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New Solution for Steady Flow over a Rotating Disk in Porous Medium with Heat Transfer

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Abstract: The purpose of this study is to implement a new analytical method which is a combination of the variational iteration method (VIM) and the Padé approximant for solving steady flow over a rotating disk in porous medium with heat transfer. The velocity and temperature profiles are shown also the influence of the porosity parameter on them is discussed in detail. The solution is compared with the numerical solution. Comparisons between the VIM-Padé and the numerical solution reveal that the new technique is a promising tool for solving the above problem. It is predicted that the VIM-Padé can have wide application in engineering problems (especially for boundary-layer and natural convection problems).

Key Words: Variational Iteration Method; Padé Approximant; Rotating Disk; Heat Transfer

1. INTRODUCTION

Large varieties of physical, chemical, and biological phenomena are governed by nonlinear evolution equations. The importance of obtaining the exact solutions, if available, of those nonlinear equations facilitates the verification of numerical solvers and aids in the stability analysis of solutions. Analytical solutions for nonlinear partial differential equations play an important role in nonlinear science, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. Many authors mainly had paid attention to study solutions of nonlinear equations by using various methods, such as the $\delta-$expansion method[1], the Adomian's decomposition method[2] and the homotopy perturbation method[3].

The VIM was first proposed by He[4,5] and systematically illustrated in 1999[6] and used to give approximate solutions of the problem of seepage flow in porous media with fractional derivatives. The VIM was successfully applied to autonomous ordinary differential equation[7], to nonlinear partial differential equations with variable coefficients[8], to Schrodinger-KdV, generalized KdV and shallow water equations[9], to Burger's and coupled Burger's equations[10], to linear Helmholtz partial differential equation[11] and other fields[12,13]. The VIM is useful to obtain exact and approximate solutions of linear and nonlinear differential equations. In this method, general Lagrange multipliers are introduced to construct correction functional for the problems. The multipliers can be identified optimally via the variational theory. There is no need of linearization or discretization, and large computational work and round-off errors is avoided. It has been used to solve effectively, easily and accurately a large class of nonlinear problems with

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approximation[14,15].

Ismail and Rabboh[16] presented a restrictive Padé approximation for the generalized Fisher and Burger-Fisher equations. The Padé approximants, that often show superior performance over series approximations, provide a successful tool and promising scheme for identical applications. Rashidi and Shahmohamadi[17] presented a novel technique for solution of three-dimensional Navier-Stokes equations for the flow near an infinite rotating disk by combination of the VIM and the Padé approximant.

The fluid flow due to an infinite rotating disk was first considered by von Karman in 1921[18]. He introduced the similarity transformations which reduced the governing partial differential equations to ordinary differential equations. Cochran[19] obtained asymptotic solutions for the steady problem formulated by von Karman and Benton[20] solved the unsteady state of this problem. Millsaps and Pohlhausen[21] considered heat transfer from a rotating disk maintained at a constant temperature for different values of Prandtl numbers in the steady state. Attia[22-24] studied the influence of an external uniform magnetic field on the flow due to a rotating disk. The effect of uniform suction or injection through a rotating porous disk on the steady hydrodynamic or hydromagnetic flow induced by the disk was investigated[25-27].

The main goal of the present study is to find the totally analytical solutions for steady flow over a rotating disk in porous medium with heat transfer using the VIM-Padé. This problem studied first by Attia[28] in 2009 and exerted the similarity solution.

2. FLOW ANALYSIS AND MATHEMATICAL FORMULATION

Set the disk in the plane $z=0$ and the space $z>0$ is equipped by a viscous incompressible fluid. An insulated infinite disk rotates about an axis perpendicular to its plane with constant angular speed ω through a porous medium where the Darcy model is assumed[29]. Otherwise the rest fluid is under pressure P_∞. The equations of steady motion are given by

