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Pricing European Call Currency Option based on Adaptive Fuzzy Numbers with Possibilistic Mean*

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Abstract: In this paper we use the fuzzy estimators based on confidence intervals in order to estimate the volatility of currency exchange rate having sample data. We model the uncertainty of the characteristics such as interest rates and volatility using adaptive fuzzy numbers and replace the fuzzy currency option price and the fuzzy volatility by the possibilistic mean value. Furthermore, a numerical example is presented, and we get the expected prices which depend on given degree of confidence.

Key words: Currency Option; Option Pricing; Possibilistic Mean Value; Fuzzy Volatility; G-K Model

1. INTRODUCTION

In statistics, as we know, there is the point estimation of a parameter. But this is not enough for us to derive safe conclusions. That is why statistics introduces confidence intervals. The disadvantage of confidence intervals is that we have to choose the probability so that the parameter for estimation should be in this interval. With this methodology and by making use of the tool of fuzzy numbers we define fuzzy estimators for any estimated parameter, using the confidence intervals. The fuzzy number that results is considered to be the statistical estimator that expresses a degree of an unbiased estimation. Since the closed form solution of the European currency options piecing model was derived by Garman and Kohlhagen (1983)based on Black and Scholes (1973),many methodologies for the currency options pricing have been proposed by using modification of Garman-Kohlhagen (G-K)model,such as Amin and Jarrow(1991),Heston(1993),bates(1996),Ekvall et al.(1997),Lim et al.(1998), Rosenberg(1998), Sarwar and Krenhbiel(2000)[1],Bollen and Rasiel(2003)[2]. In this paper we present an application of fuzzy estimators method to price European call currency option. We make use of fuzzy estimators for the volatility of exchange rate which based on statistical data to obtain the fuzzy pattern of G-K model. We use the fuzzy estimators based on confindence intervals introduced by Papadopoulos and Sfiris in [3] inorder to estimate the volatility of exchange rate knowing sample data. In this paper, we use the fuzzy

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estimators based on confidence intervals in order to estimate the volatility of currency exchange rate having sample data. we models the uncertainty of the characteristics such as interest rates and volatility using adaptive fuzzy numbers and replaces the fuzzy currency option price and the fuzzy volatility by the possibilistic mean value. Further more, a numerical example is presented, and we get the expected prices which depend on given degree of confidence.

2. FUZZY SETS PRELIMINARIES

2.1 Fuzzy Set Theory

Now we remind some facts about fuzzy sets and numbers[4]:

Let X be a universal set and $\ddot{A}$ be a fuzzy subset of X ,we denote by $\mu_{\ddot{A}}$ its membership function $\mu_{\ddot{A}} : X \to [0,1]$, and by $\ddot{A}_\alpha = \{x : \mu_{\ddot{A}}(x) \ge \alpha\}$ the α -level set $\ddot{a}_\alpha$ is a closed interval, which can be denoted by $\ddot{a}_\alpha = \left[\ddot{a}_\alpha^L, \ddot{a}_\alpha^U\right]$.

The four arithmetic operations on closed intervals are defined as follows:

$$[a,b]+[d,e]=[a+d,b+e]$$

$$[a,b]-[d,e]=[a-e,b-d]$$

$$[a,b]+[d,e]=[a+d,b+e]$$

if $0 \notin [d,e]$, then $[a,b]\cdot[d,e]=\left[\min\{ad,ae,bd,be\},\max\{ad,ae,bd,be\}\right]$

if $0 \notin [\mathrm{d},\mathrm{e}]$,then $[\mathrm{a},\mathrm{b}]\cdot[\mathrm{d},\mathrm{e}]=[\min\{\mathrm{ad},\mathrm{ae},\mathrm{bd},\mathrm{be}\},\min\{\mathrm{ad},\mathrm{ae},\mathrm{bd},\mathrm{be}\}]$

if $a,b,d,e>0,$ then $[a,b]\cdot[d,e]=[ad,be]$

if $0 \notin [d,e]$, then $[a,b]/[d,e]=[a,b]\cdot\left[\frac{1}{d},\frac{1}{e}\right]$

2.2 Triangular Fuzzy Number

The membership function of a triangular fuzzy number $\ddot{a}$ is defined by:

$$\mu_{\ddot{a}}(x)=\begin{cases}(x-a_L)/(a_c-a_L), & \text{when } a_L \le x \le a_C\\(a_R-x)/(a_R-a_C), & \text{when } a_C < x \le a_R\\0, & \text{otherwise}\end{cases}$$

which is denoted by $\ddot{a}=(a_L;a_C;a_R)$ .The α -level set of $\ddot{a}$ is then:

$$\tilde{a}_\alpha = \left[(1-\alpha)a_L + \alpha a_C, (1-\alpha)a_R + \alpha a_C\right]$$

$$\tilde{a}_\alpha^{\;L} = (1-\alpha)a_L + \alpha a_C,\ \tilde{a}_\alpha^{\;U} = (1-\alpha)a_R + \alpha a_C$$

