CSCanada
Progress in Applied Mathematics
Vol. 3, No. 2, 2012, pp. 7-15
DOI: 10.3968/j.pam.1925252820120302.1611

ISSN 1925-251X [Print]
ISSN 1925-2528 [Online]
www.cscanada.net
www.cscanada.org

First Integral Method to Study Nonlinear Evolution Equations

A. Asaraai[1]; S. Khaleghizadeh[2]; A. Samiei Paghaleh[1,*]

[1]Department of Mathematics, Faculty of Sciences, University of Guilan, Rasht, Iran
[2]Departmant of Mathematics, Payame Noor University, P. O. Box 19395-3697, Tehran, Iran
*Corresponding author.
Address: Department of Mathematics, Faculty of Sciences, University of Guilan, Rasht, Iran

Received September 10, 2011; accepted January 11, 2012

Abstract

In this paper, we apply the first integral method to generalized ZK-BBM equation and Drinefel'd-Sokolov-Wilson system and one-dimensional modified EW-Burgers equation.

The first integral method is a powerful solution method for obtaining exact solutions of some nonlinear evolution equations. This method was first proposed by Feng [8] in solving Burgers–KdV equation which is based on the ring theory of commutative algebra. This method can be applied to nonintegrable equations as well as to integrable ones.

Key words

First integral method; Generalized ZK-BBM equation ; Drinefel'd-Sokolov-Wilson system; One-dimensional modified EW-Burgers equation

A. Asaraai, S. Khaleghizadeh, A. Samiei Paghaleh (2012). First Integral Method to Study Nonlinear Evolution Equations. *Progress in Applied Mathematics*, *3*(2), 7-15. Available from: URL: http://www.cscanada.net/index.php/pam/article/view/j.pam.1925252820120302.1611 DOI: http://dx.doi.org/10.3968/j.pam.1925252820120302.1611

1. INTRODUCTION

Nonlinear evolution equations are widely used to describe complex phenomena in various sciences such as fluid physics, condensed matter, biophysics, plasma physics, nonlinear optics, quantum field theory and particle physics, etc. In recent decades, several powerful methods have been proposed to construct exact solutions for nonlinear evolution equations, such as tanh method [1-3], extended tanh method [4,5], multiple exp-function method [6], transformed rational function method [7] and so on.

In the pioneer work, Feng [8] introduced the first integral method for a reliable treatment of the nonlinear PDEs. The useful first integral method is widely used by many such as in [9-13] and by the reference therein.

Raslan [10] proposed the first integral method to solve the Fisher equation. Taghizadeh et al., [11] solved nonlinear Schrödinger equation by using the first integral method. Tascan et al., [12] used the first integral method to obtain the exact solutions of the modified Zakharov–Kuznetsov equation and ZK–MEW equation. Hosseini et al., [13] applied the first integral method to obtain the exact solutions of KdV system and Kaup–Boussinesq system and Wu–Zhang system.

The aim of this paper is to find exact soliton solutions of generalized ZK-BBM equation and Drinefel'd-Sokolov-Wilson system and one-dimensional generalized EW-Burgers equation.by the first integral method.

The paper is arranged as follows. In Section 2, we describe briefly the first integral method. In Sections 3 - 5 , we apply this method to generalized ZK-BBM equation and Drinefel'd-Sokolov-Wilson system and one-dimensional modified EW-Burgers equation.

2. FIRST INTEGRAL METHOD

Raslan summarized for using first integral method [10].

Step 1. Consider a general nonlinear PDE in the form

$$F(u, u_x, u_y, u_t, u_{xx}, u_{xy}, u_{xt}, \dots) = 0, \tag{1}$$

Using a wave variable where $\xi = k(x + ly - \lambda t)$, we can rewrite Eq. (1) in the following nonlinear ODE

$$G(u, u', u'', u''', \dots) = 0, \tag{2}$$

where the prime denotes the derivation with respect to ξ.

