



Scaling Without Conformal Invariants in the NonLocal Relativistic Quantum Systems in Living Cells

G. Petrov^{[a],*}

^[a]Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria.

*Corresponding author.

Address: Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria;
E-Mail: g_d_petrov@mail.bg

Received: October 12, 2013/accepted: December 26, 2013/Published online: April 25, 2014

Abstract: Since the 1948 the mathematical description of the so-called Casimir world as a part of the physical observed space-time in the relativistic sense is to be considered by the help of the Hamiltonian quantum field's theory and furthermore even it is based on the fine play between the continuity and the discrete too. The axiomatic-physical methods of the local quantum fields theory has given us the other possibility than the Lagrange quantum field's theory and precisely on this rigorously mathematical way to understand the singularities and the black holes, also the dark energy and the dark matter from one uniformly point of view. Aside from this, the essential difference is that external forces other than gravity, e.g. such as Casimir force, play a major role in the phenomena, i.e. there is not observed in our seeing world a local classical relativistic electromagnetic field potential $A_\mu(x)$. And also it is possible to describe the fundamental interactions between anyone concrete fundamental relativistic quantum field system with someone other or with the external and innerness material objects as a additional boundary conditions by the proving the fulfilling of the causality conditions and consider they as an external classical fields, and everyone internal background fields. At the first in his famous work "To the Electrodynamics of moved bodies", Leipzig, 1905, Einstein has proved the possibility to understand the nature from the relativistic point of view in the classical physics.

By the living cells as an object of the fundamental cryobiological researches i.e. in this case the metabolisms is minimal so that by the help of the axiomatic-physical methods by the relativistic theory of quantum fields systems considering as a Micro fields it is possible to be taken in the account the problem of a "time's arrow" at the microscopic level by the contemporary considerations of the quantum vacuum in the Casimir world as a ground state of anyone relativistic quantum system becomes a fixture by the lyophilized elementary living cells. So the possibility to understand the Hilbert space with indefinite metric for the further considerations the word elementary understands a one structure idealization of the living cells. Also the many

miracle properties of so defined living cells apparent enchanting by consideration of his functions yet are putting besides in the molecules but in the fundamental quantum field interactions between the quantum vacuum of anyone quantum fields system in the Microsoft matter and the molecules but taken in the Minkowski space-time too. Moreover it can be represented the symmetrical selfadjoint Hamiltonian operator $\tilde{\Phi}$ taken by as for simplicity for the relativistic quantum scalar fields by definition obtained as virtual (potential) element in the Hilbert functional space with indefinite metric. That is the quantum field operator obtained by everyone wave fields solution at the fixed time known as a virtual or “potential” quantum field operator acting on the virtual vacuum vector valued functional states of the Hilbert functional space with indefinite metric. So also it is realizable the possibility to be obtained the local or non local quantum force currents, i.e. which interact minimal local or global by phase integration over the field potential with the field force carrier knowing as the so called virial current i.e. that impact near local or global at the distance of interactions with the classical neighborhood in the Microsoft matter in the Minkowski space-time. The probability interpretation of the spectral family give us the physical interpretation of the observed quantum entity by the relativistic quantum systems even for the dynamically (not thermodynamically) fine structure of the ground state as potential state also as virtual vector valued functional state, t.e. as the element of the Hilbert functional space with indefinite metric by the vacuum interactions in the Casimir world. It knows yet the Casimir force today is measured with exactness by 5 %.

Precisely the impact of this force on the molecular biology (genetics) is still not clear, i.e. there is a new situation of the so called quantum cryobiology. The additional boundary conditions must be taken under account, e.g. in the cosmogony models it is not possible to consider additional boundary conditions. So also it is possible to understand better the molecules by the molecular biology as a classical object interacting with the ground state of the every one relativistic quantum field system. So also by definition it is considered the relevant operator valued functional Banach algebra or in the Schrödinger picture the vacuum wave functional as a solution of the impulse wave equation describing the some relativistic quantum system in the Minkowski space-time. With other words by the help of the so called S-matrix theory as in the non relativistic case where this theory is very gut proved we hope to understand better the nature under consideration.

It knows the following fact that it is potentially force with a long-range action at the distance or with other words asserted every experiment in this genetics domain without clearness of the role of his impact on the neighborhoods in the living cell. Just therefore this is to be taken very good under account from the point of view of the nanophysics too. May be he is the cause for not observing of the so-called Goldstein massless bosons or the quarks as it is the case by the Coulomb force between the charged particles.

Moreover the Casimir vacuum state of the relativistic quantum scalar field system may be not belonging in the operator definition's functional domain of the field's operators, but fulfill the additional causal and boundary conditions by the solving the boundary value problem for the carrier of the interaction force, the virtual fundamental scalar particles called by us scalars belonging to the domain structure of the Casimir world.

So also the Casimir vacuum in the asymptotic past at the left of the one not moved perfectly conductor plate contains then from the micro-causal point of view

propagation of the virtual particles for the initial observer understanding as referent system (a map). In the asymptotic future at the right of the same plate and the left of the second parallel moved perfectly conductor plate towards the plate at the rest with a constant velocity v the propagation of the see massive particles for the late-time observer, e.g. the Maxwell demon, and moreover at the right of the moved plate anew a propagation of the virtual relativistic quantum particles system.

Precisely the scalar massless relativistic quantum field give us then that his local algebras are unitary equivalent in the bounded domains of the locally algebras by the matter field and also they have the same structure properties which is from more great importance for the theory than the definiteness of the metric of the Hilbert functional space. So it is possible to be defined the double singularities which will be given by the ground state of local relativistic quantum scalar field system too. The symmetries and structure properties are mathematical described by the Banach algebra of the field's operators defined in the Hilbert functional space with indefinite metric. Farther the ground state is defined over dies algebra but it can be negative too as remember from the indefinite metric of the Hilbert functional space. However then there are a number of additional properties generated from the physical distinctions by the massless systems: His scale i.e. the group of the scale transformations represented by the dilatations and special conformal transformations and conformal symmetries also obtained by the group of the conformal transformations give a double singularities of the quantum systems and the vacuum state, but scale invariance does not imply necessary a conformal invariance and as well the infrared effects leaded to manifest the global structure of the relativistic quantum systems and the vacuum state. Quantum Field Theory QFT and the Renormierungs groups theory RG-groups are classified by scale invariant, Infrared IR fixed point (Wilson's philosophy). In the Doctor paper (Petrov, 1978) it is showed that the scaling behaviors of the some quantum entities are destroyed in longitudinal and conserved in the cross section's direction by fulfilling the causality condition for non forward deep inelastic scattering of leptons and hadrons. Also the scale invariance is not from the same nature as the conformal invariance by the massless quantum fields and the scale invariance lead yet not necessarily to the conformal invariance.

Furthermore the Hilbert functional space understands by means of the space of the test functions from his completion by anyone norm the possibility of the definition of the Casimir quantum vacuum state as well a ground state in the Schrödinger picture over the involutes Banach algebra of the field operators defined in the Hilbert functional space with indefinite metric. Then so one functional vector valued vacuum state can be negative as remember of the indefinite metric by definition but this is not from anyone significance for the theory. This question precisely spoken is a pure algebraically formulations of anyone relativistic quantum systems out of the Hilbert functional spaces with indefinite metric.

The theoretical underpinnings of scale without the conformal invariance in relativistic quantum physics are given in the light of the results of the non local operator's expansion on the light cone. Then the Casimir vacuum state of a given relativistic scalar quantum fields systems, precisely due to deep connections between scale-invariant theories and the recurrent scaling behaviors of the quantum entities in the Casimir world can be defined over the involutes Banach algebra of the field operators acting on the virtual vector valued state defined in the Hilbert functional space with indefinite metric. Furthermore the vacuum state in the Schrödinger picture

defined over this algebra can be negative too as remembering of the indefinite metric but that is only a one algebraic problem. It can be shown that, on scaling-invariant time like paths of the virtual quantum particles, there is a redefinition of the dilatation current by the virial current that leads to virtual generators of dilatations operators. Also just that lead to the generations of the virtual vacuum fluctuations described by the relativistic quantum fields operators created an involutes Banach algebra of the quantum field operators out of the Hilbert space with indefinite metric. Finally, it can be develop a systematic algorithm by the Casimir world for the research of scaling-invariant non space like paths of virtual particles caused by virtual fluctuations of the vacuum with a zero point energy ZPE and broken scaling-invariant time like and non space like paths of see massive particles.

Key words: Casimir effect; Time’s arrow; Relativistic quantum field systems; Elementary living cells; Lyophilization; Nanophysics; Singularities; Causal and scaling principle

Petrov, G. (2014). Scaling Without Conformal Invariants in the Non local Relativistic Quantum Systems in living Cells. *Progress in Applied Mathematics*, 7(2), 1-68. Available from <http://www.cscanada.net/index.php/pam/article/view/10.3968/5061> DOI: 10.3968/5061

1. INTRODUCTION

Following this thought it is to remark that the physical phenomena on the light cone are relativistic in the classical sense. But from the quantum point of view it is possible by interactions the directionality at a given domain’s time arrow e.g. space parameter with a broken scaling behavior of the time like paths of the virtual relativistic particles described by Einstein’s relativistic theory researched by the help of the Minkowski space-time which described simultaneously both the geometry of the special relativistic theory, and the geometry, induced on the every tangential space of anyone Lorenz manifold. So also the Minkowski space-time plays the same role for the Lorenz manifold as the Euclidian space for the Riemannian manifold. Furthermore the time parameter by definition in Minkowski space-time precisely is not so gut understanding without Lorenz transformations in the sense of the Einstein’s special relativistic theory. Then the causality principle applied just on the Minkowski space-time structure on the manifold, induced on the every tangential space of the anyone Lorenz manifold i.e. the time oriented manifold called traditional space-time give us the possibility to solve the boundary value problem from the relativistic point of view. However the gut understanding thermodynamically “time’s flow arrow” as a physical phenomena is conversely non relativistic and the time is then absolutely and precisely in force is the so-called Galileo transformation in the created by the Euclidian structure on the manifold induced on the every tangential space of everyone Riemann manifold.

Also then in our case the event points are to be thought from the geometric principles of symmetry in Minkowski space-time manifold M locally by a scale units $f(t)$ in the every fixed events points for

$$t \in (t_{-2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)}], n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n$$

by the radius 4-vector with respect to the initial observer at the rest and the event 4-point $Y_{2(n-\frac{1}{2}j)}(M, f)$, $n = 0, 1, 2, \dots; j = 0, \dots, 2n$, where $y_{-2(n-\frac{1}{2}j)}^2 = y_{2(n-\frac{1}{2}j)}^2 = y_0^2$ obtained by the Minkowski space-time. Then in this points it is even measured at the fixed time $t_{2(n-j/2)}$ right from the mirror at the rest or at the fixed time $t_{-2(n-j/2)} + 0$ left from the mirror at the rest for obtained by definition time independent scale function fulfilled the differential equation

$$df(t)/dt = x^2/(2(x y_{2(n-\frac{1}{2}j)} + y_0^2 f) - f = 0$$

at the time $t \in (t_{-2(n-j/2)}, t_{2(n-j/2)})$ and the Euclidian radius 3-vector $\bar{x} = (x^1, x^2, x^3)$ for $y^3, x^3 \in (0, L) \in (y^3_{-2(n-j/2)}, y^3_{2(n-j/2)})$. Yet then the time independent scale unit f is obtained by the equation

$$f = ((\tilde{x}y_{2(n-j/2)})y_0^{-2})((1+x^2y_0^2(\tilde{x}y_{2(n-j/2)})^{-2})^{1/2} - 1).$$

Even it is to be understand that the boundary value problem can be taken as solved by the measurements in the case of the local coordinate with respect to the observer understands both the initially coordinate system (a map) by someone scale choice under given initially conditions for the considered physical theory and the geometry following the geometric principles with respect to the causality conditions by the supposed additional boundary conditions. It is to be remarked that the mathematical meaning of the manifold is introduced at the first in the General Relativistic Theory from Einstein in his famous work.

Moreover it knows that the Minkowski space-time M^4 describe simultaneously both the Einstein special relativistic theory of the events points manifolds published at the first 1905 in Leipzig in the famous paper "To the electrodynamics of the moved bodies" and the geometry, induced by the every tangential space of anyone Lorenz manifold. Conversely in the non relativistic theories the Euclidian space describe simultaneously both the Newton mechanics of the matter point's manifolds and the geometry, induced by the every tangential space of anyone Riemann manifolds. Also it is obvious that the Minkowski space-time play the same role for the Lorenz manifold as the Euclidian space for the Riemann manifold. So this is obtained e.g. by the scaling behaviors at a time $t \in (t_{-2(n-j/2)}, t_{2(n-j/2)})$, or by the measurement at a time $t_{2(n-j/2)}$ for the space group structure obtained by $R^1(t_{2(n-j/2)}) \times R^3(\bar{y}_\perp, y^3_{2(n-j/2)})$ and scale units f at the same time. Precisely furthermore in the local cense the Lorenz structure and the Riemann structure on the manifold are equally but globally yet it must be considered of a distinct manner.