3. ADAPTIVE FUZZY NUMBERS WITH POSSIBILISTIC MEAN

Carlson and Fuller[5]referred to the next example for the possibilistic mean of a fuzzy number:

Let $S = [c, k, d]$ be a triangular fuzzy number with center $\frac{c+d}{2}$, left-width c>0 and right-width d>0,then α -level of S is computed by:

$$S[\alpha] = \left[c - (1-\alpha)\frac{c+d}{2}, c + (1-\alpha)d\right] \ \forall \alpha \in [0,1]$$

And also that:

$$E(S) = \int_0^1 \alpha\left[c - (1-\alpha)\frac{c+d}{2} + c + (1-\alpha)d\right]d\alpha = c + \frac{d - \frac{c+d}{2}}{6}$$

Following the proposition proved by Papadopoulos and Sfiris in [3], we consider the fuzzy volatility as:

$$\sigma^2(x) = \begin{cases} \frac{2-\beta}{1-\beta} - \frac{2}{1-\beta}\Phi\left(\sqrt{\frac{n-1}{2}}\left(\frac{s^2}{x} - 1\right)\right) & \text{if } \frac{s^2}{1+\Phi^{-1}\left(1-\frac{\beta}{2}\right)\sqrt{\frac{2}{n-1}}} \le x \le s^2 \\ \frac{2-\beta}{1-\beta} - \frac{2}{1-\beta}\Phi\left(\sqrt{\frac{n-1}{2}}\left(1 - \frac{s^2}{x}\right)\right) & \text{if } s^2 \le x \le \frac{s^2}{1-\Phi^{-1}\left(1-\frac{\beta}{2}\right)\sqrt{\frac{2}{n-1}}} \end{cases}$$

its α -cut is the following:

$$\sigma^2[\alpha] = \left[\frac{s^2}{1+u\sqrt{\frac{2}{n-1}}}, \frac{s^2}{1-u\sqrt{\frac{2}{n-1}}}\right]$$

where $u = \Phi^{-1}\left(1 - \left(\frac{1}{2} - \frac{\beta}{2}\right)\alpha - \frac{\beta}{2}\right)$

In the same way as above we can calculate the possibilistic mean values $E\sigma^2$ and $E\sigma$ are given as follows:

$$E\left(\sigma^2\right)=\int_0^1 \alpha\left[\frac{s^2}{1+u\sqrt{\frac{2}{n-1}}}+\frac{s^2}{1-u\sqrt{\frac{2}{n-1}}}\right]d\alpha$$

$$E\left(\sigma\right)=\int_0^1 \alpha\left[\sqrt{\frac{s^2}{1+u\sqrt{\frac{2}{n-1}}}}+\sqrt{\frac{s^2}{1-u\sqrt{\frac{2}{n-1}}}}\right]d\alpha$$

Also, let $r_1=\left[r^L{}_1,\frac{r^L{}_1+r^U{}_1}{2},r^U{}_1\right]$, $r_1=\left[r^L{}_2,\frac{r^L{}_2+r^U{}_2}{2},r^U{}_2\right]$, We get Er_1, Er_2 as:

$$E\left(r_1\right)=\int_0^1 \alpha\left[r^L{}_1-(1-\alpha)\frac{r^L{}_1+r^U{}_1}{2}+r^L{}_1+(1-\alpha)r^U{}_1\right]d\alpha$$

$$E\left(r_2\right)=\int_0^1 \alpha\left[r^L{}_2-(1-\alpha)\frac{r^L{}_2+r^U{}_2}{2}+r^L{}_2+(1-\alpha)r^U{}_2\right]d\alpha$$

According to G-K model[4] ,for d_1 we have that:

$$d_1=[\ln(E\left(s_t\right)/K)+(E\left(r_2\right)-E\left(r_1\right)+E\left(\sigma^2\right)/2)\tau]/\left(E\left(\sigma\right)\sqrt{\tau}\right)\ ,d_2=d_1-E\left(\sigma\right)\sqrt{\tau}$$

$$c_t=s_t e^{-r_1\tau}N\left(d_1\right)-Ke^{-r_2\tau}N\left(d_2\right),\tau=T-t$$

The formulas for the call options is:

$$\left[c_{t1},c_{t2}\right]=\left[s_{t1}e^{-r^U{}_1\tau}N\left(d_1\right)-Ke^{-r^L{}_2\tau}N\left(d_2\right),s_{t2}e^{-r^L{}_1\tau}N\left(d_1\right)-Ke^{-r_2{}^U\tau}N\left(d_2\right)\right],\tau=T-t$$

