

Proving the Twin Prime Conjecture

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Received: November 11, 2013/ Accepted: January 5, 2014/ Published online:
February 26, 2014

Abstract: Presented and proved symmetry primes theorem, parallelism proving the twin primes conjecture, Goldbach conjecture. Give part of the calculation.

Key words: Integer; Primes; Composite number; Theorem

Liu, D., & Liu, J. F. (2014). Proving the Twin Prime Conjecture. *Studies in Mathematical Sciences*, 8(1), 21-26. Available from URL: <http://www.cscanada.net/index.php/sms/article/view/4014> DOI: <http://dx.doi.org/10.3968/4014>

1. INTRODUCTION

Mathematicians found that primes of distance 2, there are infinitely many numbers known as twin primes conjecture ^[1-3], for example, (11, 13), (59, 61). In 1742, the German mathematician Goldbach found even greater than 4 are each equal to two prime numbers and. known as Goldbach conjecture, For example, 6 = 3+3, 8 = 3+5, Here proved:

$$L(x) \sim \frac{\pi^2(x)}{x}, \quad (x \rightarrow \infty). \quad (1)$$

Here (1) known as *twin primes conjecture*. Wherein $L(x)$ is the numbers of twin primes. And proved:

$$G(N) \sim \frac{\pi^2(N)}{N}, \quad (N \rightarrow \infty). \quad (2)$$

Here (2) known as *Goldbach conjecture*. Wherein $G(x)$ is the numbers of two primes and.

2. DISTRIBUTION DENSITY OF SYMMETRY PRIMES

Set the Integer $x = 16$, we have:

k is 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,

$k+2$ is 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,

Wherein $k+2$ is the primes have: 2, 3, 5, 7, 11, 13,

Set $k+2$ is the primes, the quantity as $\pi(x)$, we get ^[5-6]:

$$\frac{\pi(x)}{x} \quad (3)$$

Here (3) as *distribution density of symmetry primes*.

Set the composite number h , here let 0 and 1 is the composite number, we have:

h is 0, 1, 4, 6, 8, 9, 10, 12, 14,

$h+2$ is 2, 3, 6, 8, 10, 11, 12, 14, 16,

Wherein $h+2$ is primes have: 2, 3, 11. Set the h quantity as F , $h+2$ is the primes, the quantity as $F(x)$, we get:

$$\frac{F(x)}{F} \quad (4)$$

Here (4) also as *distribution density of symmetry primes*. Wherein $F = x - \pi(x) - 1$, for example, Let $x = 16$, $\pi(16) = 6$, and $F = 9$, $F(16) = 3$, by (3) and (4) get:

$$\frac{\pi(16)}{16} = \frac{6}{16} \text{ and } \frac{F(16)}{9} = \frac{3}{9}.$$

Calculate:

x ,	$\pi(x)/x$,	$F(x)/F$,
10^1 ,	0.4,	0.4,
10^2 ,	0.25,	0.23,
10^3 ,	0.168,	0.162,
10^4 ,	0.1229,	0.1168,
10^5 ,	0.09592,	0.09256,
10^6 ,	0.078498,	0.076321,
10^7 ,	0.0664579,	0.0648711,
10^8 ,	0.05761455,	0.05646461,

3. THE TWIN PRIMES

Set primes p , we have:

p is 2, 3, 5, 7, 11, 13,

$p+2$ is 4, 5, 7, 9, 13, 15,

Wherein $p+2$ is the primes have: 5, 7, 13, the twin primes (3, 5), (5, 7), (11, 13). Set $p+2$ is the primes, the quantity as $L(x)$, we can get [1]:

$$\pi(x) = F(x) + L(x) \quad (5)$$

By (5) we can prove the twin primes conjecture.

4. SYMMETRY PRIMES THEOREM

The T theorem:

$$F(x) \sim \frac{F\pi(x)}{x}, \quad (x \rightarrow \infty). \quad (6)$$

Here (6) known as *symmetry primes theorem* [5].

Proof: by (4) we get:

$$F = x - \pi(x) - 1 = x \left(1 - \frac{\pi(x) + 1}{x} \right). \quad (7)$$

By (7) can get:

$$\lim_{x \rightarrow \infty} \frac{x}{F} = 1.$$

If $F \sim x$, then $F(x) \sim \pi(x)$, by (3), (4) we can get:

$$\lim_{x \rightarrow \infty} \frac{x}{F} / \frac{\pi(x)}{F(x)} = 1. \quad (8)$$

By (8) can get:

$$\frac{F(x)}{F} \sim \frac{\pi(x)}{x}, \quad (x \rightarrow \infty). \quad (9)$$

By (9) The T theorem proved.

