quotient as the difference of the logarithms.

$\log_2\dfrac{5x^4}{\sqrt{y}\,.z^3}=\log_2 5+\log_2 x^4-(\log_2 y^{\frac{1}{2}}+\log_2 z^3)$ We use the property of logarithms in

6) to rewrite the logarithm of a product as the sum of the logarithms.

$\log_2\dfrac{5x^4}{\sqrt{y}\,z^3}=\log_2 5+\log_2 x^4-\log_2 y^{\frac{1}{2}}-\log_2 z^3$ We remove the parentheses.

$\log_2\dfrac{5x^4}{\sqrt{y}\,z^3}=\log_2 5+4\log_2 x-\dfrac{1}{2}\log_2 y-3\log_2 z$ We use the property of logarithms in

8) to rewrite the logarithm of a numbers raised to a power as the product of the power and the logarithm of the number.

10. **Your turn.** Using 9) above as models, expand each of the following expressions as much as possible. Rotate roles/tasks at each example. Work out each one step-by-step, **with a different step on each line**, just like in the models above.

a. $\log_{10}\dfrac{x^{\frac{2}{7}}}{y^5\sqrt[4]{z^3}}$,

b. $\log_8 \sqrt[4]{\dfrac{x^5 y^6}{z^3}}$,

c. $\log_5 \dfrac{\sqrt[3]{x}}{z^2 y^{-5}}$,

d. $\log_b \dfrac{16b^2}{25y^3}$.

11) **Specific Examples:**

We would like to use the properties of logarithms to write $2\log_{10}a + \dfrac{1}{4}\log_{10} b - \dfrac{1}{3} \log_{10} 8$ as a single logarithm.

$2\log_{10}a + \dfrac{1}{4}\log_{10} b - \dfrac{1}{3}\log_{10} 8$

$=\log_{10}a^2 + \log_{10}b^{\frac{1}{4}} - \log_{10}8^{\frac{1}{3}}$	We use the property of logarithms in 8) to rewrite the product of a number and the logarithm of a number as the logarithm of the number raised to a power.
$=\log_{10}a^2 + \log_{10}b^{\frac{1}{4}} - \log_{10}2$	We simplify $8^{\frac{1}{3}}=2$.
$=\log_{10}a^2 b^{\frac{1}{4}} - \log_{10}2$	We use the property in 6) to write the sum of logarithms as the logarithm of a product.
$=\log_{10}\dfrac{a^2 b^{\frac{1}{4}}}{2}$	We use the property in 7) to rewrite the difference of logarithms as the logarithm of a quotient.
$=\log_{10}\dfrac{a^2 \sqrt[4]{b}}{2}$	*We rewrite* $\sqrt[4]{b}$ *as* $b^{\frac{1}{4}}$.

12) **Your turn.** Using 11) above as models, write the following expressions as a single logarithm. Rotate roles/tasks at each example. Work out each one step-by-step, **with a different step on each line**, just like in the models above.

a. $\dfrac{2}{3}\log_5 x + \dfrac{3}{4}\log_{10} y - \dfrac{4}{5}\log_{10} z$

b. $\dfrac{1}{3}(\log_7 z + 2\log_7 (z+4)) - \log_7 (y-1)$

c. $\log_4(x^2 - x - 6) - \log_4(x^2 - 9)$

13) **Specific Examples:** We would like to use the properties of logarithms to solve for x: $\log_2(x+2) + \log_2 x = 3$

$\log_2(x+2) + \log_2 x = 3$

$\log_2(x+2)x = 3$	We use the property of logarithms in 6) to the left side of the equation to rewrite the sum of logarithm of two numbers as the logarithm of their product.
$(x+2)x = 2^3$	We write the exponential form of $\log_2(x+2)x = 3$.
$x^2 + 2x = 8$	We use the distributive property to the left side of the equation.
$x2 + 2x - 8 = 0$	We subtract 8 on both sides of the equation.
$(x+4)(x-2) = 0$	We factor $x^2 + 2x - 8$.

$x+4=0$ or $x–2=0$ We use the zero product rule.

$x= –4$ or $x=2$ We solve both equations.

x cannot be a negative number in the expression $\log_b x$. A substitution of $x=–4$ into

$\log_2(x+2)+\log_2 x = 3$ gives $\log_2(–2)+\log_2(–4)= 3$ *which contains logarithms of negative numbers,*

we cannot use $x= –4$ as a solution.

The solution of the equation is $x=2$

14) **Your turn.** Using 13) above as models, write the following expressions as a single logarithm. Rotate roles/tasks at each example. Work out each one step-by-step, **with a different step on each line**, just like in the models above.

  a. $\log_{10}(x–9)=0$,

  b. $\log_2 x+\log_2(x–2)=3$,

  c. $\log_3(x+3)–\log_3(x–1)=1$,

  d. $\log_8 \sqrt{x} + \log_8 \sqrt{5x + 2} = \dfrac{2}{3}$.