The Casimir quantum vacuum is not connected with anyone charge. Moreover his structure and symmetries are no more so narrow connected to the structure and the symmetries of the relativistic quantum system. The classes of the vacuum structure will be obtained by the dynamically classical definitions of the Casimir world and symmetrically by additionally causal and boundary conditions. The global structure of the Casimir vacuum state of anyone local relativistic quantum field system defined in a local coordinate system must be considered also in the Lorenz global geometry by fulfilling of the additional causality and boundary conditions without anyone innerness contradictoriness. The local scalar relativistic wave quantum fields even obtained in the Minkowski space-time fulfill the internal non contradictoriness too. Moreover then the observer is to be understood as a local coordinate referents system (a map), e.g. observer stayed on the mirror at the rest or on the parallel moved inertial mirror with the velocity v towards the unmoved at the rest obtained by the Lorenz transformations in the manifolds M .

The space-time interval is a dimensionless distance between two events points measured in anyone units and moreover the causal conditions is as well fulfilled by

$$dI = (dy_{2(n-j/2)} \cdot dy_{2(n-j/2)})^{1/2} f.$$

In the case of the Minkowski space-time the distance is obtained by the indefinite scalar product of the Minkowski radius 4-vector

$$y_{2(n-j/2)}^2 = y_0^2 = (ct_0)^2 - \bar{y}_\perp^2 - y^3^2$$

with respect to the initial observer and remained invariant at the same time when the Lorenz transformations are fulfilled.

The equally describing from other observer following the relativistic principles correspond to self mapping of the space-time scale manifold conserved the interval dl . By the fixed scale in the Minkowski space-time the transformation group conserved the distance d

$$d(x, y_{2(n-j/2)}) = ((x - y_{2(n-j/2)}) (x - y_{2(n-j/2)}))^{1/2}$$

by fixed points $y_{2(n-j/2)}$ is isomorphic to the half direct product $T^{3,1} \times O(3,1)$ group of transformations and the full homogeneity Lorentz transformations on the Minkowski space-time with respect to naturally action of the $O(3,1)$ group on M .

Also then from the causality and the additional boundary conditions can be supposed the solving of the boundary value problem in the relativistic sense without to consider the initially conditions for the initial observer. With other words just the question what occurs for the two plates at the time $t = 0$ is groundless. But by hook or crook in the relativistic quantum field's theory e.g. experimentally by the quantum electrodynamics QED as a experimentally gut proved theory precisely in the domains of high energy physics the utilization of the fundamental field's equations is not from major significance. It knows that in the domain of high energy physics the number of the model understandings is significant and also the role of the mathematics in this case is not to clear the bounds between the mathematical-physical models but more to obtain anyone physical theory i.e. still from the physical-axiomatically point of view QED is to be understand as a model than instead of a theory of the relativistic quantum fields systems in domains of high energy physics. The so called Standard model is from the same nature as by the QED. Also just that is from significant importance by the understanding of the Casimir world in the lyophilized elementary living cells from the theoretical point of view by the proving of the quantum field theory, e.g. QED and the Standard Model.

Yet in the same way by fulfilling of the following naturally statements defined as early as from the ancient nature-philosophy by Syrian, Egyptian and Grecian in the nature-philosophical axiomatically (also it is conferred, universal taken proposition) sense of the unity of the opposite entities

- 1) boundary and infinity
- 2) odd and even
- 3) oneness and infinity aggregate
- 4) right and left
- 5) manly and womanly
- 6) unmoved and moved
- 7) straight and curve
- 8) light and darkness
- 9) blessing and disguise
- 10) square and parallelogram

it is possibly to describe the interacting quantum relativistic field's systems in the Casimir world (boundary and infinity) becomes a fixture to the roundabout environment in lyophilized living cells and systems from the point of view of the usual axiomatic physical theory. Moreover by utilization of the idea of the vacuum as a functional ground state of the axiomatically constructed concrete fundamental relativistic quantum system in the Schrödinger picture with additional adiabatically and impulse effects can be considered the so-called micro causality. Moreover then the "time's arrow" can be understand micro causal from microscopically stand point of view by the quantum causality and localizability of the quantum entities seeing from the observer e.g. the Maxwell demon at the past $t > t_{-2n}$

and at the future time at $t = t_{2n}$ from the late-time observer for $n \rightarrow \infty$. Even then it takes not into account the thermodynamically entropies character of the time. It knows then that has his cause for the Casimir effect by the Einstein's macro causality i.e. the Casimir force in the vacuum impact over every particle as external force. Also it is the phenomena from the same nature as by the electron moved in the external classical electromagnetic field by broken vacuum symmetries of the QED e.g. the scaling behaviour of the vacuum state of the massless Dirac fundamental electron field lead to polarisation (electron-positron pair) of the vacuum by acting of the electric force on the localized massive electron.

For the light propagation in the vacuum at the microscopically level the geometrical understanding of causality Lorenz manifolds is practical from one and the same nature described by local quantum wave field systems. The former was physically understood very gut as phenomena of the quantum electrodynamics QED but not so gut from the so-called pure physics-mathematical point of view. Furthermore following the quantum character of the causality properties of the observed physical quantum entities in the domains of the high energy physics it is clear that the application of the usual mathematical analysis of the 19 Century by the necessarily analyticity representation of the causality of the quantum entities is not more sufficiently to describe this by the help of the fundamental equations for the quantum vacuum state of the relativistic quantum systems. The fundamental equations are more of no utility because just the nature of the vacuum state besides the locality is globally and it needs also the global Lorenz geometry too.

The generalized functions and more special the tempered distributions make possibly the understanding of the nature by those physical phenomena from the mathematical point of view too, e.g. without to consider the set of the measures zero as by Lebesgue's integrations. The entity of the distributions consist in them that by dropping the knowledge of the functions which define the Lebesgue's set of measure zero it is possibly to define wide class of generalized functions, included different Dirac δ -functions and his derivations. Also the physical conditions as additional causality and boundary conditions for the solution of the boundary value problem are necessary but not sufficient if there are the innerness contradictoriness bounded with the observer and the scaling problems of the group of the scale transformations in the Minkowski space-time.

At the molecular level (Mitter & Robaschik, 1999) the thermodynamic behaviour is considered by quantum electromagnetic field system with additional boundary conditions as well by the Casimir effect between the two parallel, perfectly conducting quadratic plates (side L , distance d , $L > d$), embedded in a large cube (side L) with one of the plates at face and non moved towards the other, i.e. also the case of so called Casimir effect under consideration in the sense of the local case when the Minkowski space-time is equally of 4-dimensional Euclidian space but without the considerations of the causality properties of the relativistic quantum entities given a share in the effect, e.g. relativistic supplement to the Casimir force $v/c \ll 1$ where v is the relative velocity of the moved mirror and c is the light velocity (Bordag, Petrov, & Robaschik, 1984; Petrov, 1985; Petrov, 1989). Then the boundary value problem must be considered with respect to the additional causal conditions and not implicit to be considered the initially conditions. So the time's arrow and the causality have a new understanding in the relativistic quantum physics, e.g. the Casimir energy $\omega \rightarrow -\infty$ for $t \rightarrow 0$.

Following the classical Einstein's gravitational theory Weyl in 1918 attempt to incorporate electromagnetism into the theory by gauging the metric tensor i.e. by letting

$$g_{\mu\nu} = \exp(-\gamma \int dx^\mu W_\mu(x)) g_{\mu\nu},$$

where γ was a constant and the vector field W_μ was to be identified with the electromagnetic vector potential. Although this idea was attractive, following Einstein, it was physically untenable because it would imply that the spacing of spectral lines would depend on the history of the emitting atoms, in manifest disagreement with experiment to this time by the quantum understandings of the nature. However, after the advent of Wave Mechanics in 1926, the idea was resurrected by application to other physical situations. This new observation that the usual electromagnetic differential minimal principle was equivalent to the integral minimal principle and that this was the correct version of Weyl's proposal in which the constant was chosen pure imaginary $\gamma = i/\hbar$, where \hbar is the Planck constant h divided by 2π and the electromagnetic factor was chosen to multiply more the Schrödinger wave-function which itself has not so clear physical meaning rather than Einstein metric. This observation was quite profound because it not laid the foundations for modern gauge theory but brought electromagnetism into the realm of geometry.

Our interests is the relativistic more realistic Casimir effect without the innerness contradictoriness when the one of the plates is at the rest and the other moved with a constant velocity v towards the non moved plates imbedded in the Minkowski space-time.

So the thermodynamic behavior of the elementary living cells under consideration must be considered globally by the relativistic quantum systems in the so called Casimir world too which can be better understand in the light of the considered problem in the famous paper by Einstein Zur Elektrodynamik der bewegten Körper, 1905, Leipzig, (for further considerations, see Dodonov, 2001).

It has long been presumed that, under mild assumptions, scale invariance $x^\mu \rightarrow \kappa x^\mu$, e.g. $\kappa x^\mu = t_j^r x^\mu + y_{2(n - 1/2j)}^{\mu} f_k$, implies conformal invariance in relativistic quantum field theory. Although no proof is known by the dimensions $d > 2$ in the flat space-time of the Lorenz manifolds, until very recently a credible counterexample was lacking (Fortin, et al.).

At the first it is to be considered the fixed event 4-point in the Minkowski space-time obtained by the radius 4-vector with respect to the initial observer at the rest

$$y_{2(n - 1/2(j+1))}^\mu, y_{-2(n - 1/2j)}^\mu, y_{2(n - 1/2j)}^\mu, y_{-2(n - 1/2(j-1))}^\mu, \\ m = 0, 1, 2, 3,$$

and which are obtained by the reflections and the hyperbolical turns (odd and even, right and left) of the fixed event 4-point $y_{0}^\mu = (ct_0, \bar{y})$ and the radius 3-vector $\bar{y} = (y^1, y^2, y^3)$ with the Euclidian norm $\|\bar{y}\| = (y^1^2 + y^2^2 + y^3^2)^{1/2}$ at the fixed time t_0 and the second event 4-point without reflections and hyperbolical turns $x^\mu = (ct, \bar{x})$ with the Euclidian norm $\|\bar{x}\| = (x^1^2 + x^2^2 + x^3^2)^{1/2}$ for the every one fixed time t between the two perfectly conductor plates in the coordinate Minkowski space-time \mathbf{M}^4 described both the geometry of the Einstein special relativistic theory where c is the light velocity and the geometry induced on the everyone tangential space of anyone Lorenz manifold. This is the knowing fact that the time oriented Lorenz globally geometry of the space-time give us the possibility to understand the time's arrow between the manifold's event points of the special relativistic theory in the light of the Lorenz global geometry. Moreover so it can be thought micro causal for the time belonging to this geometry where

$$t \in (t_{-2(n - 1/2j)}, t_{2(n - 1/2j)}], n = 0, 1, 2, \dots, \\ j = 0, 1, 2, \dots, 2n, by y^3, x^3 \in (0, d_0],$$

or $y^3, x^3 \in [d_0, L)$, (quadrangle and parallelogram) and n is the reflecting number of the fixed event 4-point y_{0}^μ of the Minkowski space-time M which describe the geometry induced

by the tangential space in this point of the Lorenz manifold between the unmoved and the parallel moved plate towards the plate at the rest with the constant velocity v and seeable (light and darkness) e.g. from the demon of Maxwell (blessing and disguise) at the time $t_0 = c^{-1}(y_0^2 + \bar{y}_\perp^2 + y^3)^{1/2}$ so that the moved plate is placed by $L = vt_0$. Moreover $t_0 = c^{-1} \|\bar{y}_\perp\| + \text{const}$ by

$$\lim_{y_0^2 + y^3 \rightarrow \infty} y_0 / 2 \|\bar{y}_\perp\| = \text{const} \in [0, 1] \text{ for}$$

$$y_0^2 + y^3 \rightarrow \infty, \|\bar{y}_\perp\| = -(\bar{y}_\perp^2)^{1/2} \rightarrow \infty.$$