5. PROVE THE TWIN PRIMES CONJECTURE

By (5) we get:

$$L(x) = \pi(x) - F(x). \quad (10)$$

By (6), (10) can get:

$$L(x) \sim \pi(x) - \frac{F\pi(x)}{x}, \quad (x \rightarrow \infty). \quad (11)$$

By (11) we get:

$$L(x) \sim \frac{\pi^2(x)}{x}, \quad (x \rightarrow \infty).$$

The twin primes conjecture proved. The number of twin primes. By (10) we get^[6-7]:

$$\pi(x) \sim \sum_{n=2}^x \frac{1}{\ln(n)} \quad \text{and} \quad F(x) \sim \frac{x}{\ln x}.$$

Can get:

$$L(x) \sim \sum_{n=2}^x \frac{1}{\ln(n)} - \frac{x}{\ln x}, \quad (x \rightarrow \infty).$$

Generally speaking:

$$L(x) \sim c \sum_{n=2}^x \frac{1}{\ln(n)} - c \frac{x}{\ln x}, \quad c = 1.32 \dots \dots$$

6. PROVE GOLDBACH CONJECTURE

Set even $N=16$, $k < N$, we have:

k is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
 $N-k$ is 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1.

Set $N-k$ is the numbers of primes as $\pi(N)$, we get [4]:

$$\frac{\pi(N)}{N}. \tag{12}$$

Set composite number h , the numbers of composite number as F , we have:

h is 4, 6, 8, 9, 10, 12, 14, 15,
 $N-h$ is 12, 10, 8, 7, 6, 4, 2, 1.

Set $N-h$ is the numbers of primes as $F(N)$, we get [5]26,[6]:

$$\frac{F(N)}{F}. \tag{13}$$

Set the primes p , we have [4]:

p is 2, 3, 5, 7, 11, 13,
 $N-p$ is 14, 13, 11, 9, 5, 3,

Set $N-p$ is the numbers of primes as $G(N)$, we can get:

$$\pi(N) = F(N) + G(N). \tag{14}$$

By (12), (13) can prove:

$$F(N) \sim \frac{F\pi(N)}{N}, \quad (N \rightarrow \infty). \tag{15}$$

The proof and the twin primes conjecture are the same. by (14), (15) we can get^[8]:

$$G(N) \sim \frac{\pi^2(N)}{N}, \quad (x \rightarrow \infty).$$

The Goldbach conjecture proved.

7. GOLDBACH'S CONJECTURE CALCULATION

Formulas are [3]:

$$G(N) \sim \frac{2c(N) N}{\ln^2(N)}. \tag{16}$$

Here (16) known as *Hardy formula*. Which *Laman Niu Yang factor*:

$$c(N) = \prod_{P \leq N} \frac{P(P-2)}{(P-1)^2} \prod_{P|N} \frac{P-1}{P-2}.$$

8. WANG XINYU DOUBLE SIEVE TRANSFORM

Set $p > 2$, have:

$$G(N) \sim \frac{N}{2} \prod_{P|N} \left(1 - \frac{1}{P}\right) \prod_{P \perp N} \left(1 - \frac{2}{P}\right), \quad p \leq N^{1/2}. \tag{17}$$

Here (17) known as *double sieve transform formula*. qingdao china Wang

Xinyu transform:

$$\begin{aligned} \frac{N}{2} \prod_{P|N} \frac{P-1}{P} \prod_{P \perp N} \frac{P-2}{P} &= \frac{N}{2} \frac{\prod_{P|N} \frac{P-1}{P}}{\prod_{P|N} \frac{P-2}{P}} \prod_{P|N} \frac{P-2}{P} \prod_{P \perp N} \frac{P-2}{P} \\ &= \frac{N}{2} \prod_{P|N} \frac{P-1}{P-2} \prod_{P \leq \sqrt{N}} \frac{P-2}{P} \\ &= \frac{N}{2} \prod_{P|N} \frac{P-1}{P-2} \prod_{P \leq \sqrt{N}} \frac{P-2}{P} \frac{P^2}{(P-1)^2} \frac{(P-1)^2}{P^2} \\ &= \frac{N}{2} \prod_{P|N} \frac{P-1}{P-2} \prod_{P \leq \sqrt{N}} \frac{P(P-2)}{(P-1)^2} \prod_{P \leq \sqrt{N}} \frac{(P-1)^2}{P^2}. \end{aligned}$$

Can get ^[2]:

$$\prod_{P \leq \sqrt{N}} \frac{(P-1)^2}{P^2} \sim \frac{4\pi^2(N)}{N^2}.$$

Get:

$$G(N) \sim 2 \prod_{P|N} \frac{P-1}{P-2} \prod_{P \leq \sqrt{N}} \frac{P(P-2)}{(P-1)^2} \frac{N}{\ln^2 N}. \tag{18}$$

Here (18) also double sieve transform formula. And Hardy formulas are the same.

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