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)+\frac{\partial p}{\partial r}=\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\mu}{K}u, \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}-\frac{uv}{r}\right)=\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\mu}{K}v, \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)+\frac{\partial p}{\partial z}=\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\mu}{K}w, \tag{4}$$

where u,v and w are velocity components in the directions of r,φ and z respectively. p is the pressure, μ is the coefficient of viscosity, ρ is the density of the fluid and K is the Darcy permeability. Attia[28] introduced von Karman transformations[18]

$$u=r\omega F,\quad v=r\omega G,\quad w=\sqrt{\omega\upsilon}H,\quad z=\sqrt{\upsilon/\omega}\eta,\quad p-p_\infty=-p\upsilon\omega P, \tag{5}$$

where η is a non-dimensional distance measured along the axis of rotation, F,G,H and P are non-dimensional functions of η and υ is the kinematic viscosity of the fluid, $\upsilon=\mu/\rho$ [28]. Upon substitution of Eq. (5) into the Navier-Stokes equations, a set of similarity non-linear ordinary differential equations are obtained

$$\frac{dH}{d\eta}+2F=0, \tag{6}$$

$$\frac{d^2F}{d\eta^2} - H\frac{dF}{d\eta} - F^2 + G^2 - M\,F = 0, \tag{7}$$

$$\frac{d^2G}{d\eta^2} - H\frac{dG}{d\eta} - 2F\,G - M\,G = 0, \tag{8}$$

$$\frac{d^2H}{d\eta^2} - H\frac{dH}{d\eta} + \frac{dP}{d\eta} - M\,H = 0, \tag{9}$$

$M = \upsilon / K\omega$ is the porosity parameter. The boundary conditions are as follow

$$F(0) = 0, \quad G(0) = 1, \quad H(0) = 0, \tag{10a}$$

$$F(\infty) = 0, \quad G(\infty) = 0, \quad P(\infty) = 0, \tag{10b}$$

Eq. (10a) indicates the no-slip condition of viscous flow on the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in Eq. (10b)[28]. Eq. (9) can be used to compute the pressure distribution.

The energy equation without the dissipation terms takes the form[21]

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) - k\left(\frac{d^2T}{\partial z^2} + \frac{d^2T}{\partial r^2} + \frac{1}{r}\frac{dT}{\partial r}\right) = 0, \tag{11}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The non-dimensional variable θ introduced as follow

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{12}$$

where T_w and T_∞ are the temperature of the surface of the disk and the temperature at large distances from the disk, respectively. The energy equation is reduced by using Eqs. (5) and (12)

$$\frac{1}{\Pr}\frac{d^2\theta}{d\eta^2} - H\frac{d\theta}{d\eta} = 0, \tag{13}$$

where $\Pr$ is the Prandtl number, $\Pr = c_p\mu_k / k$. The boundary conditions are

$$\theta(0) = 1, \tag{14a}$$

$$\theta(\infty) = 0. \tag{14b}$$

3. BASIC CONCEPTS OF THE VIM

To illustrate the basic concepts of the VIM, we consider the following differential equation:

$$Lu + Nu = g(t), \tag{15}$$

where L, N and $g(t)$ are the linear operator, the nonlinear operator and a heterogeneous term, respectively. The VIM was proposed by He where a correction functional for Eq. (15) can be written as

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda[Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)]\,\mathrm{d}\tau, \qquad n \ge 0. \tag{16}$$

It is obvious that the successive approximations u_j, $j \ge 0$ can be established by determining λ, a general Lagrangian multiplier, which can be identified optimally via the variational theory. The function $\tilde{u}_n$ is a restricted variation which means $\delta \tilde{u}_n = 0$. Therefore, we first determine the Lagrange multiplier λ that will be identified optimally via integration by parts. The successive approximations $u_{n+1}(t),\ n \ge 0$ of the solution $u(t)$ will be readily obtained upon using the obtained Lagrange multiplier and by using any selective function u_0. When λ determined, then several approximations $u_j(t),\ j \ge 0,$ follow immediately. Consequently, the exact solution may be obtained by using

$$u = \lim_{n\to\infty} u_n. \tag{17}$$

4. THE VIM SOLUTION

To solve the system of Eqs. (6)–(9) and (13) by means of the VIM, we construct the following correction functionals:

$$H_{n+1}(\eta) = H_n(\eta) + \int_0^\eta \lambda_1 \left(\frac{\partial H_n(\tau)}{\partial \tau} + 2\tilde{F}_n(\tau) \right) \mathrm{d}\tau, \tag{18}$$