Step 2. Suppose that the solution of ODE (2) can be written as follows:

$$u(x, y, t) = u(\xi) = f(\xi). \tag{3}$$

Step 3. We introduce a new independent variable

$$X(\xi) = f(\xi), \quad Y = \frac{\partial f(\xi)}{\partial \xi}, \tag{4}$$

which leads a system of nonlinear ordinary differential equations

$$\begin{aligned} \frac{\partial X(\xi)}{\partial \xi} &= Y(\xi), \\ \frac{\partial Y(\xi)}{\partial \xi} &= F_1(X(\xi), Y(\xi)). \end{aligned} \tag{5}$$

Step 4. By the qualitative theory of ordinary differential equations [14] , if we can find the integrals to Eq. (5) under the same conditions, then the general solutions to Eq. (5) can be solved directly. However, in general, it is really difficult for us to realize this even for one first integral, because for a given plane autonomous system, there is no systematic theory that can tell us how to find its first integrals, nor is there a logical way for telling us what these first integrals are. We will apply the Division Theorem to obtain one first integral to Eq.(5) which reduces Eq.(2) to a first order integrable ordinary differential equation. An exact solution to Eq. (1) is then obtained by solving this equation. Now, let us recall the Division Theorem:

Division Theorem. Suppose that $P(w, z)$ and $Q(w, z)$ are polynomials in $C[w, z]$, and $P(w, z)$ is irreducible in $C[w, z]$. If $Q(w, z)$vanishes at all zero points of $P(w, z)$, then there exists a polynomial $G(w, z)$ in $C[w, z]$ such that

$$Q(w, z) = P(w, z)G(w, z).$$

3. GENERALIZED ZK-BBM EQUATION

Consider the generalized ZK-BBM equation [15]

$$u_t + u_x + a(u^3)_x + b(u_{xt} + u_{yy})_x = 0, \tag{6}$$

where a, b are real constants.

By make the transformation

$$u(x, y, t) = f(\xi), \quad \xi = k(x + ly - \lambda t), \tag{7}$$

the generalized ZK-BBM equation becomes

$$(1 - \lambda) f'(\xi) + 3a f^2(\xi) f'(\xi) + bk^2(l^2 - \lambda) f'''(\xi) = 0. \tag{8}$$

Integrating (6) with respect to ξ,then we have

$$(1-\lambda)f(\xi)+af^3(\xi)+bk^2(l^2-\lambda)f''(\xi)=R, \tag{9}$$

where R is integration constant.

Using (4) and (5), we get

$$\dot{X}(\xi)=Y(\xi), \tag{10}$$

$$\dot{Y}(\xi)=\frac{(\lambda-1)}{bk^2(l^2-\lambda)}X(\xi)-\frac{a}{bk^2(l^2-\lambda)}(X(\xi))^3+\frac{R}{bk^2(l^2-\lambda)}.$$

According to the first integral method, we suppose the X(ξ) and $Y(\xi)$, are the nontrivial solutions of (10) also

$$Q(X,Y)=\sum_{i=0}^{m}a_i(X)Y^i=0,$$

is an irreducible polynomial in the complex domain $C[X,Y]$, such that

$$Q(X(\xi),Y(\xi))=\sum_{i=0}^{m}a_i(X(\xi))Y^i(\xi)=0, \tag{11}$$

where $a_i(X)(i=0,1,...,m)$,are polynomials of $X$and $a_m(X)\neq 0$. Eq. (11) is called the first integral to (10). Due to the Division Theorem, there exists a polynomial$g(X)+h(X)Y$,in the complex domain $C[X,Y]$, such that

$$\frac{dQ}{d\xi}=\frac{dQ}{dX}.\frac{dX}{d\xi}+\frac{dQ}{dY}.\frac{dY}{d\xi}=(g(X)+h(X)Y)\sum_{i=0}^{m}a_i(X)Y^i. \tag{12}$$

In this example, we take two different cases, assuming that $m=1$, and $m=2$, in (11).