Furthermore for the mirror fixed 4-points $y^\mu_{-2(n-1/2j)}$ and $y^\mu_{2(n-1/2j)}$ it can be defined a light like vectors \tilde{x}^μ and \tilde{x}^μ in the Minkowski space-time by the distinguishing marks “l” = left and “r” = right obtained by the following relations

$$\tilde{x}^\mu = x^\mu + y^\mu_{2(n-1/2j)} f = x^\mu + y^\mu_{2(n-1/2j)}$$

where

$$f = ((\tilde{x}y_{2(n-1/2j)})y_0^{-2})((1 - x^2y_0^2(\tilde{x}y_{2(n-1/2j)})^{-2})^{1/2} - 1)$$

for t const and for the fixed time $t_{2(n-1/2j)}$. Moreover then the light-like 4-vector $\tilde{x}^\mu = x^\mu - y^\mu_{2(n-1/2j)}$ is obtained by the equation $f + 1 = 0$ and vice versa and this led to a indefinite product which in this case is obtained as a light-like interval $(x^\mu - y^\mu_{2(n-1/2j)})^2 = 0$ used by the calculation of the Casimir effect by $x^\mu \rightarrow y_0^\mu$ for massless quantum scalar field. Furthermore also $\tilde{x}^\mu = x^\mu + y^\mu_{2(n-1/2j)} f$, with a time-independent scale function

$$f = ((xy_{-2(n-1/2j)})y_0^{-2})((1 - x^2y_0^2(xy_{-2(n-1/2j)})^{-2})^{1/2} - 1)$$

for the fixed time $t = t_{-2(n-1/2j)}$ and

$$t \in (t_{-2(n-1/2j)}, t_{2(n-1/2j)}],$$

$$n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n.$$

Moreover by setting

$$0 \leq \kappa' \leq t \leq \kappa \leq 1,$$

explicitly it can be defined by the fulfilling of the dilatation's invariance the Minkowski space-time non local radius 4-vector by the following relations

$$kx^\mu = t_j^r x^\mu + y_{2(n-1/2j)}^m f_k,$$

where

$$f_k = y_0^{-2}(y_{2(n-1/2j)} t_j^r \tilde{x})((1 + k^2 x^2 y_0^2 (y_{2(n-1/2j)} t_j^r \tilde{x})^{-2})^{1/2} - 1)$$

and

$$kx^\mu = t_j^r \tilde{x}^\mu + 1/2 (x + t_j^r \tilde{x}) f_k, \text{ where } f_k = 1/2 y_0^{-2} (x t_j^r \tilde{x}) ((1 + 4k^2 x^2 y_0^2 (x t_j^r \tilde{x})^{-2})^{1/2} - 1),$$

$$k^r x^\mu = t_j^l \tilde{x}^\mu + y_{-2(n-1/2j)}^m f_k, \text{ where } f_k = 1/2 y_0^{-2} (y_{-2(n-1/2j)} t_j^l \tilde{x}) ((1 + k^2 x^2 y_0^2 (y_{-2(n-1/2j)} t_j^l \tilde{x})^{-2})^{1/2} - 1)$$

and

$$k^r x^\mu = t_j^l \tilde{x}^\mu + 1/2 (x + t_j^l \tilde{x}) f_k, \text{ where } f_k = 1/2 y_0^{-2} (x t_j^l \tilde{x}) ((1 + 4k^2 x^2 y_0^2 (x t_j^l \tilde{x})^{-2})^{1/2} - 1)$$

for

$$y_0^2 = y_{2(n-1/2j)}^2 = 1/4 (x + t_j^r \tilde{x})^2 = y_{-2(n-1/2j)}^2 = 1/4 (x + t_j^l \tilde{x})^2, t \in (t_{-2(n-1/2j)}, t_{2(n-1/2j)}],$$

$$n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n,$$

$$k^r x^\mu = t_{2n-1}^l \tilde{x}^\mu + 1/2 (x + t_{2n-1}^l \tilde{x}) f_k;$$

for

$$t \in (t_1, t_1], t_1 = t_0, j = 2n-1,$$

$$f_k = \frac{1}{2}y_0^{-2}(xt_{2n-1}\tilde{x})$$

$$((1 + 4k^2x^2y_0^2(xt_{2n-1}\tilde{x})^{-2})^{1/2} - 1)$$

for

$$y_{-1}^2 = y_0^2 = y_1^2 = \frac{1}{4}(x + t_{2n-1}\tilde{x})^2 = \frac{1}{4}(x + t_{2n-1}\tilde{x})^2$$

Then by the fixed vectors $\frac{1}{2}(x + t_j^r\tilde{x})^\mu$ and $y_{2(n-\frac{1}{2}j)}^\mu$ and by $(x - y_{2(n-\frac{1}{2}j)})^2 = 0$ as that is the case by calculation of the Casimir effect follows $y_0^2 - (xt_j^r\tilde{x}) = 0$ for $(\frac{1}{2}(x + t_j^r\tilde{x}))^2 = y_0^2$ and this led to $(\frac{1}{2}(x - t_j^r\tilde{x}))^2 = 0$ too.

Precisely also it is to remark that from the invariance group, if the scale units are changed from point to point, it is obtained for the dilatations and the special conformal transformations in the Minkowski space-time by the following relations

$$x^{\mu} = (x^{\mu} - x^2 y_{2(n-\frac{1}{2}j)}^{\mu})/\sigma(x),$$

where

$$\sigma(x) = 1 - 2x_{\mu}y_{2(n-\frac{1}{2}j)}^{\mu} + x^2 y_0^2 = (1 - x^2)(1 - y_0^2) + (x - y_{2(n-\frac{1}{2}j)})^2$$

and

$$x^{\mu} = (x^{\mu} - x^2(\frac{1}{2}(x + t_j^r\tilde{x})^{\mu})/\sigma(x) \text{ by obtained}$$

$$\sigma(x) = 1 - x_{\mu}(x + t_j^r\tilde{x})^{\mu} + x^2 y_0^2 = (1 - x^2)(1 - y_0^2) + (\frac{1}{2}(x - t_j^r\tilde{x}))^2$$

$$t \in (t_{-2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)}], n = 0, 1, 2, \dots,$$

$$j = 0, 1, 2, \dots, 2n,$$

also by $x^2 = x^2/\sigma$ are the former suppositions fulfilled on the light cone in the Minkowski space-time.

Furthermore in the impulse Minkowski space-time and fixed heat impulse 4-vector $k^\mu = (\omega/c, \vec{0}_\perp, k^3)$ and the impulse 4-vector $q^\mu = (q^0, \vec{q}_\perp, q^3)$ as that was the case by $\omega = 0$ in the dissertation paper of G. Petrov 1978 by studying of the causality properties of the form factors in the non forward Compton scattering by deep inelastic scattering of leptons and hadrons by means of the following relation $q_k^\mu = q^\mu - k^\mu$ and $q_k^\mu = q^\mu + k^\mu$ so that $dq_k^\mu = dq_k^\mu = dq^\mu$ by fixed k^μ it can be defined by impulse scale unit function f

$$\tilde{q}^\mu = q^\mu + k^\mu f \text{ with } f = k^2(kq)/((1 - q^2k^2/(kq)^2)^{1/2} - 1)$$

and then also if the light-like impulse 4-vector $\tilde{q}^\mu = q^\mu - k^\mu$ is obtained by the equation $f + 1 = 0$ and vice versa which lead to $\tilde{q}^2 = (q^\mu - k^\mu)^2 = 0$.

Further it can be defined

$$q_k^\mu = \tilde{q}^\mu + k^\mu f_k \text{ with } f_k = k^2(k\tilde{q})/((1 + k^2q^2k^2/(k\tilde{q})^2)^{1/2} - 1),$$

$$q_k^\mu = \tilde{q}^\mu + k^\mu f_k \text{ with } f_k = k^2(k\tilde{q})/((1 + k^2q^2k^2/(k\tilde{q})^2)^{1/2} - 1),$$

and where $q^\mu = \frac{1}{2}(q_k^\mu + q_k^\mu)$ and $k^\mu = \frac{1}{2}(q_k^\mu - q_k^\mu)$ is the fixed heat impulse 4-vector by $\tilde{q}^2 = \tilde{q}^2 = 0$, and for

$$q_k^2 \in (-\infty, 0], q_k^2 \in [0, \infty), q^2 \in (-\infty, \infty), k^2 \in (-\infty, \infty),$$

and by the $\kappa > \kappa' \in [0, 1], q_k^2 = k^2q^2, q^2 > 0, q_k^2 = k'^2q^2, q^2 < 0$.

The quadrate of the heat impulse 4-vector in the referent system at the rest $k^2 = m_d^2c^2 \in (-\infty, \infty)$, and also by fixed dark mass $m_d = c^{-1}(\omega^2/c^2 - k^2)^{1/2}$ it can be chosen as

the so called dark impulse in the referent system at the rest. Moreover the dark Energy in the referent system at the rest $m_d c^2 = \omega + \text{const}$ where for $k^2 \rightarrow \infty$ and $\omega \rightarrow -\infty$ the $\lim(-k^2/2\omega) = 1/2 m \dot{z}^2 = \text{const} \in [0, 1]$ is the kinetically energy of the virtual scalar particles in the Casimir vacuum state defined as a virtual state in the Hilbert functional space with indefinite metric. So also $m \dot{z}$ is the kinetic impulse obtained by the Casimir force of this relativistic quantum system consisted by the particles number $n \rightarrow \infty$ of the virtual scalars. So it can be obtained that the mass of the virtual scalar particle $m = m_d / (1 - \dot{z}^2/c^2)^{1/2}$.

Moreover also the number of the virtual scalars in the case of the heat relativistic quantum system which consist from this scalars particles and the his impulse in the referent system at the rest so that also $m_d c = i(U_d + F_c V_d)$ where U_d is the inner energy F_c is the Casimir force per unit surface area and the V_d is the volume of the system can be considered rather as the invariant quantitative parameters of the heat relativistic quantum systems.

Furthermore for the vacuum fluctuations the Casimir energy can be obtained by

$$\omega = c(k^2 + k^3)^{1/2},$$

where the Casimir vacuum energy ω is calculated by the Casimir effect for the relativistic quantum field system and can be positive or negative in dependence from the topology of the additional boundary conditions to the initial conditions, i.e. by solving the boundary value problem.

It is obtained by the difference

$$\omega = 1/2(E_k - E_{k'}) = 1/2(E_{\alpha_k} - E_{\alpha_{k'}}).$$

$\omega/c = (k^2 + k^3)^{1/2} = k^3 + \text{const}$ and even by $k^3 \rightarrow -\infty, k^2 \rightarrow -\infty$, where by the longitudinal impulse $k^3 = m \dot{z}$ of the virtual scalars is the $\lim(k^2/2m \dot{z}) = \text{const} \in [0, 1]$. Also the Casimir energy ω is equally to the impulse k^3 in the longitudinal direction except of the const. Moreover it can be defined the mass of the so called moved fundamental scalar particle of the solution of the Klein-Gordon equation of "matter" local scalar field $\varphi(x)$.

It is remarkable to choose from the special conform group invariance

$$q^\mu = (q^\mu - q^2 k^\mu) / \sigma(q),$$

where

$$\sigma(q) = 1 - 2q^\mu k_\mu + q^2 k^2 = (1 - q^2)(1 - k^2) + (q^\mu - k^\mu)^2 = (1 - q^2)(1 - k^2) + q_{k^2}.$$

Moreover the following bounded open domains of double cons can be defined

${}^1D = {}^1D_{\kappa x, t_j^x} = V_{t_j^x}^+ \cap V_{\kappa x}^-$ with the basis $S_{\kappa x, t_j^x}$ and the axis $[kx^\mu, t_j^x]$, ${}^rD = {}^rD_{\kappa' x, t_j^x} = V_{t_j^x}^+ \cap V_{\kappa' x}^-$ with the basis $S_{\kappa' x, t_j^x}$ and the axis $[k'x^\mu, t_j^x]$, where on compact subsets of the domain 1D it can be obtained the Green function by

$$k^2 = m_d^2 c^2,$$

$$D(kx, m_d) = \int d^4 q \cdot \exp[-iqkx] / (q^2 - m_d^2 - i\varepsilon) \text{ or}$$

$$D(q_{\kappa x}, \kappa(x^2)^{1/2}) = D(q - k, \kappa(x^2)^{1/2}) = \int d^4 kx \cdot \exp[iq_{\kappa} kx] / ((\kappa x)^2 - \kappa^2 x^2 - i\varepsilon),$$

so that for $\kappa \rightarrow 0$ and that is also $\kappa^2 x^2 = 0$ it is

$$D(q + k) = \int d^4 kx \cdot \exp[iq_{\kappa} kx] / (\kappa x)^2 - i\varepsilon \text{ and } \kappa x^\mu \rightarrow \tilde{\kappa} x^\mu$$

Also for $\kappa' \rightarrow 0$ and that is also

$$\kappa'^2 x'^2 = 0 \text{ it is } D(q - k) = \int d^4 kx \cdot \exp[iq_{\kappa'} k'x] / (\kappa' x')^2 - i\varepsilon$$

and $k'x^m \rightarrow \tilde{\kappa}' x^m$.

At the first it can be reviewed the circumstances for the non local quantum field theory also precisely the scaling behaviors without to consider conformal invariance. The

most general form of the non local dilatation current operator $\tilde{D}_\mu(\kappa x)$ is obtained by the operator equation for the non-local operators fulfilled on the light cone for $\kappa' \rightarrow 0$ and $\kappa'^2 x^2 = 0$ also by $q_k^2 \rightarrow -\infty$,

$$\tilde{D}_\mu(\kappa x) = \kappa^2 x^\nu \tilde{T}_{\mu\nu}(\kappa x, \kappa' x) - \tilde{V}_\mu(\kappa x),$$

where $\tilde{T}_{\mu\nu}(\kappa x, \kappa' x)$ is the non local operator of a anyone symmetric energy-momentum tensor of the relativistic quantum scalar field system and $\tilde{V}_\mu(\kappa x)$, the non local operator of the virial current.

Also for the vacuum expectation value of the tensor of the averaged energy-momentum and the virial current between the scalar field states at the fixed 4-point it can be obtained

$$T_{\mu\nu}(\kappa x, \kappa' x) = \langle y_{2(n-\frac{1}{2}j)} | : \tilde{T}_{\mu\nu}(\kappa x, \kappa' x) : | y_{2(n-\frac{1}{2}j)} \tilde{n} \rangle,$$

and

$$V^\mu(\kappa x) = \langle y_{2(n-\frac{1}{2}j)} | : \tilde{V}^\mu(\kappa x) : | y_{2(n-\frac{1}{2}j)} \tilde{n} \rangle,$$

$$t \in (t_{2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)}], n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n.$$

And further for consideration of the energy-momentum tensor of the relativistic scalar quantum field just must be obtained the quantum scalar mass field.