$$F_{n+1}(\eta) = F_n(\eta) + \int_0^\eta \lambda_2 \left(\frac{\partial^2 F_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{F}_n(\tau)}{\partial \tau} - (\tilde{F}_n(\tau))^2 + (\tilde{G}_n(\tau))^2 - M\,\tilde{F}(\tau) \right) \mathrm{d}\tau, \tag{19}$$

$$G_{n+1}(\eta) = G_n(\eta) + \int_0^\eta \lambda_3 \left(\frac{\partial^2 G_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{G}_n(\tau)}{\partial \tau} - \tilde{F}_n(\tau)\tilde{G}_n(\tau) - M\,\tilde{G}(\tau) \right) \mathrm{d}\tau, \tag{20}$$

$$P_{n+1}(\eta) = P_n(\eta) + \int_0^\eta \lambda_4 \left(\frac{\partial P_n(\tau)}{\partial \tau} + \frac{\partial^2 \tilde{H}_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{H}_n(\tau)}{\partial \tau} - M\,\tilde{H}_n(\tau) \right) \mathrm{d}\tau, \tag{21}$$

$$\theta_{n+1}(\eta) = \theta_n(\eta) + \int_0^\eta \lambda_5 \left(\frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{\theta}_n(\tau)}{\partial \tau} \right) \mathrm{d}\tau, \tag{22}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 are the general Lagrangian multipliers and $\tilde{H}_n(\tau),\ \tilde{F}_n(\tau),\ \tilde{G}_n(\tau)$ and $\tilde{\theta}_n(\tau)$ are considered as restricted variations. To find the optimal values of $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5, we have

$$\delta H_{n+1}(\eta) = \delta H_n(\eta) + \delta \int_0^\eta \lambda_1 \left(\frac{\partial H_n(\tau)}{\partial \tau} + 2\tilde{F}_n(\tau) \right) \mathrm{d}\tau, \tag{23}$$

$$\delta F_{n+1}(\eta) = \delta F_n(\eta) + \delta \int_0^\eta \lambda_2 \left(\frac{\partial^2 F_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{F}_n(\tau)}{\partial \tau} - (\tilde{F}_n(\tau))^2 + (\tilde{G}_n(\tau))^2 - M\,\tilde{F}(\tau) \right) \mathrm{d}\tau, \tag{24}$$

$$\delta G_{n+1}(\eta) = \delta G_n(\eta) + \delta \int_0^\eta \lambda_3 \left(\frac{\partial^2 G_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{G}_n(\tau)}{\partial \tau} - \tilde{F}_n(\tau)\tilde{G}_n(\tau) - M\,\tilde{G}(\tau) \right) \mathrm{d}\tau, \tag{25}$$

$$\delta P_{n+1}(\eta) = \delta P_n(\eta) + \delta \int_0^\eta \lambda_4 \left(\frac{\partial P_n(\tau)}{\partial \tau} + \frac{\partial^2 \tilde{H}_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{H}_n(\tau)}{\partial \tau} - M\,\tilde{H}_n(\tau) \right) \mathrm{d}\tau, \tag{26}$$

$$\delta \theta_{n+1}(\eta) = \delta \theta_n(\eta) + \delta \int_0^\eta \lambda_5 \left(\frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta_n(\tau)}{\partial \tau^2} - \tilde{H}_n(\tau) \frac{\partial \tilde{\theta}_n(\tau)}{\partial \tau} \right) \mathrm{d}\tau, \tag{27}$$

or

$$\delta H_{n+1}(\eta)=\delta H_n(\eta)+\delta\int_0^\eta \lambda_1\left(\frac{\partial H_n(\tau)}{\partial\tau}\right)\mathrm{d}\tau, \tag{28}$$

$$\delta F_{n+1}(\eta)=\delta F_n(\eta)+\delta\int_0^\eta \lambda_2\left(\frac{\partial^2 F_n(\tau)}{\partial\tau^2}\right)\mathrm{d}\tau, \tag{29}$$

$$\delta G_{n+1}(\eta)=\delta G_n(\eta)+\delta\int_0^\eta \lambda_3\left(\frac{\partial^2 G_n(\tau)}{\partial\tau^2}\right)\mathrm{d}\tau, \tag{30}$$

$$\delta P_{n+1}(\eta)=\delta P_n(\eta)+\delta\int_0^\eta \lambda_4\left(\frac{\partial P_n(\tau)}{\partial\tau}\right)\mathrm{d}\tau, \tag{31}$$

$$\delta \theta_{n+1}(\eta)=\delta \theta_n(\eta)+\delta\int_0^\eta \lambda_5\left(\frac{1}{\Pr}\frac{\partial^2 \theta_n(\tau)}{\partial\tau^2}\right)\mathrm{d}\tau. \tag{32}$$