Case A: Suppose that $m=1$,by comparing with the coefficients of $Y^i(i=2,1,0)$of both sides of (12), we have

$$\dot{a}_1(X)=h(X)a_1(X), \tag{13}$$

$$\dot{a}_0(X)=g(X)a_1(X)+h(X)a_0(X), \tag{14}$$

$$a_1(X)[\frac{(\lambda-1)}{bk^2(l^2-\lambda)}X-\frac{a}{bk^2(l^2-\lambda)}X^3+\frac{R}{bk^2(l^2-\lambda)}]=g(X)a_0(X). \tag{15}$$

Since $a_i(X)(i=0,1)$ are polynomials, then from (13) we deduce that $a_1(X)$ is constant and $h(X)=0$. For simplicity, take $a_1(X)=1$. Balancing the degrees of $g(X)$ and $a_0(X)$,we conclude that $\deg(g(X))=1$, only. Suppose that $g(X)=A_1X+B_0$,then we find $a_0(X)$.

$$a_0(X)=A_0+B_0X+\frac{1}{2}A_1X^2, \tag{16}$$

where A_0 is arbitrary integration constant. Substituting $a_0(X)$, $a_1(X)$ and $g(X)$in the last equation in (15) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$B_0=0,\quad A_0=-\frac{2a+A_1^2bk^2(l^2-1)}{2A_1bak^2},\quad \lambda=\frac{2a+A_1^2bk^2l^2}{A_1^2bk^2},\quad R=0, \tag{17}$$

where A_1, l and k are arbitrary constants.

Using the conditions (17) in (11), we obtain

$$Y(\xi)=-\frac{A_1}{2}X^2(\xi)+\frac{2a+A_1^2bk^2(l^2-1)}{2A_1bak^2}. \tag{18}$$

Combining (18) with (10), we obtain the exact solution to equation (9) and then the exact solution to generalized ZK-BBM equation can be written as

$$u(x,y,t) = \sqrt{\frac{2}{bk^2A_1^2} + \frac{(l^2-1)}{a}}\tanh[\frac{A_1}{2}\sqrt{\frac{2}{bk^2A_1^2} + \frac{(l^2-1)}{a}}(k(x+ly-(l^2+\frac{2a}{A_1^2bk^2})t)+\xi_0)],$$

where ξ_0 is an arbitrary constant.

Case B: Suppose that $m = 2$, by equating with the coefficients of $Y^i(i = 3,2,1,0)$ of both sides of (12), we have

$$\dot{a}_2(X) = h(X)a_2(X), \tag{19}$$

$$\dot{a}_1(X) = g(X)a_2(X) + h(X)a_1(X), \tag{20}$$

$$\begin{aligned}\dot{a}_0(X) = &-2a_2(X)[\tfrac{(\lambda-1)}{bk^2(l^2-\lambda)}X - \tfrac{a}{bk^2(l^2-\lambda)}X^3 + \tfrac{R}{bk^2(l^2-\lambda)}] \\ &+g(X)a_1(X) + h(X)a_0(X),\end{aligned} \tag{21}$$

$$a_1(X)[\frac{(\lambda-1)}{bk^2(l^2-\lambda)}X - \frac{a}{bk^2(l^2-\lambda)}X^3 + \frac{R}{bk^2(l^2-\lambda)}] = g(X)a_0(X). \tag{22}$$