4. CONCLUSION

We designed an innovative group- and project/problem-based learning in a College Algebra class that combined three different types of teaching and learning. Our collaborative project/problem-based learning has some type of hybrid learning component which effectiveness is well known (Petrocino, 2004; Hiebert, 1996). We used a group setting environment which has been proven to be efficient in number of papers (Toolin, 2004; Barron et al., 1998; Petrocino, 2004) to challenge and stimulate students, especially weaker students and encourage them to learn from each other. Our teaching and learning approach is based on projects/problems done in the classroom in a group setting. Our approach engaged the students in deep learning by encouraging them to write and explain all the steps of their reasoning. The tasks and roles are rotated to motivate and give a sense of power and control to the students. The grades of the students in the collaborative project/problem-based learning class were higher by one step than the grades of the students in the traditional, lecture-type class. The students in the experimental group seemed, in general, to be more responsive and appreciative that they learned from each other. Our group- and project/problem-based learning can not only be used in a hybrid learning environment but can actually make it more effective. It may also be very helpful to instructors who want to use a group- and project/problem-based learning but are hesitant because of its time consuming aspect and/or the lack of participation of all the group members. Our group- and project/problem-based learning also helped in the horizontal learning with students improving their general mathematical writing, management and leadership skills.

5. ACKNOWLEDGMENTS

REFERENCES

[01]   Alseweed, M. (2013). Students' achievement and attitude toward using traditional learning, blended learning, and virtual classes learning in teaching and learning at the university level. *Studies in Literature and Language*, *6*(1), 65-73.

[02]   Barron, B. J. S., Schwartz, D. L., Vye, N. J., Moore, A., Zech, L., & Bransford, J. D. (1998). Doing with understanding: Lessons from research on problem- and project-based learning. *The Journal of the Learning Sciences*, *7*(3/4), 271-311.

[03]   Blumenfeld, P. C., Soloway, E., Marx, R. W., Krajcik, J. S., Guzdial, M., & Palincsar, A. (1991). Motivating project-based learning: Sustaining the doing, supporting the learning. *Educational Psychologist*, *26*, 369-398.

[04]   Bransford, J., Brown, A., & Cocking, R. (1999). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.

[05]   Byrne, M. (2013). Learning to cooperate: One researcher's experiences building effective cooperative groups with students. *Abstracts of American Mathematical Society*, *34*(1), 171, p.532.

[06]   Collins, A., Brown, S., & Newman, G. (1989). Cognitive apprenticeship: Teaching the craft of reading, writing, and mathematics. In L. B. Resnick (Ed.)*, Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp.453-494). Hillsdale, NJ: Erlbaum.

[07]   Dahlgren, M., & Öberg, G. (2001). Questioning to learn and learning to question: Structure and function of problem-based learning scenarios in environmental science education. *Higher Education*, *41*(3), 263-282.

[08]   Hancock, D. (2004). Cooperative learning and peer orientation effects on monitoring and achievement. *The Journal of Educational Research*, *97*(3), 159-166.

[09]   Helle, L., Tynjälä, P., & Olkinuora, E. (2006). Project-based learning in post-secondary education: Theory, practice and rubber sling shots. *Higher Education*, *51*(2), 287-314.

[10]   Hiebert, J., Carpenter, P., Fennema, E., Fuson, K., Human, P., Murray, H., … Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, *25*(4), 12-21.

[11] Kolodner, J., Camp, P., Crismond, D., Fasse, B., Gray, J., Holbrook, J., … Ryan, M. (2003). Problem-based learning meets case-based reasoning in the middle-school science classroom: Putting learning by *design^tm* into practice. *The Journal of the Learning Sciences*, *12*(4), 495-547.

[12] Laffey, J., Tupper, T., Musser, D., & Wedman, J. (1998). A computer-mediated support system for project-based learning. *Educational Technology Research and Development*, *46*(1), 73-86.

[13] Marx, R. W., Blumenfeld, P. C., Blunk, M., Crawford, B., Kelly, B., & Mills, K. (1994). Enacting project-based science: Experiences of four middle grade teachers. *The Elementary School Journal*, *94*(5), 517-538.

[14] Marx, R. W., Blumenfeld, P. C., Krajcik, J. S., & Soloway, E. (1997). Enacting project-based science. *The Elementary School Journal*, *97*(4), 442-359.

[15] Meyer, D., Turner, J., & Spencer, C. (1997). Challenge in a mathematics classroom: Students' motivation and strategies in project-based learning. *The Elementary School Journal, 97*(5), 501-521.

[16] Petrosino, A. (2004). Integrating curriculum, instruction, and assessment in project-based instruction: A case study of an experienced teacher. *Journal of Science Education and Technology*, *13*(4), 447-460.

[17] Prescott, J., & Smith, D. (1987). A project-based approach to competitive analysis. *Strategic Management Journal*, *8*(5), 411-423.

[18] Toolin, R. (2004). Striking a balance between innovation and standards: A study of teachers implementing project-based approaches to teaching science. *Journal of Science Education and Technology*, *13*(2), 179-187.

[19] Williams, T. (2003). Learning from projects. *The Journal of the Operational Research Society*, *54*(5), 443-451.