If it is supposed that the energy must be positive then the solution of the non local scalar field is restricted on the dark mass hyperboloid i.e. following

$$\varphi(\kappa x) = \int d^4 q q(q^0) \delta(q^2 - m_d^2) \exp[-iq\kappa x] \varphi(q).$$

Moreover $(\partial_{\kappa x}^2 - m_d^2)\varphi(\kappa x) = 0$ is the “matter” wave Klein-Gordon equation and also precisely it is

$$\int d^4 q q(q^0) \delta(q^2 - m_d^2) \exp[-iq\kappa x] (q^2 - m_d^2) \varphi(q_\kappa) = 0.$$

For the positive time also as well by the Casimir effect the dark impulse $m_d c$ in the referent system at the rest by

$$k^0 = \omega/c = \frac{1}{2}(q_\kappa^0 - q_{\kappa'}^0)^{\frac{1}{2}}$$

$$q^0 = (q_\kappa^0 + \omega/c)^{\frac{1}{2}} = (q_{\kappa'}^0 - \omega/c)^{\frac{1}{2}}$$

can be negative or positive for $q^0 \rightarrow \infty$ and $\omega \rightarrow \pm\infty$ too.

$$\varphi(q) = \int d^4 \kappa x \theta(\kappa x^0) \delta((\kappa x)^2 - \kappa^2 x^2) \exp[-iq\kappa x] \varphi(\kappa x),$$

so that

$$(\partial_q^2 - \kappa^2 x^2) \varphi(q) = 0, \quad \kappa^2 x^2 = q^{-2}, \quad \kappa'^2 x^2 = q^{-2}$$

can be considered as a quadrate of the non local radius 4-vector κx^μ or $\kappa' x^\mu$ in Minkowski coordinate space-time which describe the tangential space-time on the anyone point of the hyperboloid in the Lorenz manifold of the “matter” scalar field.

Moreover for the fixed time it can be obtained for the state vector $|\alpha_\kappa\rangle = |q_{a_k}\rangle$ the scale function for the 4-impulse q_{a_k} in the impulse Minkowski space by defined

$$\alpha_\kappa(q_{a_k}) = \int d^4 y_{2(n-\frac{1}{2}j)} \theta(y_{2(n-\frac{1}{2}j)}^0) \delta(y_{2(n-\frac{1}{2}j)}^2 - x t_j^r \tilde{x}) \exp[iq_{a_k} y_{2(n-\frac{1}{2}j)}] \alpha_\kappa(y_{2(n-\frac{1}{2}j)}),$$

so that it is fulfilled the equation

$$(\partial_{q_{a_k}}^2 - x t_j^r \tilde{x}) \alpha_\kappa(q_{a_k}) = 0,$$

$$t \in (t_{2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)}], n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n.$$

Moreover for the Hilbert functional space with indefinite metric it is obtained for the fixed event 4-point $y_{2(n-\frac{1}{2}j)}$ in the Minkowski space-time the one field vector valued state

$$|\varphi_j\rangle = |y_{2(n-\frac{1}{2}j)}\rangle = \int d^4 q_a \theta(q_a^0) \delta(q_a^2 - m_d^2) \exp[-iq_a y_{2(n-\frac{1}{2}j)}] |q_a\rangle$$

and $q_a^2 = y_0^{-2}$ can be considered as a quadrate of the 4-impulse vector in Minkowski impulse space which lay on the mass hyperboloid of the “matter” scalar field.

Furthermore it is possible the Casimir energy to be obtained by the

$$q_a^m = \frac{1}{2}(q_{a_k}^m + q_{a_{k'}}^m) \text{ and } (q_{a_k}^m q_{a_{k,m}})^{\frac{1}{2}} = \omega/c = (q_a^2 + k^3^2 - (q_{a_k}^m q_{a_{k,m}})^{\frac{1}{2}})^{\frac{1}{2}} \\ = (q_a^2 + ((q_{a_k}^m q_{a_{k,m}})^{\frac{1}{2}} - k^2) - (q_{a_k}^m q_{a_{k,m}})^{\frac{1}{2}})^{\frac{1}{2}} = (q_a^2 - k^2)^{\frac{1}{2}}.$$

So also for $\omega = 0$ it follows

$$q_a^2 = m_d^2 \text{ by } c = 1.$$

Furthermore it can be defined the heat 4-vector of the impulse k^m

$$k^m = \frac{1}{2}(q_k^m - q_{k'}^m) = \frac{1}{2}(q_{a_k}^m - q_{a_{k'}}^m) = (\omega/c, \bar{0}_\perp, k^3)$$

and further

$$q_a^\mu = \frac{1}{2}(q_{a_k}^m + q_{a_{k'}}^m) = (q_a^0, \bar{0}) = (E_d/c, \bar{0}_\perp) \text{ and} \\ q_{a_k}^m = (E_d/c + \omega/c, \bar{0}_\perp, k^3), \\ q_{a_{k'}}^m = (E_d/c - \omega/c, \bar{0}_\perp, -k^3).$$

For the non-local Wick’s operator product it can be obtained the non local normal ordered operator product

$$:\varphi(q_\kappa)\varphi(q_{\kappa'}) : = \int d^4 \kappa x d^4 \kappa' x' \exp[-iq_\kappa \kappa x - iq_{\kappa'} \kappa' x'] : \varphi(\kappa x) \varphi(\kappa' x) :.$$

The Wick’s non-local operator of the tensor of energy-momentum $\tilde{T}_{\mu\nu}$ for the quantum relativistic scalar field system can be defined by the non local normal ordered operator product.

$$:\tilde{T}_{\mu\nu}(q_\kappa, q_{\kappa'}) : = \int d^4 \kappa x d^4 \kappa' x' \exp[-iq_\kappa \kappa x - iq_{\kappa'} \kappa' x'] : \tilde{T}_{\mu\nu}(\kappa x, \kappa' x) :.$$

Moreover the non local tensor $T_{\mu\nu}(\kappa x, \kappa' x)$ of energy momentum is obtained by the invariant entities T’s and the localization for the $T_{\mu\nu}(\kappa x, \kappa' x)$ is obtained for κ and κ' tended towards zero also the localizability must be proven for the invariant entities T’s explicit determined the averaged tensor $T_{\mu\nu}$ of energy-momentum by the follows definition

$$T_{\mu\nu}(\kappa x, \kappa' x) = (g_{\mu\alpha} - \kappa_\mu \kappa_\alpha / (\kappa x)^2) (g_{\nu\beta} - \kappa'_\nu \kappa'_\beta / (\kappa' x)^2) (g^{\alpha\beta} T_0 + y_{2(n-\frac{1}{2}j)}^\alpha y_{2(n-\frac{1}{2}j)}^\beta T_1 + \\ y_{-2(n-\frac{1}{2}j)}^\alpha y_{-2(n-\frac{1}{2}j)}^\beta T_2 + \frac{1}{2} (y_{2(n-\frac{1}{2}j)}^\alpha y_{-2(n-\frac{1}{2}j)}^\beta + y_{-2(n-\frac{1}{2}j)}^\alpha y_{2(n-\frac{1}{2}j)}^\beta) T_3), \\ t \in (t_{-2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)}], \\ n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n.$$

Moreover vice versa the localizability condition in the coordinate Minkowski space-time for the energy-momentum tensor will be fulfilled if T’s fulfils the so called analytically conditions and are localized for the vacuum without particles by κ and κ' tended towards zero also the following conditions for the Minkowski space-time radius 4-vector are fulfilled by definition

$$\kappa^\nu x^\nu T_{\mu\nu}(\kappa x, \kappa' x) = \kappa^\mu x^\mu T_{\mu\nu}(\kappa x, \kappa' x) = 0,$$

or in the impulse Minkowski space-time it follows

$$\partial_{q_\kappa}^\mu T_{\mu\nu}(q_\kappa, q_{\kappa'}) = \partial_{q_{\kappa'}}^\nu T_{\mu\nu}(q_\kappa, q_{\kappa'}) = 0.$$

Also it is clear that by averaging of the operators in this case the non local dilation current fulfill the equation

$$D_\mu(\kappa x) = -V_\mu(\kappa x).$$

Then $T_{0\nu}(q_\kappa, q_\kappa)$ are a 4-impulse and $T_{00}(q_\kappa, q_\kappa)$ is the Hamiltonian of the relativistic quantum fields system obtained by the invariant entities T^s .

If for $\bar{y}_\perp \rightarrow 0$ the following identity is in force

$$y_\mu^0 = (|xt_j^r \tilde{x}|, \bar{o}_\perp, (xt_j^r \tilde{x} - y_0^2)^{1/2}),$$

$$|\bar{y}_\perp| \leq |\bar{x}_\perp| = ((\kappa x^0)^2 - x^3^2 - \kappa^2 x^2)^{1/2} = ((ct)^2 - x^3^2)^{1/2}$$

for $\kappa^2 x^2 = 0$ in the local case for $\kappa \rightarrow 0$. Then also for $x^3 \in (0, L)$, $\dot{z} = y^3/t$, $\cos(\bar{x}, \bar{x}) = \cos\theta$. So also there is the causality condition $x^\mu \rightarrow y_\mu^0$ and $y^3 \in (0, L)$, where $L = vt_0$,

$$t \in (t_{-2(n-1/2j)}, t_{2(n-1/2j)}], n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n.$$

Moreover it is possibly to consider a case where the surface S as the kind of the domain of definition of the boundary scale function $f(\bar{x}_\perp, x^3, t) \in {}^1D$ is time independent for

$$t \in (t_{-2(n-1/2j)}, t_{2(n-1/2j)}],$$

$$n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n,$$

$x^3 \in (-\infty, 0]$ and $f(\bar{x}_\perp)_{x^3, ct} = \text{const}$ by fixed x^3, t on the remaining boundary kind of the domain.

Also further it is possibly to consider a so called lexicographic order for the events 4-point

$$y_{2(n-1/2(j+1))}^\mu \text{ and } y_{-2(n-1/2j)}^\mu; y_{2(n-1/2j)}^\mu \text{ and } y_{-2(n-1/2(j-1))}^\mu,$$

$$n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n,$$

and without the changing of y_ν by the reflections and the hyperbolically turns .

Furthermore by means of the following relation and fixed Minkowski space impulse 4-vector $k^\mu = (\omega/c, \bar{k}_\perp, k^3)$ where ω is the Casimir energy characterised by the spectre of the energy by so called "zero fluctuations" and fixed $k^3 = c^{-1}(\omega^2 - \bar{k}_\perp^2 - k^2)^{1/2} = c^{-1}\omega + \text{const}$ for $|\bar{k}_\perp| \rightarrow 0$ and $|\bar{y}_\perp| \rightarrow \infty$ and $\lim[(-k^2)/2\omega] = \text{const} \in [0, 1]$ for $k^2 \rightarrow \infty, \omega \rightarrow -\infty$ for $t_0 \rightarrow -0$ obtained by the calculation of the Casimir energy. Precisely the impulse k^3 is equally of the Casimir energy except of a const. At the first by Casimir energy trended towards the zero $\omega = (k^2 + k^3^2)^{1/2} \rightarrow -0$ also by the calculation of the Casimir energy that is $t_0 \rightarrow -\infty$ and so yet $k^2 \rightarrow -k^3^2$ in the Doctor paper of G. Petrov (1978) by the prove of the causality properties of the form factors by the virtual non forward Compton effect in the deep non forward scattering of the leptons and hadrons for central interacting virtual particles i.e. $|\bar{y}_\perp| \rightarrow \infty, ct_0 \rightarrow \text{fixed}$ and without the idea of the Casimir energy but after all by fixed impulse k^3 .

Actually, the only in this way it is to be possible the extension of the symmetry of the theory to the super symmetry without renouncing to the analyticity of the entities to be proved theoretical of the so called analyticity of the quantum entities as a effect of the analytical representation of the causality properties by fulfilled kinematical relations between the same entities, e.g. for the dark mass m_d and energy E_d , so that $q^2 = m_d^2 c^2$ analogously to the form factors too. So the extra boson super symmetry is an effect of the causality properties of the theory. In the relativistic S-matrix theory it was defined rigorously by the axiomatic way from N.N. Bogolubov, and then the local quantum field theory is analytic since it is causal everywhere except by restriction for the discrete values selected by the fulfilled kinematical relations between the theoretical entities as effect of his causal properties and describing the observed quantities by the experiment too.