Since $a_i(X)(i = 0,1)$ are polynomials, then from (19) we deduce that $a_2(X)$, is constant and $h(X) = 0$. For simplicity, take $a_2(X) = 1$. Balancing the degrees $g(X), a_1(X)$ and $a_0(X)$, we conclude that $\deg(g(X)) = 1$, only. Suppose that $g(X) = A_1X + B_0$, then we find $a_0(X)$and $a_1(X)$ as

$$a_1(X) = A_0 + B_0X + \frac{1}{2}A_1X^2, \tag{23}$$

$$\begin{aligned}a_0(X) = d &+ (B_0A_0 - \frac{2R}{bk^2(l^2-\lambda)})X + \frac{1}{2}(B_0^2 + A_0A_1 + \frac{2(1-\lambda)}{bk^2(l^2-\lambda)})X^2 \\ &+ \frac{1}{2}A_1B_0X^3 + \frac{1}{4}(\frac{A_1^2}{2} + \frac{2a}{bk^2(l^2-\lambda)})X^4,\end{aligned} \tag{24}$$

where d is arbitrary integration constant. Substituting $a_0(X), a_1(X)$ and $g(X)$, in the last equation in (22) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it with aid Maple, we obtain

$$\begin{aligned}&B_0 = 0, \quad A_0 = -\frac{8a + A_1^2bk^2(l^2-1)}{2A_1bak^2}, \quad \lambda = \frac{8a + A_1^2bk^2l^2}{A_1^2bk^2}, \\ &R = 0, \quad d = \frac{(8a + A_1^2bk^2(l^2-1))^2}{16k^4a^2b^2A_1^2},\end{aligned} \tag{25}$$

where A_1, l and k are arbitrary constants.

Using the conditions (25) in (12), we obtain

$$Y(\xi) = -\frac{A_1}{4}X^2(\xi) + \frac{8a + A_1^2bk^2(l^2-1)}{2A_1bak^2}. \tag{26}$$

Combining (26) with (10), we obtain the exact solution to equation (9) and then the exact solution to generalized ZK-BBM equation can be written as

$$u(x,y,t) = \sqrt{\frac{8}{bk^2A_1^2} + \frac{(l^2-1)}{a}}\tanh[\frac{A_1}{4}\sqrt{\frac{8}{bk^2A_1^2} + \frac{(l^2-1)}{a}}(k(x+ly-(l^2+\frac{8a}{A_1^2bk^2})t)+\xi_0)],$$

where ξ_0 is an arbitrary constant.

4. DRINEFEL'D-SOKOLOV-WILSON SYSTEM

Consider the Drinefel'd-Sokolov-Wilson system

$$\begin{aligned} &u_t + pvv_x = 0, \\ &v_t + qv_{xxx} + ruv_x + su_x v = 0, \end{aligned} \tag{27}$$

where p, q, r and s are non-zero constants.

When $p = 2, \quad q = -1$ and $r = s = 3$, the Eq. (27) become the nonlinear Drinfeld-Sokolov System

$$\begin{aligned} &u_t + (v^2)_x = 0, \\ &v_t - v_{xxx} + 3uv_x + 3u_x v = 0. \end{aligned}$$

By considering the wave transformations

$$u(x, y) = u(\xi), \quad v(x, t) = v(\xi), \quad \xi = x - ct, \tag{28}$$

we change Eq. (27) into a system of ODEs given by

$$\begin{aligned} &-cu' + pvv' = 0, && (a) \\ &-cv' + qv''' + ruv' + su'v = 0. && (b) \end{aligned} \tag{29}$$

Integrating (29)(a) with respect to ξ, then we have

$$-cu + \frac{p}{2}v^2 = R_1, \tag{30}$$

where R_1 is integration constant. Rewrite this equation as follows

$$u(\xi) = \frac{p}{2c}v^2 - \frac{R_1}{c}. \tag{31}$$

Inserting Eq. (31) into Eq. (29) (b) yields

$$qv''' - (c + \frac{rR_1}{c})v' + (\frac{pr}{2c} + \frac{sp}{c})v^2 v' = 0. \tag{32}$$