3.2 The Step to Find the Price of European Call Currency Option

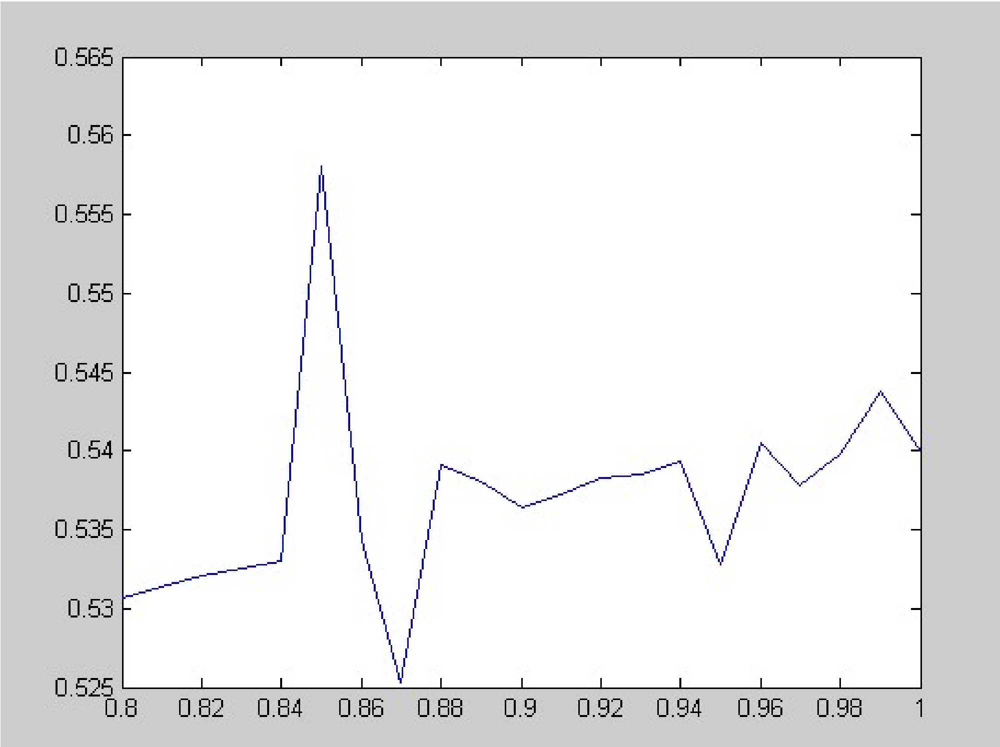
We have the following five steps in order to find the price of European call currency option:

1. We compute the α -level of the adaptive fuzzy number $S, S[\alpha]$.
2. We compute the possibilitic mean values $E\left(S\right),E\left(\sigma\right),E\left(\sigma^2\right),E\left(r_1\right),E\left(r_2\right)$.
3. We compute d_1 and d_2.
4. We compute $\Phi\left(d_1\right),\Phi\left(d_2\right)$.
5. We use the G-K model for the option we want to price.

4. EMPIRICAL EXAMPLE

In this section, our model is tested with the daily market price data of EUR/CNY. These market data come from [6] and cover the period 9-27-2010 to 11-15-2010 as table 1. we calculate $s^2 = 0.013287$, the spot exchange rate is around 9.2418 , and assume the strike price is K=9.12 with one year to expiry .Let $\beta = 0.01$.The triangular fuzzy numbers are $\tilde{r}_1 = (1.09\%, 1.1\%, 1.11\%)$;

$\tilde{r}_2 = (2.4\%, 2.5\%, 2.6\%)$; $\tilde{s}_t = (9.2398, 9.2418, 9.2438)$ .we average the interval [0.8, 1] into 20 equal parts, that is $\alpha = 0.8, 0.81, 0.82 \cdots 1$, Based on every α ,we calculate the expected value of fuzzy currency option price as the following Fig:



For $\alpha = 0.95$,it means that the call currency option price will lie in the closed interval [0.53,0.535] with belief degree 0.95.From another point of view, if a financial analyst is comfortable with this belief degree 0.95, then he (she) can pick any value from the closed interval [0.53,0.535] as the option price for his(her)later use.

5. CONCLUSION

Owing to the fluctuation of financial market from time to time, the data sometimes cannot be expected in a precise sense. Therefore, the fuzzy sets theory provides a useful tool for conquering this kind of impreciseness. The fuzzy pattern of G-K model are proposed in this paper. We present a fuzzy pattern of currency option pricing. The fuzzy volatility is obtained from exchange rate following the proposition proved by Papadopoulos and Sfiris. Using the possibilistic mean method, we get the expected option price with the given confidence degree in the empirical example.

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[6] http://www.123cha.com/hl/?q=100&from=EUR&to=CNY&s=EURCNY#symbol=EURCNY=X; range=3m.