Also at the time $t \rightarrow t_{-2(n-1/2j)} + 0$,

$$n \rightarrow \infty, \bar{x}_\perp^2 \rightarrow \infty, x^0 \geq 0,$$

for $|\bar{y}_\perp| \leq |\bar{x}_\perp| \leq R = ((ct)^2 - x^2)^{1/2} = ct + \text{const}$ and $\lim(-x^2/2ct) = \text{const} \in [0, 1]$ for $x^2 \rightarrow -\infty$, $t \rightarrow \infty$, $t \in (t_{2(n-1/2j)}, t_{2(n+1/2j)})$, $n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n$ it can be defined the quadrate for the 4-impulse q^μ in the referent inertial system

$$q^2 = 1/4((\kappa q_k^\mu + \kappa' q_k'^\mu)^2 + (\kappa q_k^\mu - \kappa' q_k'^\mu)^2)$$

and the Casimir vacuum energy (calculated for the relativistic scalar quantum field system by Bordag, Petrov, & Robaschik 1984; Petrov, 1989, where the Casimir energy $\omega \sim t_0^{-3}$) is also

$$\omega/c = 1/2((\kappa q_k^\mu + \kappa' q_k'^\mu)^2 - (\kappa q_k^\mu - \kappa' q_k'^\mu)^2)^{1/2} = (\kappa q_k^\mu \kappa' q_k'^\mu)^{1/2}.$$

Then it is possible to be defined the quadrate of the heat 4-vector k in the inertial referent system by $\bar{y}_\perp^2 \rightarrow \infty$ and $\bar{k}_\perp^2 \rightarrow 0$ and the Casimir vacuum energy is obtained for

$$q_k^\mu = (q^0, \bar{q}_\perp q^3 + k^3) \text{ and}$$

$$q_k'^\mu = (q^0, \bar{q}_\perp q_3 - k_3) \text{ by } \omega/c = (\kappa \kappa' (q^2 + k^3)^2)^{1/2}.$$

So also by definition it can be obtained the heat 4-impulse by the help of the calculated Casimir energy

$$k^\mu = 1/2 (q_k^\mu - q_k'^\mu) = ((q_k^\mu q_k'^\mu)^{1/2},$$

$$\bar{0}_\perp, (q_k^\mu q_k'^\mu - k^2)^{1/2}) = (\omega/(\kappa' \kappa)^{1/2} c,$$

$$\bar{0}_\perp, (\omega/(\kappa' \kappa)^{1/2} c + \text{const})),$$

by the $\lim(-k^2/2(q_k^\mu q_k'^\mu)^{1/2}) = \text{const} \in [0, 1]$, for $k^2 \rightarrow \infty$ and $(q_k^\mu q_k'^\mu)^{1/2} \rightarrow -\infty$.

Furthermore it is

$${}^1\tilde{q}^2 = {}^r\tilde{q}^2 = 0,$$

$$0 < q_k^2 = k^2 q^2, 0 > q_k'^2 = k'^2 q'^2.$$

Moreover it can be defined for fixed $k^3 = m\dot{z}$, and $k^2 \rightarrow \infty$ $k^2 \in (-\infty, \infty)$ the follows for the dark energy $E_d = \omega/(\kappa \kappa')^{1/2}$

$$q^2 = (\omega^2/\kappa \kappa' c^2 - k^3^2) = k^2,$$

so that, from the so called dark matter it follows

$$m_d c^2 = (\omega^2/\kappa \kappa' - c^2 k^3^2)^{1/2} = \omega/(\kappa \kappa')^{1/2} + \text{const for}$$

$\lim(-\kappa \kappa' c^2 k^3^2/2\omega) = \text{const} \in [0, 1]$ by $k^3^2 \rightarrow \infty$ and $\omega \rightarrow -\infty$. So the zero point energy ZPE of the dark matter $m_d c^2$ of the sea virtual particles in the inertial center of mass referent system is the dark energy of the infinity dark matter except for a const.

Moreover by Casimir energy $\omega \rightarrow -\infty$ and $\bar{q}_\perp^2 \rightarrow 0$ for $\bar{x}_\perp^2 \rightarrow \infty$, also $\omega/c = (q^2 + k^3^2)^{1/2}$

and the $\lim(-k^2/2\omega) = \text{const} \in [0, 1]$ for $k^2 \rightarrow \infty$ and $\omega \rightarrow -\infty$ the positive energy is the dark energy E_d of the infinity dark matter except for the vacuum kinetic energy of the vacuum fluctuations obtained by the const $\in [0, 1]$.

For $|\bar{q}_\perp - \bar{u}_\perp|$, and if $\psi(\bar{u}_\perp, k^3, \lambda)$ is then anyone spectral function considered by the possible analyticity representation of the causality conditions for the invariant entities T_0 it can be obtained the spectral representation by energy of the referent inertial system at the

fixed time t of scalar see particles $E_d = cq^0$ for the fixed impulse $q^3 = k^3 = m\dot{z}$ and for $\bar{q}_\perp \neq 0$ and $\bar{q}_\perp^2 \rightarrow 0$ for $\bar{x}_\perp^2 \rightarrow \infty$ but $\omega^2/c^2 - k^2 = k^3^2$ by fulfilling of the corresponding kinematical conditions for the invariant T 's the following spectral representation is possible

$$T_0(q^0, \bar{q}_\perp, k^3) = \int_0^\infty d\lambda^2 \int d\bar{u}_\perp \delta(q^0 - (\bar{q}_\perp - \bar{u}_\perp)^2 - k^3 - \lambda^2),$$

$$\psi(\bar{u}_\perp, k^3, \lambda) = \int_0^\infty d\lambda^2 \int d\bar{u}_\perp \delta(q^0 - \omega^2/c^2 - (\bar{q}_\perp - \bar{u}_\perp)^2 + k^2 - \lambda^2),$$

where $\psi(\bar{y}_\perp, k^3, \lambda) = \int d\bar{u}_\perp \exp(-i \bar{y}_\perp \bar{u}_\perp) \psi(\bar{u}_\perp, k^3, \lambda^2)$ can be considered as a solution of everyone differential equation taken from the potential theory of the cylindrical wave propagation.

Here $k^3 = m\dot{z}$ is the impulse in the longitudinal direction of the scalar see particles (Pterophyllum scalare) with the mass m at the time t and c is the light velocity in vacuum. By the fixed Casimir energy ω and obtained from the masses as effect of the super selections principle by the introduction of the "fermionic" symmetries, i.e. symmetries whose generators are anticommuting objects but neutral and called by us scalarino also furthermore it can be spoken from the super symmetric point of view about a "fermionization" and "bosonization" of relativistic scalar quantum field system.

The arbitrariness of the phase of vector valued one quantum field's functional state obtained by the quantization of the field function $\varphi(x)$ is the usual method to obtain the reel existing interactions taken in account the invariance. The partly ordered events can be introduced by the help of the relations given by $x^\mu > y_{2(n-\nu_{sj})}^\mu$ then and only then if $x^0 > y_{2(n-\nu_{sj})}^0$ and $(x - y_{2(n-j/2)}) \cdot (x - y_{2(n-j/2)}) > 0$, i.e. the event 4-point x^μ is "more latest" than the event 4-point $y_{2(n-j/2)}^\mu$ and the relative vector $(x - y_{2(n-j/2)})^\mu$ is time-like. The transformation in the time-space manifold $\varphi(x): M \rightarrow M$ for them the relations above means $\varphi(x) > \varphi(y_{2(n-j/2)})$ by fixed point $y_{2(n-j/2)}$ and vice versa is called causal automorphism of the space-time with respect to the local coordinate system. The causal automorphisms forms a group for them it is fulfilled the so called Zeman's theorem for the group of the full causal avtomorphisms of the Minkowski space-time the so called half direct product $T^{3,1} \times (\Lambda \uparrow \times D)$ where $\Lambda \uparrow$ is the orthochronic Lorentz transformations and D the dilatations group $x^\mu \rightarrow \kappa x^\mu$, for $x^\mu \in M^4$, κ belong to the multiplicity group of the reel numbers different from zero.

Then this is typical consideration. There is a dynamic equilibrium in which the mass at the rest of the virtual scalar particles at the fixed time stabilizes the so called Higgs boson which has a mass in classes of vacuum ground-state orbit in the Casimir world. It seems that the very stability of matter itself in this case appears to depend on an underlying sea of scalar field energy by the "zero vacuum fluctuations" of the Casimir quantum field state. The Casimir effect has been posited as a force produced solely by interaction of the quantum field ground state in the vacuum with additional causal and boundary conditions. The vacuum fluctuations are fundamentally based upon the interaction of the relativistic quantum fundamental field system with the classical objects, which has been predicted to be "signed into law" someday soon, since so far no violations have been found. This may lead everyone to believe that though it is random, it can no longer be called "spontaneous emission" but instead should properly be labelled "stimulated emission" much like laser light is stimulated emission, even though there is a random quality to it.

If the Hilbert functional space is constructed by the anyone number of the fundamental field function vector valued states $\varphi(y_{2(n-\nu_{sj})})|0\rangle = |\varphi_j\rangle = |y_{2(n-\nu_{sj})}\rangle$ defined in the space-time

as a event 4-point $y_{-2(n-\frac{1}{2}j)}$ for the geometry described by the Minkowsky space-time and the anyone vector valued states obtained in the event x^μ -point with $\mu = 0, 1, 2, 3$ by the relation $\alpha(x)|0\rangle = |\alpha\rangle = |x\rangle$ which is to be considered by the definition as a field operator valued functional α acting on the anyone virtual vector valued state $|\varphi_j\rangle \in H$ where H is the so called Hilbert functional space with indefinite metric and

$$|\alpha(t_{-2(n-\frac{1}{2}j)}, \mathbf{y})\rangle = \tilde{\alpha} \cdot |\varphi_j\rangle = \alpha|\varphi_j\rangle = \int d^4x \delta(x - y_{2(n-\frac{1}{2}j)}) |\varphi_j(y_{2(n-\frac{1}{2}j)})\rangle \alpha^j(x) ,$$

by the summation over the repetitions of the above and down indices.

So also it is possible to be defined by integration over the functional measure $D\alpha$

$$\begin{aligned} |\varphi_j, t\rangle &= \int |\alpha\rangle D\alpha \langle \alpha | \varphi_j, t \rangle \\ &= \int |\alpha\rangle \langle \varphi_j, t | \alpha \rangle D\alpha \\ &= \int |\alpha\rangle \Psi^*_\alpha(\varphi_j, t) D\alpha , \end{aligned}$$

where $\Psi^*_\alpha(\varphi_j, t)$ is the conjugate Schrödinger wave functional fulfilled the impulse wave functional equation. Moreover $\alpha(t, \mathbf{x})$ can be considered as a scale function fulfilled by time condition $|t - t_{-2(n-\frac{1}{2}j)}| \rightarrow \varepsilon$ the equation

$$\partial_t \alpha(t, \mathbf{x}) = \|\varphi(x)\|^2 / (2(\varphi(x)\varphi(y_{-2(n-\frac{1}{2}j)})) + \varphi^2(y_0)) \alpha(t_{-2(n-\frac{1}{2}j)}, \mathbf{x}) - |\alpha(t_{-2(n-\frac{1}{2}j)}, \mathbf{x})\rangle = 0$$

by $t \in (t_{-2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)})$, $n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n$ and $\partial_t = \partial/\partial t$ so also further, for $\bar{x}_\perp \rightarrow \bar{y}_\perp$, $x^3 = y^3_{-2(n-\frac{1}{2}j)}$, the time independent components of the anyone vector valued state by the fixed $t_{-2(n-\frac{1}{2}j)}$

$$|\alpha(t_{-2(n-\frac{1}{2}j)}, \mathbf{x})\rangle = \int dt \delta(t - t_{-2(n-\frac{1}{2}j)}) |\varphi(y_{-2(n-\frac{1}{2}j)})\rangle \alpha^j(t, \mathbf{x}) \in H,$$

where H is knowing as the Hilbert functional space with indefinite metric, are obtained by

$$|\alpha(t_{-2(n-\frac{1}{2}j)}, \mathbf{x})\rangle = \varphi^2(y_0) (\varphi^1(x)\varphi(y_{-2(n-\frac{1}{2}j)})) ((1 + (\|\varphi(x)\|^2 \varphi^2(y_0)) (\varphi(x)\varphi(y_{-2(n-\frac{1}{2}j)}))^2)^{\frac{1}{2}} - 1).$$

Also so it can be defined

$$|\varphi(x)\rangle = |\varphi^1(x)\rangle + |\varphi(y_{-2(n-\frac{1}{2}j)})\rangle |\alpha(t_{-2(n-\frac{1}{2}j)}, \mathbf{x})\rangle \text{ and } |\varphi(x)\rangle^2 = \|\varphi(x)\|^2.$$

Moreover for $\partial_t \alpha(t, \mathbf{x}) \neq 0$ and by definition

$$\pi(x)|0\rangle = \partial_t \varphi(t, \mathbf{x})|0\rangle = |\pi\rangle$$

and further by $|t - t_{-2(n-\frac{1}{2}j)}| \rightarrow \varepsilon$ and

$$\partial_t \tilde{\alpha} |\pi_j\rangle = \sum \partial_t \alpha^j |\pi_j\rangle \text{ for } \partial_t \varphi(y_{-2(n-\frac{1}{2}j)})|0\rangle = |\pi_j\rangle,$$

where over j must be summed by repetition of the above and down indices and the fundamental vector states $|\pi_j\rangle$ create the Hilbert functional state with indefinite metric.