Integrating Eq. (32) once leads to

$$qv'' - (c + \frac{rR_1}{c})v + (\frac{pr}{6c} + \frac{sp}{3c})v^3 = R_2, \tag{33}$$

where R_2 is an integration constant. Rewrite this second-order ordinary differential equation as follows

$$v'' - (\frac{c}{q} + \frac{rR_1}{qc})v + (\frac{pr}{6qc} + \frac{sp}{3qc})v^3 - \frac{R_2}{q} = 0. \tag{34}$$

Using (5) and (6), we get

$$\begin{aligned} &\dot{X}(\xi) = Y(\xi), \\ &\dot{Y}(\xi) = (\frac{c}{q} + \frac{rR_1}{qc})X(\xi) - (\frac{pr}{6qc} + \frac{sp}{3qc})(X(\xi))^3 + \frac{R_2}{q}. \end{aligned} \tag{35}$$

Eq. (11) is called the first integral to Eq. (35). Suppose that $m = 1$,by comparing with the coefficients of $Y^i (i = 2, 1, 0)$of both sides of (12), we have

$$\dot{a}_1(X) = h(X)a_1(X), \tag{36}$$

$$\dot{a}_0(X) = g(X)a_1(X) + h(X)a_0(X), \tag{37}$$

$$a_1(X)[(\frac{c}{q} + \frac{rR_1}{qc})X - (\frac{pr}{6qc} + \frac{sp}{3qc})X^3 + \frac{R_2}{q}] = g(X)a_0(X). \tag{38}$$

Since $a_i(X)(i = 0, 1)$ are polynomials, then from (36) we deduce that $a_1(X)$ is constant and $h(X) = 0$. For simplicity, take $a_1(X) = 1$. Balancing the degrees of $g(X)$ and $a_0(X)$,we conclude that $\deg(g(X)) = 1$, only. Suppose that $g(X) = A_1X + B_0$,then we find $a_0(X)$.

$$a_0(X) = A_0 + B_0X + \frac{1}{2}A_1X^2, \tag{39}$$

where A_0 is arbitrary integration constant. Substituting $a_0(X)$, $a_1(X)$ and $g(X)$ in the last equation in (38) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$\begin{aligned} &B_0 = 0, \quad A_1 = \frac{\sqrt{-3pqc(r+2s)}}{3qc}, \quad R_1 = -\frac{3c^2 - A_0\sqrt{-3pqc(r+2s)}}{3r}, \\ &R_2 = 0, \end{aligned} \tag{40}$$

where A_0 and c are arbitrary constants.

$$\begin{aligned} &B_0 = 0, \quad A_1 = -\frac{\sqrt{-3pqc(r+2s)}}{3qc}, \quad R_1 = -\frac{3c^2 + A_0\sqrt{-3pqc(r+2s)}}{3r}, \\ &R_2 = 0, \end{aligned} \tag{41}$$

where A_0 and c are arbitrary constants.

Using the conditions (40) in (11), we obtain

$$Y(\xi) = -\frac{\sqrt{-3pqc(r+2s)}}{6qc}X^2(\xi) - A_0. \tag{42}$$

Combining (42) with (35), we obtain the exact solution to equation (34) and then the exact solution to Drinefel'd-Sokolov-Wilson system can be written as

$$u(x,t) = \frac{3c^2 - A_0\sqrt{-3pqc(r+2s)}}{3rc} - \frac{A_0}{c}\sqrt{-\frac{3pqc}{(r+2s)}}\tan^2(\sqrt{\frac{A_0\sqrt{-3pqc(r+2s)}}{6qc}}(x - ct + \xi_0)),$$

$$v(x,t) = -\sqrt{-\frac{2A_0}{p}\sqrt{-\frac{3pqc}{(r+2s)}}}\tan(\sqrt{\frac{A_0\sqrt{-3pqc(r+2s)}}{6qc}}(x - ct + \xi_0)).$$