Its stationary conditions can be obtained as follows

$$1+\lambda_1(\tau)\big|_{\tau=\eta}=0,\quad \lambda_1'(\tau)=0, \tag{33}$$

$$1-\lambda_2'(\tau)\big|_{\tau=\eta}=0,\quad \lambda_2(\tau)\big|_{\tau=\eta}=0,\quad \lambda_2''(\tau)=0, \tag{34}$$

$$1-\lambda_3'(\tau)\big|_{\tau=\eta}=0,\quad \lambda_3(\tau)\big|_{\tau=\eta}=0,\quad \lambda_3''(\tau)=0, \tag{35}$$

$$1+\lambda_4(\tau)\big|_{\tau=\eta}=0,\quad \lambda_4'(\tau)=0, \tag{36}$$

$$1-\lambda_5'(\tau)\big|_{\tau=\eta}=0,\quad \lambda_5(\tau)\big|_{\tau=\eta}=0,\quad \lambda_5''(\tau)=0. \tag{37}$$

The Lagrange multipliers, can be identified as follows

$$\lambda_1=\lambda_4=-1,\quad \lambda_2=\lambda_3=\lambda_5=(\tau-\eta), \tag{38}$$

and the following variational iteration formulas can be obtained

$$H_{n+1}(\eta)=H_n(\eta)-\int_0^\eta\left(\frac{\partial H_n(\tau)}{\partial\tau}+2F_n(\tau)\right)\mathrm{d}\tau, \tag{39}$$

$$F_{n+1}(\eta)=F_n(\eta)+\int_0^\eta(\tau-\eta)\left(\frac{\partial^2 F_n(\tau)}{\partial\tau^2}-H_n(\tau)\frac{\partial F_n(\tau)}{\partial\tau}-(F_n(\tau))^2+(G_n(\tau))^2-M\,F(\tau)\right)\mathrm{d}\tau, \tag{40}$$

$$G_{n+1}(\eta)=G_n(\eta)+\int_0^\eta(\tau-\eta)\left(\frac{\partial^2 G_n(\tau)}{\partial\tau^2}-H_n(\tau)\frac{\partial G_n(\tau)}{\partial\tau}-F_n(\tau)G_n(\tau)-M\,G(\tau)\right)\mathrm{d}\tau, \tag{41}$$

$$P_{n+1}(\eta)=P_n(\eta)-\int_0^\eta\left(\frac{\partial P_n(\tau)}{\partial\tau}+\frac{\partial^2 H_n(\tau)}{\partial\tau^2}-H_n(\tau)\frac{\partial H_n(\tau)}{\partial\tau}-M\,H_n(\tau)\right)\mathrm{d}\tau, \tag{42}$$

$$\theta_{n+1}(\eta)=\theta_n(\eta)+\int_0^\eta(\tau-\eta)\left(\frac{1}{\Pr}\frac{\partial^2 \theta_n(\tau)}{\partial\tau^2}-H_n(\tau)\frac{\partial \theta_n(\tau)}{\partial\tau}\right)\mathrm{d}\tau. \tag{43}$$

Now we must start with arbitrary initial approximations. Therefore according to (10a) and (14a) it is straight-forward to choose initial guesses

$$H_0(\eta)=0,\quad F_0(\eta)=\eta,\quad G_0(\eta)=1-\eta,\quad P_0(\eta)=0,\quad \theta_0(\eta)=1-\eta. \tag{44}$$

By the above iteration formulas (39)–(43) and the initial guesses (44), we can obtain the following results:

$$H_1(\eta)=-\eta^2, \tag{45}$$

$$F_1(\eta)=\eta-\frac{\eta^2}{2}+\frac{\eta^3}{3}+\frac{M\eta^3}{6}, \tag{46}$$

$$G_1(\eta)=1-\eta-\frac{1}{6}\eta^2\left(\eta^2+(M-2)\eta-3M\right), \tag{47}$$

$$P_1=0, \tag{48}$$

$$\theta_1(\eta)=1-\eta, \tag{49}$$

After the 10th iteration, we apply the padé approximant to the obtained results (see [17] and [30]).