2. Main Result

Let it be given the virtual (potential) vector valued functional one quantum field state in the Hilbert functional space obtained on the coordinate Minkowski space-time. Then the virtual one field state $\varphi_{\alpha_{\kappa}}|0\rangle = |\varphi(\alpha_{\kappa})\rangle$ is obtained by the acting of the scalar field operator $\Phi_{\alpha_{\kappa}}$ on the virtual vacuum vector valued functional state of Hilbert space with indefinite metric obtained over the anyone Banach operator algebra

$$|\varphi(\alpha_{\kappa})\rangle = |y_{-2(n-\frac{1}{2}j)}\rangle.$$

Also it is

$$|\alpha_{\kappa}\rangle = \int d_4k \exp[ky_{-2(n-\frac{1}{2}j)}] \alpha_{\kappa}(k)|0\rangle.$$

Then $|\alpha_{\kappa}\rangle$ is a anyone state vector of the functional Hilbert space with indefinite metric

building by the help of anyone number of the fundamental state vectors

$$\tilde{\Phi}_j(\varphi(y_{-2(n-\frac{1}{2}j)}))|0\rangle = |\varphi_j\rangle = \int d_4k \exp[ky_{-2(n-\frac{1}{2}j)}] \varphi_j(k) |0\rangle,$$

where $d_4k = d^4k/(2\pi)^4$ and $\exp[ky_{-2(n-\frac{1}{2}j)}]$ can be generalized by the time independent function

$$\alpha_{\kappa^j}(\mathbf{x}, t_{-2(n-\frac{1}{2}j)}) = \alpha_{\kappa^j} = \langle \varphi^j | \alpha_{\kappa^j} \rangle.$$

The entities α_{κ^j} are called the counter invariant components of the anyone state vector

$$|\alpha_{\kappa^j}\rangle = \Sigma \int dx \delta(x - y_{-2(n-\frac{1}{2}j)}) \alpha_{\kappa^j}(x) |\varphi_j\rangle.$$

Furthermore the mathematical quantum vacuum functional vector valued state $|0\rangle$ is defined by the Casimir energy of the “vacuum fluctuations” of the so called “zero point energy” ZPE of the virtual Hilbert space vector valued one quantum field state at the fixed time

$$t \in (t_{-2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)}],$$

$$n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n, \text{ for } \bar{q}_\pm \neq 0.$$

Moreover by $\partial_\mu = \partial_{\tilde{x}^\mu} = \partial/\partial(\tilde{x}^\mu)$

$$d\alpha(\tilde{x}) = d\tilde{x}^\mu \partial_\mu \alpha(\tilde{x}) = d\tilde{x}^\mu A_\mu(\tilde{x})$$

$$d^2\alpha(\tilde{x}) = (\partial_\nu \alpha(\tilde{x}) \partial_\mu \alpha(\tilde{x}) - (\partial_\mu \alpha(\tilde{x}) \partial_\nu \alpha(\tilde{x}))) d\tilde{x}^\nu d\tilde{x}^\mu$$

$$= F_{\nu\mu} d\tilde{x}^\nu d\tilde{x}^\mu.$$

Also for $A_\mu(\tilde{x}) = 0 = \partial_\mu \alpha(\tilde{x})$,

$$d\alpha(\tilde{x}) = (\kappa \partial^\mu \varphi(x))^2 (2(\partial_\mu \varphi(y_{2(n-\frac{1}{2}j)})) \partial^\mu \varphi(\kappa x)) + (\partial^\nu \varphi(y_0))^2 \alpha_\tau^{-1} - \alpha_\tau = 0,$$

and it is obtained for

$$\alpha_\tau^2 - 2\alpha_\tau (\partial_\mu \varphi(y_{2(n-\frac{1}{2}j)}) \partial^\mu \varphi(\kappa x)) / (\partial^\nu \varphi(y_0))^2 + (\kappa \partial^\mu \varphi(x))^2 = 0,$$

$$\alpha_\tau = (\partial_\mu \varphi(y_{2(n-\frac{1}{2}j)}) \partial^\mu \varphi(\kappa x)) / (\partial^\nu \varphi(y_0))^2 ((1 - ((\kappa \partial^\mu \varphi(x))^2 (\partial^\nu \varphi(y_0))^2) / (\partial_\mu \varphi(y_{2(n-\frac{1}{2}j)}) \partial^\mu \varphi(\kappa x))^2)^{1/2} - 1).$$

It is also obtained $|\partial^\mu \varphi(\tilde{x}^\mu)\rangle = |\partial^\mu \varphi(\kappa x)\rangle + |\partial^\mu \varphi(y_{2(n-\frac{1}{2}j)})\rangle |\alpha_\tau\rangle$ and also for $|\partial^\mu \varphi(\tilde{x}^\mu)\rangle^2 = 0$ it is $|\partial^\mu \varphi(\kappa x)\rangle^2 = \kappa^2 |\partial^\mu \varphi(x)\rangle^2$.

Also for $|\partial^\mu \varphi(\tilde{x}^\mu)\rangle = |\partial^\mu \varphi(x)\rangle - |\partial^\mu \varphi(y_{2(n-\frac{1}{2}j)})\rangle$ is $|\alpha_\tau\rangle + 1 = 0$ and vice versa. Even it is obtained by

$$|\partial^\mu \varphi^\tau_j\rangle = \partial^\mu \varphi(\tilde{x}^\mu),$$

$$\tilde{a}(\tilde{x}) |0\rangle = |\alpha_\tau\rangle = \Sigma a_\tau^j |\partial^\mu \varphi^\tau_j\rangle,$$

where the vector valued functional state $|\alpha_\tau\rangle$ is anyone state of the Hilbert functional space with indefinite metric moreover α_τ^j are his counter invariants components and the non local functional state $|\partial^\mu \varphi^\tau_j\rangle$ create this space.

Further the non local symmetries vacuum averaged Wicks product can be obtained for the locality condition by $\kappa \rightarrow 0$

$$\text{Sym}(|\partial_\mu \varphi(\kappa x)\rangle |\partial_\nu \varphi(y_{2(n-\frac{1}{2}j)})\rangle) = ((g_{\mu\nu} - \kappa x_\mu \kappa x_\nu / (\kappa x)^2) (-i/4\pi^2) (\kappa x^\mu - y_{2(n-\frac{1}{2}j)}^\mu)^2)$$

so that for the local case when $\kappa \rightarrow 0$ follows the singularity by

$$\kappa x^\mu \rightarrow \tilde{x}^\mu = \kappa x^\mu - y_{2(n-\frac{1}{2}j)}^\mu \text{ and } 0 = (\kappa x^\mu - y_{2(n-\frac{1}{2}j)}^\mu)^2, \\ (\kappa x)^2 = \kappa^2 x^2 = 0.$$

Further although by fulfilling the causality condition follows

$$\kappa x^\mu \text{Sym} (\partial_\mu \varphi(\kappa x) \partial_\nu \varphi(y_{2(n-1/2)})) = 0.$$

Moreover the virtual vector valued functional quantum one field state $|\varphi_j\rangle$ with invariant field components φ_j is obtained by the vacuum Ψ -functional $\Psi_{\alpha_{\kappa'}}^*(\varphi_{\alpha_{\kappa'}}, t)$ defined over the Banach algebra of the quantum field operators obtained for the following relation

$$\begin{aligned} \varphi(y_{-2(n-1/2)}) &= |y_{-2(n-1/2)}^\mu\rangle = |\varphi_{\alpha_{\kappa'}}\rangle \\ &= \int |\alpha_{\kappa'}\rangle D\alpha_{\kappa'} \langle \alpha_{\kappa'}, t | y_{-2(n-1/2)}^\mu \rangle \\ &= \int |\alpha_{\kappa'}\rangle \langle \varphi_{\alpha_{\kappa'}} | \alpha_{\kappa'}, t \rangle D\alpha_{\kappa'} \\ &= \int |\alpha_{\kappa'}\rangle \Psi_{\alpha_{\kappa'}}^*(\varphi_{\alpha_{\kappa'}}, t) D\alpha_{\kappa'}, \end{aligned}$$

where

$$0 \leq \kappa' \leq \kappa \leq 1, \\ \alpha_{\kappa'} = \alpha_{\kappa'}(\bar{y}_\perp)_{y^3_{-2(n-1/2)}}^{ct_{-2(n-1/2)}},$$

for the functional integral measure

$$D\alpha_{\kappa'} = \prod [d^2\bar{y}_\perp],$$

So $\Psi_{\alpha_{\kappa'}}^*(\varphi_{\alpha_{\kappa'}}, t) = \langle \varphi_{\alpha_{\kappa'}} | \alpha_{\kappa'}, t \rangle$ and also it is $\Psi_{\alpha_{\kappa'}}(\alpha_{\kappa'}, t) = \langle \alpha_{\kappa'}, t | \varphi_{\alpha_{\kappa'}} \rangle$ so that from

$$\begin{aligned} |\alpha_{\kappa'}, t\rangle &= \int |\varphi_{\alpha_{\kappa'}}\rangle D\varphi_{\alpha_{\kappa'}} \langle \varphi_{\alpha_{\kappa'}} | \alpha_{\kappa'}, t \rangle \\ &= \int |\varphi_{\alpha_{\kappa'}}\rangle \langle \alpha_{\kappa'}, t | \varphi_{\alpha_{\kappa'}}\rangle D\varphi_{\alpha_{\kappa'}} \\ &= \int |\varphi_{\alpha_{\kappa'}}\rangle \Psi_{\varphi_{\alpha_{\kappa'}}}^*(\alpha_{\kappa'}, t) D\varphi_{\alpha_{\kappa'}}, \end{aligned}$$

where $\varphi_{\alpha_{\kappa'}}$ are the so called counter invariant components of the anyone vector valued states $|\alpha_{\kappa'}^j\rangle \varphi_j = |\varphi\rangle$. The fundamental vector states $|\varphi_j\rangle$ creates the Hilbert functional space with indefinite metric. Also it can be obtained the vector valued state by $|\varphi\rangle = |\alpha_{\kappa'}^j\rangle \varphi_j$ with his invariants components φ_j . Moreover the $\alpha_{\kappa'}^j$ are used as the time independent scale functions for $\bar{x}_\perp \rightarrow \bar{y}_\perp$ and fixed

$$\kappa' x^3 = y^3_{-2(n-1/2)}, \quad \kappa' x^0 = y^0_{-2(n-1/2)}.$$

It is to be remarked that over the repetitions of the above and down indices is to be summed.

Furthermore for a time $t \in (t_{-2(n-1/2)}, t_{2(n-1/2)})$ there are in force the relations $\varphi(\kappa' x) = \varphi(\alpha_{\kappa'})$, $\pi(\kappa' x) = \pi(\partial_{ct}\alpha_{\kappa'})$, and for $\kappa' \rightarrow 0$ $\varphi(\kappa' x) = \varphi(\tau_j^1 \tilde{x})$ for the time independent scale functions states

$$|\alpha_{\kappa'}\rangle = \alpha_{\kappa'}(\bar{y}_\perp)_{y^3_{-2(n-1/2)}}^{ct_{-2(n-1/2)}} = f_{\kappa'}^1(\bar{y}_\perp) = 0,$$

and $\pi(\kappa' x) = \pi(\tau_j^1 \tilde{x})$ for the time independent scale functions

$$|\partial_{ct}\alpha_{\kappa'}\rangle = \partial_{ct}\alpha_{\kappa'}(\bar{y}_\perp)_{y^3_{-2(n-1/2)}}^{ct_{-2(n-1/2)}} = f_{\kappa'}^2(\bar{y}_\perp) = 0,$$

and $\bar{f}_{\kappa'}(\bar{y}_\perp) = (f_{\kappa'}^1(\bar{x}_\perp), f_{\kappa'}^2(\bar{x}_\perp)) = 0$ by the additional causality properties and boundary condition for

$$t = t_{-2(n-1/2)}, \\ \kappa' x^3 \in (-\infty, y^3_{-2(n-1/2)}) U (y^3_{-2(n-1/2)}, 0],$$

$$\bar{y}_\perp = (y^1, y^2) \in \delta\Omega_t = S,$$

and moreover for $\bar{y}_\perp \in \Omega_t$, $\dot{\mathbf{R}}^2$ follows $\bar{\partial}_\perp \mathbf{f}_\kappa(\bar{y}_\perp) = 0$ too, and $\bar{\partial}_\perp = (\partial_{y^1}, \partial_{y^2})$.