Similarly, in the case of (41), from (11), we obtain

$$Y(\xi) = -A_0 + \frac{\sqrt{-3pqc(r+2s)}}{6qc}X^2(\xi), \tag{43}$$

and then the exact solution of the Drinefel'd-Sokolov-Wilson system can be written as

$$u(x,t) = \frac{3c^2 + A_0\sqrt{-3pqc(r+2s)}}{3rc} - \frac{A_0}{c}\sqrt{-\frac{3pqc}{(r+2s)}}\tanh^2(\sqrt{\frac{A_0\sqrt{-3pqc(r+2s)}}{6qc}}(x - ct + \xi_0)),$$

$$v(x,t) = -\sqrt{-\frac{2A_0}{p}\sqrt{-\frac{3pqc}{(r+2s)}}}\tanh(\sqrt{\frac{A_0\sqrt{-3pqc(r+2s)}}{6qc}}(x - ct + \xi_0)).$$

5. ONE-DIMENSIONAL MODIFIED EW-BURGERS EQUATION

Let us consider one-dimensional modified EW-Burgers equation [16]

$$u_t + au^2u_x - \delta u_{xx} - \mu u_{xxt} = 0, \tag{44}$$

where a, δ, μ are real constants.

We use the wave transformation

$$u(x,t) = f(\xi), \quad \xi = x - ct. \tag{45}$$

Substituting (45) into (44), we obtain ordinary differential equation:

$$-cf'(\xi) + af^2(\xi)f'(\xi) - \delta f''(\xi) + \mu cf'''(\xi) = 0. \tag{46}$$

Integrating Eq. (46) with respect to ξ, then we have

$$-cf(\xi) + \frac{a}{3}f^3(\xi) - \delta f'(\xi) + \mu cf''(\xi) = R, \tag{47}$$

where R is integration constant.

Using (4) and (5), we get

$$\dot{X}(\xi) = Y(\xi), \tag{48}$$
$$\dot{Y}(\xi) = \frac{\delta}{\mu c}Y(\xi) - \frac{a}{3\mu c}(X(\xi))^3 + \frac{1}{\mu}X(\xi) + \frac{R}{\mu c}.$$

Eq. (11) is called the first integral to Eq. (48). In this example, we take two different cases, assuming that $m = 1$, and $m = 2$, in (11).

Case A: Suppose that $m = 1$,by comparing with the coefficients of $Y^i(i = 2, 1, 0)$of both sides of (12), we have

$$\dot{a}_1(X) = h(X)a_1(X), \tag{49}$$

$$\dot{a}_0(X) = (-\frac{\delta}{\mu c} + g(X))a_1(X) + h(X)a_0(X), \tag{50}$$

$$a_1(X)(-\frac{a}{3\mu c}X^3 + \frac{1}{\mu}X + \frac{R}{\mu c}) = g(X)a_0(X). \tag{51}$$

Since $a_i(X)(i = 0, 1)$ are polynomials, then from (49) we deduce that $a_1(X)$ is constant and $h(X) = 0$. For simplicity, take $a_1(X) = 1$. Balancing the degrees of $g(X)$ and $a_0(X)$,we conclude that $\deg(g(X)) = 1$, only. Suppose that $g(X) = A_1X + B_0$,then we find $a_0(X)$.

$$a_0(X) = A_0 + (B_0 - \frac{\delta}{\mu c})X + \frac{1}{2}A_1X^2, \tag{52}$$

where A_0 is arbitrary integration constant. Substituting $a_0(X), a_1(X)$ and $g(X)$ in the last equation in (51) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$A_0 = \frac{(\delta^2\mu A_1^4 + 2a^2)}{2a^2\mu A_1}, \quad B_0 = -\frac{\delta A_1^2}{a}, \quad c = -\frac{2}{3}\frac{a}{A_1^2\mu},$$
$$R = \frac{\delta(\delta^2\mu A_1^4 + 2a^2)}{3a^2\mu A_1}, \tag{53}$$

where A_1 and is an arbitrary constant.