For $M=0$ and $\Pr=0.7$, the VIM-Padé solutions are as follows:

$$\mathrm{H}(\eta)_{[4,5]}=(-0.5124\,\eta^2-0.000709725\,\eta^3-0.0100011\,\eta^4)\,/\,(1+0.644746\,\eta +0.237953\,\eta^2+0.0505405\,\eta^3+0.00681915\,\eta^4+0.000280046\,\eta^5), \tag{50}$$

$$\mathrm{F}(\eta)_{[4,8]}=(0.5124\,\eta-0.274009\,\eta^2+0.276441\,\eta^3-0.0341633\,\eta^4)\,/\,(1+0.436192\,\eta+0.56219\,\eta^2+0.365289\,\eta^3+0.174719\,\eta^4 +0.0540587\,\eta^5+0.0129953\,\eta^6+0.00201287\,\eta^7+0.000288859\,\eta^8), \tag{51}$$

$$\mathrm{G}(\eta)_{[1,5]}=(1+0.564031\,\eta)\,/\,(1+1.17383\,\eta+0.715802\,\eta^2 +0.263796\,\eta^3+0.0678037\,\eta^4+0.00579835\,\eta^5), \tag{52}$$

$$\mathrm{P}(\eta)_{[1,3]}=(1.0362\,\eta)\,/\,(1+0.965064\,\eta+0.539018\eta^2+0.071849\,\eta^3), \tag{53}$$

$$\theta(\eta)_{[1,5]}=(1+0.0198273\,\eta)\,/\,(1+0.342827\,\eta+0.110733\,\eta^2 +0.0357668\,\eta^3+0.00208179\,\eta^4+0.00149051\,\eta^5). \tag{54}$$

For $M=0.5$ and $\Pr=0.7$, the VIM-Padé solutions are as follows:

$$\mathrm{H}(\eta)_{[4,4]}=(-0.385\,\eta^2+0.0129432\,\eta^3-0.0051507\,\eta^4)\,/\,(1+0.832182\,\eta +0.325549\,\eta^2+0.0691014\,\eta^3+0.00692846\,\eta^4), \tag{55}$$

$$\mathrm{F}(\eta)_{[1,7]}=(0.385\,\eta)\,/\,(1+1.2987\,\eta+0.869958\,\eta^2+0.386831\,\eta^3+0.127323\,\eta^4 +0.0334909\,\eta^5+0.00770411\,\eta^6+0.00176691\,\eta^7), \tag{56}$$

$$\mathrm{G}(\eta)_{[1,7]}=(1+0.619893\,\eta)\,/\,(1+1.46689\,\eta+0.992459\,\eta^2+0.416139\,\eta^3 +0.119733\,\eta^4+0.0257693\,\eta^5+0.00419788\,\eta^6+0.000499608\,\eta^7), \tag{57}$$

$$\mathrm{P}(\eta)_{[1,3]}=(0.862+0.878514\,\eta)\,/\,(1+1.205712\eta+0.795562\eta^2+0.316266\eta^3), \tag{58}$$

$$\theta(\eta)_{[1,5]}=(1+0.191469\,\eta)\,/\,(1+0.414469\,\eta+0.0924265\,\eta^2 +0.0206111\,\eta^3-0.000242975\,\eta^4+0.000746562\,\eta^5). \tag{59}$$

