

The Application of Probability Method on Mathematical Analysis

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Abstract

This paper summarizes the applications of probability method on mathematical analysis, including a class of generalized integral and acquiring the sum of infinite series so that contact up the mathematical analysis and probability and statistics knowledge, explain the inner contacts among different branches of mathematics in some discussions.

Key words

Probability method; Mathematical analysis; Generalized integral; The sum of infinite series

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1. INTRODUCTION

The normal distribution and the exponential distribution are the most commonly used distributions of probability and statistic, whose expectation, variance and moments function are expressed in form of infinite interval generalized integral. How to solve some mathematical analysis problems using them? This paper will give some examples to explain, including a class of generalized integral and acquiring the sum of infinite series and the limit of multi-integral. This paper is organized as follows: some related properties of normal distribution and exponential distribution will be given in section 2. we will explain the application of the two distribution's moment to solve some generalized integral, and give six important integral formula and its applications in examples. In part 3, acquiring the sum of infinite series using probability method, some examples will also be showed in this section.

2. THE PROPERTIES OF NORMAL AND EXPONENTIAL DISTRIBUTION

2.1 Definitions

Definition 1[1]. It is said that a random variable X has a normal distribution with mean μ and variance σ^2 if

X has a continuous distribution for which the $p.d.f.f(x)$ is as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \text{ for } -\infty < x < \infty$$

Definition 2[1]It is said that a random X has an exponential distribution with parameter $\lambda (\lambda > 0)$ if X has a continuous distribution for which the $p.d.f.f(x)$ is as follows:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

2.2 The Properties

(1) The k -order moment of normal distribution is:

$$E(X) = \int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \mu$$

$$\begin{aligned} E(X^2) &= D(X) + (E(X))^2 \\ &= \int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx \\ &= \sigma^2 + \mu^2 \end{aligned}$$

We can get some important integral formulas are:

$$\int_{-\infty}^{+\infty} x \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \sqrt{2\pi}\mu\sigma \quad (1)$$

$$\int_{-\infty}^{+\infty} x^2 \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \sqrt{2\pi}\sigma(\sigma^2 + \mu^2) \quad (2)$$

We also can get an integral formula from the properties of $p.d.f$, that is:

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = 1$$

So

$$\int_{-\infty}^{+\infty} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \sqrt{2\pi}\sigma \quad (3)$$

(2) The k -order moment of exponential distribution is:

The same as exponential distribution, some useful formulas are given:

$$\int_0^{+\infty} e^{-\lambda x} dx = \frac{1}{\lambda} \quad (4)$$

$$\int_0^{+\infty} x e^{-\lambda x} dx = \frac{1}{\lambda^2} \quad (5)$$

$$\int_0^{+\infty} x^2 e^{-\lambda x} dx = \frac{2}{\lambda^3} \quad (6)$$

2.3 Examples

Example 1. To calculate generalized integral $\int_{-\infty}^{+\infty} (x-2)e^{-\frac{(x-6)^2}{6}} dx$

Solution:

$$\begin{aligned} \int_{-\infty}^{+\infty} (x-2)e^{-\frac{(x-6)^2}{6}} dx &= \int_{-\infty}^{+\infty} xe^{-\frac{(x-6)^2}{6}} dx - \int_{-\infty}^{+\infty} 2e^{-\frac{(x-6)^2}{6}} dx \\ &\stackrel{(1)(3)}{\underset{\substack{\mu=6 \\ \sigma=\sqrt{3}}}{=}} \sqrt{2\pi}6\sqrt{3} - 2\sqrt{2\pi}\sqrt{3} \\ &= 4\sqrt{6\pi} \end{aligned}$$

3. ACQUIRING THE SUM OF INFINITE SERIES USING PROBABILITY METHOD

By the idea of constructing a stochastic model to obtain the sum of infinite series, such as:

Example 2[2]: to calculate the sum of infinite series

$$\begin{aligned} &\frac{a^2}{k^2} + \left(1 - \frac{a^2}{k^2}\right) \frac{a^2}{(k+1)^2} + \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \frac{a^2}{(k+2)^2} + \\ &\dots + \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \dots \left(1 - \frac{a^2}{(k+n-2)^2}\right) \frac{a^2}{(k+n-1)^2} + \dots \end{aligned}$$

Solution: Consider the following model: suppose that there are k balls in the bag with the same size, shape, but different color, where a white balls ($0 < a < k, a, k \in N$), $k-a$ black balls. There are twice back to touch the ball, if the white ball is removed, then that trial is successful; otherwise considered a failure, in this case, then put into a black ball and take out two balls again, so keep on going, as well as infinite number, find the probability of a successful test.

In fact, suppose A denotes the event is successful, and A_n denotes the first test successfully at n -th times, with the probability formula of independent events,

$$\begin{aligned} P(A_1) &= \frac{a^2}{k^2}, P(A_2) = \left(1 - \frac{a^2}{k^2}\right) \frac{a^2}{(k+1)^2}, \dots \\ P(A_n) &= \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \dots \left(1 - \frac{a^2}{(k+n-2)^2}\right) \frac{a^2}{(k+n-1)^2}, \dots \end{aligned}$$

And $A = A_1 \cup A_2 \cup \dots \cup A_n \cup \dots$ where $\{A_n\}$ are exclusive, so

$$\begin{aligned} P(A) &= P(A_1) + P(A_2) + \dots + P(A_n) + \dots \\ &= \frac{a^2}{k^2} + \left(1 - \frac{a^2}{k^2}\right) \frac{a^2}{(k+1)^2} + \dots + \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \dots \left(1 - \frac{a^2}{(k+n-2)^2}\right) + \dots \end{aligned}$$

On the other hand, with the probability of opposite event,

$$\begin{aligned} P(\bar{A}) &= P(\bar{A}_1)P(\bar{A}_2) \dots P(\bar{A}_n) \dots \\ &= \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \dots \left(1 - \frac{a^2}{(k+n-1)^2}\right) \dots \\ &= \lim_{n \rightarrow \infty} \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \dots \left(1 - \frac{a^2}{(k+n-1)^2}\right) \dots \end{aligned}$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \frac{k-a}{k} \frac{k+a}{k} \frac{k-a+1}{k+1} \frac{k+1+a}{k+1} \cdots \frac{k}{k+a} \frac{k+a+a}{k+a} \frac{k+1}{k+a+1} \\
 &\quad \frac{k+1+a+a}{k+a+1} \cdots \frac{k-a+n-1}{k+n-1} \frac{k+n-1-a}{k+n-1} \\
 &= \frac{k-a}{k} \cdots \frac{k-a+1}{k+1} \frac{k-a+2}{k+2} \frac{k-a+3}{k+3} \cdots \frac{k-1}{k+a-1} \\
 P(A) &= 1 - P(\bar{A}) = 1 - \frac{k-a}{k} \cdots \frac{k-a+1}{k+1} \frac{k-a+2}{k+2} \frac{k-a+3}{k+3} \cdots \frac{k-1}{k+a-1}
 \end{aligned}$$

So

$$\begin{aligned}
 &\frac{a^2}{k^2} + \left(1 - \frac{a^2}{k^2}\right) \frac{a^2}{(k+1)^2} + \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \frac{a^2}{(k+2)^2} + \\
 &\quad \cdots + \left(1 - \frac{a^2}{k^2}\right) \left(1 - \frac{a^2}{(k+1)^2}\right) \cdots \left(1 - \frac{a^2}{(k+n-2)^2}\right) \frac{a^2}{(k+n-1)^2} + \cdots \\
 &= 1 - \frac{k-a}{k} \cdots \frac{k-a+1}{k+1} \frac{k-a+2}{k+2} \frac{k-a+3}{k+3} \cdots \frac{k-1}{k+a-1}
 \end{aligned}$$

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