Moreover if on the boundary surface S for $\kappa^0 x^0 = ct$, and the fulfilled additional causality condition

$(y^{0^2} - y^{3^2})^{1/2} > |\bar{y}_\perp|$ can be supposed $\alpha_{\kappa'}(\bar{y}_\perp, \kappa^0 x^0, ct) = \text{const}$ or $\pi(\partial_{ct} \alpha_{\kappa'}) = \partial_{ct} \varphi(\kappa^0 x) = \delta_{\alpha_{\kappa'}} \varphi(\alpha_{\kappa'}) \partial_{ct} \alpha_{\kappa'}(\bar{y}_\perp, \kappa^0 x^0, ct) = 0$ so that

$$\partial_{ct} \alpha_{\kappa'}(\bar{y}_\perp, \kappa^0 x^0, ct) = 0, \quad (1)$$

and the same follows for $\partial_{ct} \alpha_{\kappa'}(\bar{y}_\perp, \kappa^0 x^0, ct) = \text{const}$ or $\pi(\kappa^0 x) = \text{const}$, so that for $\kappa^0 x^0 = x^3$,

$\partial_{ct} \pi(\kappa^0 x) = \delta_{\partial_{ct} \alpha_{\kappa'}} \pi(\partial_{ct} \alpha_{\kappa'}) \partial_{ct}^2 \alpha_{\kappa'}(\bar{x}_\perp, x^3, ct) = 0$, so that

$$\partial_{ct}^2 \alpha_{\kappa'}(\bar{x}_\perp, \kappa^0 x^0, ct) = 0. \quad (2)$$

Moreover by definition from $\partial_{ct} \varphi(\kappa^0 x) = \pi(\kappa^0 x)$ and $\partial_{ct} \pi(\kappa^0 x) - \Delta \varphi(\kappa^0 x) = 0$, i.e. $\Delta \varphi(\kappa^0 x) = 0$ by $\bar{\partial}_\perp^2 + \partial_{x^3}^2 = \Delta$ is fulfilled too.

Yet

$$\begin{aligned} \partial_{ct} \alpha_{\kappa'}^+(\bar{y}_\perp, y^{3-2(n-1/2j)}, ct_{-2(n-1/2j)}) &= \partial_{ct} \alpha_{\kappa'}(\bar{y}_\perp, y^{3-2(n-1/2(j+1))}, ct_{-2(n-1/2(j+1))}) \\ &+ v \partial_{x^3} \alpha_{\kappa'}(\bar{y}_\perp, y^{3-2(n-1/2(j+1))}, ct_{-2(n-1/2(j+1))}). \end{aligned}$$

Furthermore from the following equation for the non free simple connected vacuum surface of the relativistic quantum fields system given above and from the fulfilled Equation (1) and Equation (2) follows the following equation by definition

$$\partial_{ct} \alpha_{\kappa'}(\bar{y}_\perp, \kappa^0 x^0, ct) = \kappa^2 \|\varphi(x)\|^2 / (2(\varphi(y_{-2(n-1/2j)}))\varphi(\tilde{tx})) + \varphi^2(y_0) \alpha_{\kappa'} - \alpha_{\kappa'} = 0$$

and

$$\partial_{ct}^2 \alpha_{\kappa'}(\bar{y}_\perp, \kappa^0 x^0, ct) = \kappa^2 \|\pi(x)\|^2 / (2(\pi(y_{-2(n-1/2j)}))\pi(\tilde{tx})) + \pi^2(y_0) \partial_{ct} \alpha_{\kappa'} - \partial_{ct} \alpha_{\kappa'} = 0, \quad (3)$$

$$\bar{y}_\perp \in \mathbf{R}^2 \text{ and } (\tilde{tx})^2 = 0, (\kappa^0 x)^2 = \kappa^2 x^2, (\kappa x)^2 = \kappa^2 x^2, y_{-2(n-1/2j)}^2 = y_{2(n-1/2(j+1))}^2 = y_0^2,$$

and

$$\varphi^2(y_{-2(n-1/2j)}) = \varphi^2(y_0), \pi^2(y_{-2(n-1/2j)}) = \pi^2(y_0).$$

Then also from Equation (1) can be obtained

$$\begin{aligned} f_{\kappa'}^1(\bar{y}_\perp) &= \alpha_{\kappa'}(\bar{y}_\perp) y^{3-2(n-1/2(j-\kappa))}, ct_{-2(n-1/2(j-\kappa))} \\ &= (\varphi(y_{-2(n-1/2j)})\varphi(\tilde{tx}))\varphi^2(y_0)((1+(\kappa^2\|\varphi(x)\|^2)\varphi^2(y_0)) \\ &\quad (\varphi(y_{-2(n-1/2j)})\varphi(\tilde{tx}))^{-2})^{1/2} - 1), \end{aligned} \quad (4)$$

and from Equation (2)

$$\begin{aligned} f_{\kappa'}^2(\bar{x}_\perp) &= \partial_{ct} \alpha_{\kappa'}(\bar{y}_\perp) y^{3-2(n-1/2(j-\kappa))}, ct_{-2(n-1/2(j-\kappa))} \\ &= (\pi(y_{-2(n-1/2j)})\pi(\tilde{tx}))\pi(y_0)^2((1+(\kappa^2\|\pi(x)\|^2)\pi^2(y_0)) \end{aligned}$$

$$\cdot (\pi(y_{-2(n-1/2)})\pi(\tilde{\tau x}))^{-2/2} - 1). \tag{5}$$

$$\begin{aligned} \partial_{ct}\alpha(\tilde{\tau x}) &= \|\varphi(x)\|^2 / (2(\varphi(y_{-2(n-1/2)}))\varphi(x)) - \varphi^2(y_0)\alpha_\tau^j - \alpha_\tau^j, \\ \alpha_\tau^j &= \varphi^2(y_0)(\varphi(x)\varphi(y_{2(n-1/2)}))((1 - \|\varphi(x)\|^2\varphi^2(y_0))(\varphi(x)\varphi(y_{2(n-1/2)}))^{-2/2} - 1) \end{aligned}$$

and for

$$\begin{aligned} \partial_{ct}^2\alpha(\tau x) &= \|\pi(x)\|^2 / (2(\pi(y_{-2(n-1/2)}))\pi(x)) + \pi^2(y_0)\partial_{ct}\alpha_\tau^j - \partial_{ct}\alpha_\tau^j = 0, \\ \partial_{ct}\alpha_\tau^j &= \pi^2(y_0)(\pi(y_{-2(n-1/2)})\pi(x))((1 - (\|\pi(x)\|^2\pi^2(y_0))(\pi(x)\pi(y_{-2(n-1/2)}))^{-2/2} - 1)). \end{aligned}$$

Moreover the function $\mathbf{f}_\kappa(\bar{y}_\perp)$ is taken from potential theory by $\bar{x}_\perp \rightarrow \bar{y}_\perp$ and from

$$\partial_{ct}\alpha_\tau(\bar{y}_\perp, \kappa^2 x^3, ct) = \kappa^2 \|\varphi(x)\|^2 / (2(\varphi(y_{-2(n-1/2)}))\varphi(\tau x)) + \varphi^2(y_0)\alpha_{\kappa^2}^j - \alpha_{\kappa^2}^j = 0$$

and

$$\partial_{ct}^2\alpha_{\kappa^2}(\bar{y}_\perp, \kappa^2 x^3, ct) = \kappa^2 \|\pi(x)\|^2 / (2(\pi(y_{-2(n-1/2)}))\pi(\tau x)) + \pi^2(y_0)\partial_{ct}\alpha_{\kappa^2}^j - \partial_{ct}\alpha_{\kappa^2}^j = 0,$$

as a solution of the equation following

$$\bar{\partial}_\perp^2 \mathbf{f}_\kappa(\bar{y}_\perp) + \lambda_\kappa \mathbf{f}_\kappa(\bar{y}_\perp) = \bar{\partial}_\perp \varphi(\kappa^2 x)_{y^3} \varphi_{-2(n-1/2(j-\kappa^2))}^{ct} \varphi_{-2(n-1/2(j-\kappa^2))} \bar{y}_\perp \in \Omega_t, \tag{6}$$

$$\bar{\partial}_\perp \mathbf{f}_\kappa(\bar{y}_\perp) = 0, \bar{y}_\perp \in \Omega_t, \tag{7}$$

$$\mathbf{f}_\kappa(\bar{y}_\perp) = 0, \bar{y}_\perp \in \delta\Omega_t = S, \tag{8}$$

for additional causal condition for

$$|\bar{x}_\perp| \rightarrow |\bar{y}_\perp| \text{ and } |\bar{y}_\perp| < (y^0 - y^3)^{1/2}, \tag{9}$$

where

$$\varphi(\kappa^2 x)_{y^3} \varphi_{-2(n-1/2)}^{ct} \varphi_{-2(n-1/2)}$$

is anyone non local scalar field function with the norm

$$\kappa^2 \|\varphi(x)\| = |\varphi(\kappa^2 x)_{y^3} \varphi_{-2(n-1/2)}^{ct} \varphi_{-2(n-1/2)}|$$

fulfilled the given additional causal and boundary condition for fixed

$$\kappa^2 x^0 = ct = y^0_{-2(n-1/2)},$$

$$\kappa^2 x^3 = x^3 = y^3_{-2(n-1/2)},$$

so that the norm $\|\bar{\partial}_\perp \mathbf{f}_\kappa\|$ is given by the double product

$$\|\bar{\partial}_\perp \mathbf{f}_\kappa\|^2 = (\bar{\partial}_\perp \mathbf{f}_\kappa(\bar{y}_\perp), \bar{\partial}_\perp \mathbf{f}_\kappa(\bar{y}_\perp)), \tag{10}$$

and for the minimum of the norm $\|\mathbf{f}_\kappa\|$ is the minimal value of $\lambda_\kappa = \lambda_1$ by the fulfilling of the additional causality and boundary conditions (7) and (8) and by $\|\mathbf{f}_\kappa\| = 1$ where $\|\mathbf{f}_\kappa\|$ is the norm defined by the help of the equation

$$(\mathbf{f}_\kappa, \mathbf{g}_\kappa) = \int (\mathbf{f}'_\kappa, \mathbf{g}'_\kappa) d\bar{y}_\perp, \tag{11}$$

$$\Omega_{t,x^3}$$

included the double product

$$(\bar{\partial}_\perp \mathbf{f}_\kappa, \bar{\partial}_\perp \mathbf{g}_\kappa) = \int (\bar{\partial}_\perp \mathbf{f}'_\kappa, \bar{\partial}_\perp \mathbf{g}'_\kappa) d\bar{y}_\perp,$$

$$\Omega_{t,x^3}$$

obtained by the definition

$$(\bar{\partial}_\perp \mathbf{f}_{\kappa'} : \bar{\partial}_\perp \mathbf{g}_{\kappa'}) = \sum_{k,j=1}^2 (\partial_j \mathbf{f}_{\kappa'k})(\partial_j \mathbf{g}_{\kappa'k})$$

and $\mathbf{f}_{\kappa'}$ is orthogonal transposed of \mathbf{f}_{κ} .

Then it can be defined by $|\varphi(\tau x)| = |\varphi(x)| = 0$ and $\varphi^2 = \varphi^2(y_{-2(n-1/2j)}) = \varphi^2(y_0) \neq 0$, $\kappa'^2 |\varphi(x)|^2 = \varphi^2(\kappa'x)$ or by the Banach impulse scalar field for $\|\pi(\tau x)\| = 0$,

$$\begin{aligned} \pi^2(y_{-2(n-1/2j)}) &= \pi^2(y_0) \neq 0, \pi^2(\kappa'x) = \kappa'^2 \|\pi(x)\|^2, \\ \varphi(\alpha_{\kappa'}^j) &= \varphi(\kappa'x) = \varphi(\tau x) + \varphi_j(y_{-2(n-1/2j)}) \mathbf{f}_{\kappa'}^j, \end{aligned} \tag{12}$$

$$\text{or} \quad \pi(\partial_{ct} \alpha_{\kappa'}^j) = \pi(\kappa'x) = \pi(\tau x) + \pi_j(y_{-2(n-1/2j)}) \mathbf{f}_{\kappa'}^j, \tag{13}$$

with following

$$|\varphi(\kappa'x)| = \kappa' \|\varphi(\kappa'x)\| \text{ or } |\pi(\kappa'x)| = \kappa' \|\pi(\kappa'x)\|,$$

where $\|\varphi\|$ and $\|\pi\|$ are norms of the real closed Schwarz space also following from $S_R(\mathbf{M}) = S^+(\mathbf{M}) + S(\mathbf{M})$ obtained by the reduction from Equation (3) following from the fixing of the coordinates by Equation (2) for odd or even functions depending by the fixed coordinate variable x^0, x^3 and defined scalar product

$$(f_{\kappa'}^1, f_{\kappa'}^1)_{\tilde{L}} = (\alpha_{\kappa'}, \alpha_{\kappa'})_{\varphi} \text{ for } f_{\kappa'}^1, f_{\kappa'}^1 \in \tilde{L}^+ \text{ or } f_{\kappa'}^2, f_{\kappa'}^2 \in \tilde{L}^-$$

and extended by an isometric image $\tilde{L}^+(\mathbf{M}) \rightarrow L_{\varphi}(\mathbf{R}^2) = S_R(\mathbf{R}^2)^{\|\varphi\|}$ and $\tilde{L}^-(\mathbf{M}) \rightarrow L_{\pi}(\mathbf{R}^2) = S_R(\mathbf{R}^2)^{\|\pi\|}$ for L_{φ}, L_{π} from the Sobolev's spaces with fractional numbers of the indices.