For $M=1$ and $\Pr=0.7$, the VIM-Padé solutions are as follows:

$$\mathrm{H}(\eta)_{[4,5]} = (-0.31\ \eta^2 + 0.0159129\ \eta^3 - 0.0119675\ \eta^4) \,/\, (1 + 1.02394\ \eta + 0.482085\ \eta^2 + 0.129043\ \eta^3 + 0.0195857\ \eta^4 + 0.00141672\ \eta^5), \tag{60}$$

$$\mathrm{F}(\eta)_{[3,7]} = (0.31\ \eta - 0.631111\ \eta^2 + 0.0893737\ \eta^3) \,/\, (1 - 0.42294\ \eta - 1.70891\ \eta^2 - 1.49028\ \eta^3 - 0.735346\ \eta^4 - 0.239268\ \eta^5 - 0.0532043\ \eta^6 - 0.00739484\ \eta^7), \tag{61}$$

$$\mathrm{G}(\eta)_{[2,6]} = (1 + 0.40573\ \eta - 0.0246593\ \eta^2) \,/\, (1 + 1.47373\ \eta + 1.04928\ \eta^2 + 0.458437\ \eta^3 + 0.144264\ \eta^4 + 0.0326342\ \eta^5 + 0.0061114\ \eta^6), \tag{62}$$

$$\mathrm{P}(\eta)_{[1,3]} = (15.21 + 0.825729\eta) \,/\, (1 + 1.3822\eta + 0.968333\eta^2 + 0.473635\eta^3) \tag{63}$$

$$\theta(\eta)_{[2,6]} = (1 + 0.4318\ \eta + 0.00722049\ \eta^2) \,/\, (1 + 0.6018\ \eta + 0.109527\ \eta^2 + 0.0186195\ \eta^3 + 0.000194859\ \eta^4 + 0.000384961\ \eta^5 + 0.000155439\ \eta^6). \tag{64}$$

5. RESULTS AND DISCUSSION

Graphical representation of results is very useful to demonstrate the efficiency and accuracy of the VIM-Padé for above problem. This section describes the influence of some interesting parameters on the velocity and temperature fields. In particular, attention has been focused to the variations of the porosity parameter M on the velocity, pressure and temperature fields, respectively. Figures 1-3 show the normalized velocity profiles $H(\eta)$, $F(\eta)$ and $G(\eta)$ obtained by the VIM-Padé in comparison with the numerical. From these figures, one can see a very good agreement between the purely analytical results of the VIM–Padé and numerical results. Figure 4 indicates the influence of the porosity parameter M on the temperature and the thermal boundary-layer thickness. Figure 5 shows the normalized pressure profile obtained by the VIM-Padé for different value of the porosity parameter M . Derivatives of functions at $\eta = 0$ for different value of the porosity parameter M are listed in Table 1.

Table 1: Derivatives of functions at $\eta = 0$

VIM-Padé solutions	M		
	0	0.5	1
$F'(0)$	0.51240	0.38544	0.30920
$G'(0)$	–0.60981	–0.84715	–1.06851
$\theta'(0)$	–0.32298	–0.22310	–0.16944

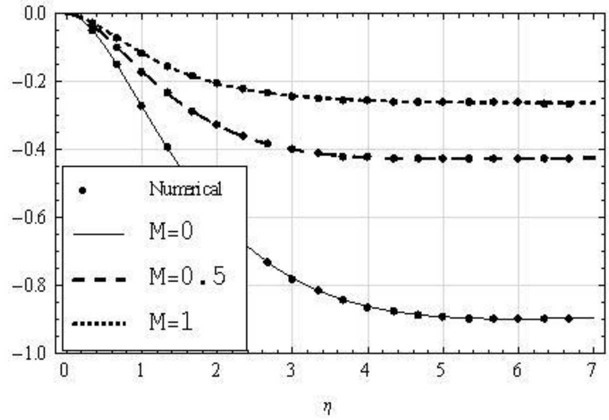

Figure 1: The profile of $H(\eta)$ obtained by the 10th-order approximation of the VIM-Padé in comparison with the numerical solution for different value of the porosity parameter M

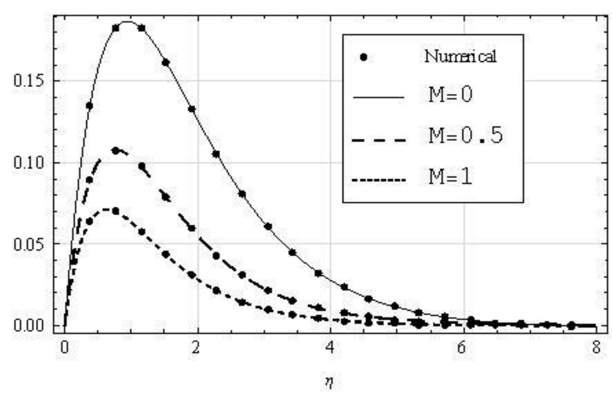

Figure 2: The profile of $F(\eta)$ obtained by the 10th-order approximation of the VIM-Padé in comparison with the numerical solution for different value of the porosity parameter M