Using the conditions (53) in (11), we obtain

$$Y(\xi) = -\frac{(\delta^2\mu A_1^4 + 2a^2)}{2a^2\mu A_1} - \frac{\delta A_1^2}{2a}X(\xi) - \frac{A_1}{2}X^2(\xi). \tag{54}$$

Combining (54) with (48), we obtain the exact solution to equation (47) and then the exact solution to one-dimensional modified EW-Burgers equation can be written as

$$u(x,t) = -\frac{\delta A_1}{2a} - \frac{\sqrt{(3\delta^2 A_1^4\mu^2 + 8a^2\mu)}}{2a\mu A_1}\tan\left(\frac{\sqrt{(3\delta^2 A_1^4\mu^2 + 8a^2\mu)}}{4a\mu}(x + \frac{2a}{3A_1^2\mu}t + \xi_0)\right).$$

Case B: Suppose that $m = 2$, by equating with the coefficients of $Y^i (i = 3, 2, 1, 0)$ of both sides of (12), we have

$$\dot{a}_2(X) = h(X)a_2(X), \tag{55}$$

$$\dot{a}_1(X) = (-\frac{2\delta}{\mu c} + g(X))a_2(X) + h(X)a_1(X), \tag{56}$$

$$\dot{a}_0(X) = -2a_2(X)[-\frac{a}{3\mu c}X^3 + \frac{1}{\mu}X + \frac{R}{\mu c}] + (-\frac{\delta}{\mu c} + g(X))a_1(X) + h(X)a_0(X), \tag{57}$$

$$a_1(X)[-\frac{a}{3\mu c}X^3 + \frac{1}{\mu}X + \frac{R}{\mu c}] = g(X)a_0(X). \tag{58}$$

Since $a_i(X)(i = 0, 1)$ are polynomials, then from (55) we deduce that $a_2(X)$, is constant and $h(X) = 0$. For simplicity, take $a_2(X) = 1$. Balancing the degrees $g(X), a_1(X)$ and $a_0(X)$, we conclude that $\deg(g(X)) = 1$, only. Suppose that $g(X) = A_1X + B_0$, then we find $a_0(X)$ and $a_1(X)$ as

$$a_1(X) = A_0 + B_0X + \frac{1}{2}A_1X^2, \tag{59}$$

$$a_0(X) = d + (B_0A_0 - \frac{\delta A_0}{\mu c} - \frac{2R}{\mu c})X + (\frac{1}{2}(B_0 - \frac{\delta}{\mu c})(B_0 - \frac{2\delta}{\mu c}) + (\frac{1}{2}A_0A_1 - \frac{1}{\mu})X^2 + (\frac{1}{2}A_1B_0 - \frac{5\delta A_1}{6\mu c})X^3 + (\frac{A_1^2}{8} + \frac{a}{6\mu c})X^4, \tag{60}$$

where d is arbitrary integration constant. Substituting $a_0(X), a_1(X)$ and $g(X)$, in the last equation in (58) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it with aid Maple, we obtain

$$\begin{aligned} &B_0 = -\frac{\delta A_1^2}{2a}, \quad A_0 = \frac{(A_1^4\delta^2\mu + 32a^2)}{8a^2\mu A_1}, \quad c = -\frac{8}{3}\frac{a}{\mu A_1^2}, \\ &R = \frac{\delta(A_1^4\delta^2\mu + 32a^2)}{24a^2\mu A_1}, \quad d = \frac{(A_1^4\delta^2\mu + 32a^2)^2}{256a^4\mu^2A_1^2}, \end{aligned} \tag{61}$$

where A_1 and is an arbitrary constant.