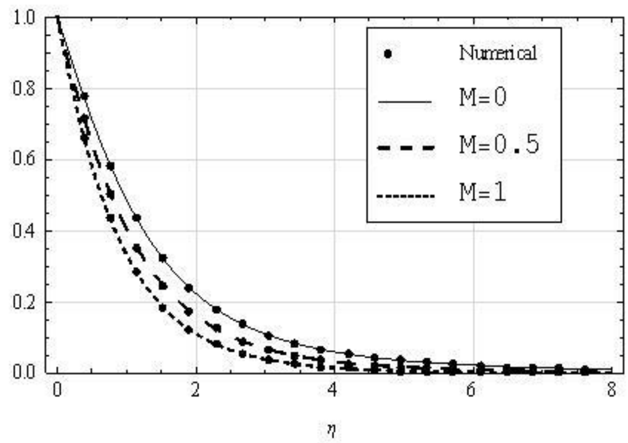

Figure 3: The profile of $G(\eta)$ obtained by the 10th-order approximation of the VIM-Padé in comparison with the numerical solution for different value of the porosity parameter M

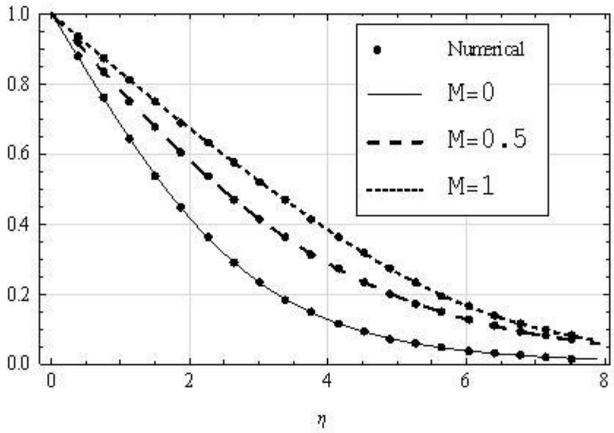

Figure 4: The profile of $\theta(\eta)$ obtained by the 10th-order approximation of the VIM-Padé in comparison with the numerical solution for different value of the porosity parameter M

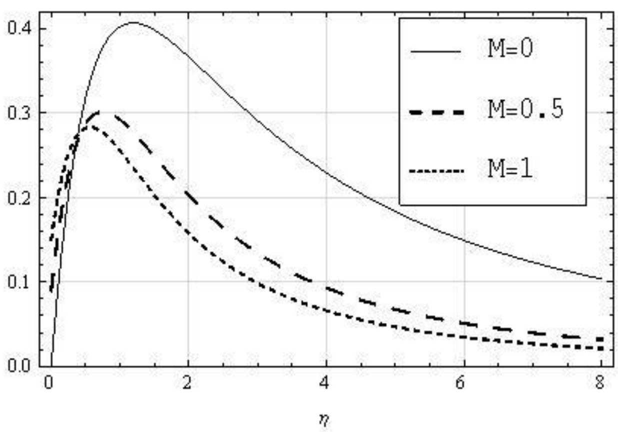

Figure 5: The profile of $P(\eta)$ obtained by the 10th-order approximation of the VIM-Padé for different value of the porosity parameter M

6. CONCLUSIONS

In this letter, combination of the variational iteration method and the Padé approximant was used for finding the totally analytic solutions of the system of nonlinear ordinary differential equations derived from similarity transform for the steady flow over a rotating disk in porous medium with heat transfer. The VIM-Padé was used in a direct way without using linearization, perturbation or restrictive assumptions. The validity of our solutions is verified by the numerical results. Consequently, the present success of the VIM-Padé for the highly nonlinear problem of steady flow over a rotating disk in porous medium with heat transfer verifies that the method is a useful tool for nonlinear problems in science and engineering.

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