Using the conditions (61) in (11), we obtain

$$Y(\xi) = -\frac{A_1^4\delta^2\mu + 32a^2 + 2\delta A_1^3 a\mu X(\xi) + 4a^2\mu A_1^2X^2(\xi)}{16a^2\mu A_1}. \tag{62}$$

Combining (62) with (48), we obtain the exact solution to equation (47) and then the exact solution to one-dimensional modified EW-Burgers equation can be written as

$$u(x,t) = -\frac{\delta A_1}{4a} - \frac{\sqrt{(3\delta^2 A_1^4 \mu^2 + 128a^2\mu)}}{4a\mu A_1} \tan(\frac{\sqrt{(3\delta^2 A_1^4 \mu^2 + 128a^2\mu)}}{16a\mu}(x + \frac{8a}{3A_1^2\mu}t + \xi_0)).$$

6. CONCLUSION

In this paper, the first integral method is applied successfully for solving generalized ZK-BBM equation and Drinefel'd-Sokolov-Wilson system and one-dimensional modified EW-Burgers equation. The results show that this method is efficient in finding the exact solutions of nonlinear differential equations.

REFERENCES

[1] Malfliet, W. (1992). Solitary Wave Solutions of Nonlinear Wave Equations. *Am. J. Phys, 60*(7), 650-654.

[2] Malfliet, W., Hereman, W. (1996). The Tanh Method: I. Exact Solutions of Nonlinear Evolution and Wave Equations. *Phys. Scripta, 54*, 563-568.

[3] Malfliet, W., Hereman, W. (1996). The Tanh Method: II. Perturbation Technique for Conservative Systems. *Phys. Scripta, 54*, 569-575.

[4] Ma, W. X., Fuchssteiner, B. (1996). Explicit and Exact Solutions to a Kolmogorov–Petrovskii–Piskunov Equation. *Internat. J. Non-Linear Mech 31*, 329–338.

[5] Fan, E. (2000). Extended Tanh-function Method and Its Applications to Nonlinear Equations, *Phys. Lett. A., 277*(4-5), 212-218.

[6] Ma, W. X. Huang, T. W., Zhang, Y. (2010). A Multiple Exp-function Method for Nonlinear Differential Equations and Its Application. *Phys. Scr., 82*, 065003.

[7] Ma, W. X., Lee, J.-H. (2009). A Transformed Rational Function Method and Exact Solutions to the (3+1)-dimensional Jimbo-Miwa Equation. *Chaos Solitons Fract., 42*, 1356-1363.

[8] Feng, Z. S. (2002). The First Integral Method to Study the Burgers-Korteweg-de Vries Equation. *J.Phys. A., 35*(2), 343-349.

[9] Feng, Z. S., Wang, X. H. (2002) . The First Integral Method to the Two-dimensional Burgers-KdV Equation. *Phys. Lett. A., 308*, 173-178.

[10] Raslan, K. R. (2008) . The First Integral Method for Solving Some Important Nonlinear Partial Differential Equations. *Nonlinear Dynam, 53*, 281.

[11] Taghizadeh, N., Mirzazadeh, M., Farahrooz, F. (2011) . Exact Solutions of the Nonlinear Schrödinger Equation by the First Integral Method. *J. Math. Anal. Appl, 374*, 549-553.

[12] Tascan, F., Bekir, A. and Koparan, M. (2009). Travelling Wave Solutions of Nonlinear Evolutions by Using the First Integral Method. *Commun. None. Sci. Numer.Simul., 14*, 1810-1815.

[13] Hosseini, K., Ansari, R., Gholamin, P. (2012). Exact Solutions of Some Nonlinear Systems of Partial Differential Equations by Using the First Integral Method. *J. Math. Anal. Appl., 387*, 807–814.

[14] Bourbaki, N. (1972). Commutative Algebra. Addison-Wesley, Paris.

[15] Li, H., Zhang, J. (2009). The Auxiliary Elliptic-like Equation and the Exp-function Method. *Indian Academy of Sciences, 72(6)*, 915-925.

[16] Hamdi, S., Enright, WH., Schiesser, WE. Gottlieb, JJ. (2003). Exact Solutions of the Generalized Equal Width Wave Equation. *ICCSA, 2*, 725-734.