Further if by fixed variables

$$f_{\kappa'}^2(\bar{y}_\perp) = \partial_{ct} \alpha_{\kappa'}^j(y_\perp)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}} = \partial_{ct} \alpha_{\kappa'}^j(\bar{y}_\perp) = 0$$

would hold for the additional causality and boundary conditions for $\bar{y}_\perp \in \delta\Omega_t = S$ at the right, and by defined

$$d_t(\cdot) = \partial_t(\cdot) + \dot{z} \partial_x^3(\cdot) \text{ and } \dot{z} = \partial y^3, \tag{14}$$

on free surface S placed in Minkowski space-time for

$$\begin{aligned} ct &= \kappa' x^0 = ct_{2(n-1/2(j+1))}, \\ x^3 &= \kappa' x^3 = y^3_{2(n-1/2(j+1))}, \\ \partial y^3 &= \partial y^3_{2(n-1/2(j+1))}, \end{aligned}$$

follow the impulse equations for fulfilled additional causality and boundary condition $y_\perp \in \delta\Omega_t = S$ on the fixed surface S by

$$\dot{z} \partial_x^3 \alpha_{\kappa'}(\bar{y}_\perp)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}} = 0. \tag{15}$$

Also by the definition it is in force the equation

$$d_{ct} \alpha_{\kappa'}(\bar{y}_\perp)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}} = d_{ct} \alpha_{\kappa'}(\bar{y}_\perp)_{y^3_{2(n-1/2(j+1))}, ct_{2(n-1/2(j+1))}}$$

and obtained by the definition for the time $t = t_{-2(n-1/2j)}$

$$\begin{aligned} f_{\kappa'}^{2+}(\bar{y}_\perp) &= \partial_{ct} a_{\kappa'}^+(\bar{y}_\perp)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}} \\ &= f_{\kappa'}^2(\bar{y}_\perp) + \dot{z} \partial_x^3 \alpha_{\kappa'}(\bar{y}_\perp)_{y^3_{2(n-1/2(j+1))}, ct_{2(n-1/2(j+1))}}. \end{aligned} \tag{16}$$

It is assumed the local relativistic quantum scalar wave field system under consideration to have additional causality and boundary conditions on the generic

surface S for his ground state. In this case the so called Casimir vacuum, fixed or moved with a constant velocity v parallel towards the fixed one boundary, which do surgery, bifurcate and separate the singularity by virtual particles of the relativistic quantum system in the Minkowski manifold of the event points from some others vacuum state as by Casimir effect of the quantum vacuum states for the relativistic quantum fields f . That has the property that any virtual quantum particle which is once on the generic surface S remains on it and fulfilled every one additional causality and boundary conditions on this local relativistic scalar quantum system with a vacuum state, described by the one field operator valued functional $A(f)$ for the local test function $f \in \tilde{L}^+$ or $f \in \tilde{L}^-$. Then the solution of the Klein-Gordon wave equation is obtained by covariant statement

$$\begin{aligned} \square f(x^\mu) &= (\partial_{ct}^2 - (\Delta + m^2))f(x^\mu) \\ &= (\partial_{ct}^2 - (\bar{\partial}_\perp^2 + \partial_z^2 + m^2))f(x^\mu) \\ &= 0, \end{aligned} \tag{17}$$

where \square is a d’Lembertian and

$$\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2 = \bar{\partial}_\perp^2 + \partial_z^2$$

is a Laplacian differential operator by additional causal properties and boundary conditions.

So also

$$f(\bar{y}_\perp, x^3, ct)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}} = f^1_{\kappa}(\bar{y}_\perp) = a_\kappa(\bar{y}_\perp)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}}$$

and

$$\partial_{ct} f(\bar{y}_\perp, x^3, ct)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}} = f^2_{\kappa}(\bar{y}_\perp) = \partial_{ct} a_\kappa(\bar{y}_\perp)_{y^3_{-2(n-1/2j)}, ct_{-2(n-1/2j)}}$$

by the causality condition

$$|\bar{y}_\perp| < (y^0^2 - y^3^2)^{1/2},$$

where $t \in (t_{-2(n-1/2j)}, t_{2(n-1/2j)})$, $n = 0, 1, 2, \dots; j = 0, 1, 2, \dots, 2n$.

Also the ground states of the local relativistic quantum fields system defined in the Minkowski space-time fulfilled every one additional causal and boundary conditions interact at the large distance with the boundary surface S by the help of the non local fundamental virtual quantum particles and so the vacuum state has a globally features, e.g. the Casimir force calculated from the Casimir energy of the vacuum “zero fluctuations”¹ and the Minkowski space-time described the geometry induced on the every tangential space on the anyone Lorenz manifold that create the globally Lorenz geometry.

Examples of such boundary surfaces S with additional causality properties by a kind of the boundary of importance for the living cells are those in which the surface of a fixed mirror at the initial time $t = 0$ by the referent inertial system at the rest (a map) and is in contact with the local quantum relativistic scalar fields system with additional causality properties in his simple connected vacuum region—the bottom of the sea of the virtual

¹ Actually, this property is a consequence of the basic assumption by relativistic local quantum wave field theory that the wave front of the relativistic local quantum wave field system by his ground state propagate on the light hyper plane in any contact space (also called “dispersions relations”) and can be described mathematically as a non local virtual topological deformation or fluctuation which depends continuously on the time t .

(potential) non local scalar quantum field particles, for example—and the generic free surface of the parallel moved mirror with a constant velocity v towards the fixed one or the free vacuum surface of the local quantum scalar particles of the wave field in contact with the moved mirror parallel towards the fixed one—the free and localizable vacuum region, described in the non local case by the conjugate impulse Schrödinger wave functional given by vector valued states $|\varphi_j\rangle$ at a given time t created Hilbert functional space with indefinite metric. It is

$$\Psi_{\alpha_\kappa}^{\varphi_j}(\varphi_j, t) = \langle \varphi_j | \alpha_\kappa, t \rangle$$

and

$$\Psi_{\varphi}(\alpha_\kappa, t) = \langle \alpha_\kappa | \varphi, t \rangle$$

where $\alpha_\kappa \rangle = |\varphi_j\rangle \alpha_\kappa^j$ and $|\varphi\rangle = |\alpha_\kappa^j\rangle \varphi_j$ summed up by repetition of the above and down indices and

$$t \in [t_{-2(n - \frac{1}{2}(j-\kappa))}, t_{2(n - \frac{1}{2}(j+\kappa))}], n = 0, 1, 2, \dots; j = 0, 1, 2, \dots, 2n,$$

with additional causality condition

$$|\bar{y}_\perp| < (y^0{}^2 - y^3{}^2)^{1/2}.$$

Moreover by given fulfilled operators equation in the case of the canonical Hamiltonian local relativistic quantum scalar field system in a statement of a equally times

$$x^0 = ct = \kappa x^0 = ct_{2(n - \frac{1}{2}j)} = ct_{2(n - \frac{1}{2}(j+1))} = (\kappa^2 x^2 + \bar{y}_\perp^2 + (y^{3(2(n - \frac{1}{2}(j+1)))})^2)^{1/2}$$

by the definition of the canonical quantum local field $\varphi(x)$ and impulse $\pi(x)$, $x^\mu \in \mathbf{M}^4$, $\mu = 0, 1, 2, 3$, corresponding to implicit operator valued covariant field tempered functional $A(f)$, where $f \in S_R(\mathbf{M}^4)$ is a boundary test function of this reel Swartz space, fulfilled all Whitman axioms and acting in the functional Hilbert space \hat{H}_F with a Fok's space's construction, e.g. a direct sum of symmetries tensor power of one relativistic quantum field's Hilbert space \hat{H}_1 with indefinite metric

$$\hat{H} = \hat{H}_F(\hat{H}_1) = \bigoplus_{n=0}^{\infty} \text{sym } \hat{H}_1^{\otimes n}$$

can be given by the formulas,

$$\begin{aligned} \varphi(\alpha) &= A(f^1) = \int K(x)\alpha(x)d^4x, \\ \pi(\partial_{ct}\alpha) &= A(f^2) = \int \pi(x)\partial_{ct}\alpha(x)d^4x, \end{aligned}$$

by fulfilled Klein-Gordon equation in a covariate statement for massive and massless scalar fields

$$\begin{aligned} K_m A(f) &= A(K_m f) = \int K_m \varphi(x)\alpha(x)d^4x = -\int \varphi(x)K_m \alpha(x)d^4x = 0 \\ KA(f) &= A(Kf) = \int K\varphi(x)\alpha(x)d^4x = -\int \varphi(x)K\alpha(x)d^4x = 0, \end{aligned} \tag{18}$$

with

$$\begin{aligned} K_m &= (\square + m^2) = -\partial_{ct}^2 + (\Delta + m^2), \\ K &= \square = -\partial_{ct}^2 + \Delta, \Delta = \partial_x^2 + \partial_y^2 + \partial_z^2, \end{aligned} \tag{19}$$

Furthermore from the operator's equations for the local quantum fields system represented by the tempered distributions follows the impulse equation of the impulse operator's equation for the free vacuum state surface of the non local quantum scalar field system at the left of the free surface S placed in Minkowski space-time

$$\pi^+(\partial_{ct}\alpha(\bar{y}_\perp y^3_{2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)})) = A(\partial_{ct}f(\bar{y}_\perp)) = \pi(\partial_{ct}\alpha_\kappa(\bar{y}_\perp) y^3_{2(n-\frac{1}{2}(j+1)}, t_{2(n-\frac{1}{2}(j+1))})} + \tilde{z}\partial_{x^3}\varphi(\alpha_\kappa(\bar{y}_\perp) y^3_{2(n-\frac{1}{2}(j+1)}, t_{2(n-\frac{1}{2}(j+1))})}) \tag{20}$$

can be given the impulse Schrödinger equation for the quantum scalar field vacuum functional $\Psi_\varphi(\alpha, t)$ by a given generic surface S .

By the definition the canonical non local field $\varphi(\kappa x)$ and impulse $\pi(\kappa x)$, $\kappa x^\mu \in M^4$, $\mu = 0, 1, 2, 3$, corresponding to implicit operator valued covariant field tempered functional $A(\mathbf{f}_\kappa)$, where

$$\mathbf{f}_\kappa(\bar{y}_\perp) = (f^1_\kappa(\bar{y}_\perp, f^2_\kappa(\cdot y_\perp)) \in S_R(\mathbf{M}^4)$$

is a boundary test function of this reel Swartz space defined in Minkowski space, fulfilled all Whitman axioms by fulfilled time product of the interacting field's operator in the functional Hilbert space \hat{H} with indefinite metric and a Fok's space's construction, e.g. a direct sum of symmetries tensor power by one quantum scalar fields space \hat{H}_1 :

$$\hat{H} = \hat{H}_F(\hat{H}_1) = \bigoplus_{n=0}^{\infty} \text{sym } \hat{H}_1^{\times n} \tag{21}$$

and can be given for physical representation of the relativistic scalar quantum field system by the formulas,

$$\begin{aligned} \varphi(\alpha_\kappa) &= A(f^1_\kappa) = \int \varphi(\kappa x) \alpha(\kappa x) d^4 \kappa x, \tag{22} \\ \pi(\partial_{ct}\alpha_\kappa) &= A(f^2_\kappa) = \int \pi(\kappa x) \partial_{ct}\alpha(\kappa x) d^4 \kappa x, \tag{23} \\ \partial_\mu \varphi(\tau x) (\partial_{ct}\alpha_\kappa) &= \int \partial_\mu \varphi(\tau x) \partial_{ct}\alpha(\tau x) d^4 \tau x \end{aligned}$$

by fulfilled Klein-Gordon equation with a current operator sources in a covariant statement for massive and massless scalar fields

$$K_m A(f_\kappa) = A(K_m f_\kappa) = A(J), \tag{24}$$

$$K A(f) = A(K f) = 0. \tag{25}$$

Furthermore the matrix elements of the current $\langle \varphi | J(\tilde{q}_\tau) | 0 \rangle$, i.e. in the local case for the Wick product of the field operators

$$\langle 0 | K_m A(\varphi f) | 0 \rangle = \delta(x - y_{2(n-\frac{1}{2}j)}) = \langle \varphi | A(J) | 0 \rangle, \tag{26}$$

have a singularities at $\tilde{q}_\tau^2 = 0$ which can be interpreted as a presence of the massless scalars Goldstones bosons in the ground state of the relativistic scalar quantum field system in the Hilbert space \hat{H} with indefinite metric. Also from dies point of view when we have a zero temperature too the “Einstein condensation” in a ground state has on the light cone a δ -function behaviour in the impulse Minkowski space as by the ideal gas in the vacuum and by the Casimir world go over state more realistic with a interacting quantum vacuum state. But this resemblance is only formal and by going over the physical representation the scalar massless Goldstones bosons disappears. This is one of the indications of the Higgs-mechanisms, e.g. effect of the mass preservation from the vector fields by spontaneous broken gauge group (or the scalar Goldstones bosons are “swallowing up”) and so it is to show, that the Casimir force is to be obtained by quantum electromagnetic field system with a massless real photon and asymmetrical Casimir vacuum state where the scaling behaviour by the fermions as a fundamental quantum particles or by the Higgs massive boson as a

fundamental scalar quantum particle in the Standard Model with the generic boundary conditions S is broken. That is the cause to be observed a massive scalar particles following the theorem of Goldstone and the Higgs mechanism.

For simplicity here we have considered a domain of space-time containing any one massless scalar field $j(x)$ defined at the point of the Minkowski space-time at the fixed time t . Further a concrete massless field $\varphi(y^0, \bar{y})$ is considered as a Hilbert valued vector state obeying the impulse wave equation in a Hilbert space, defined over the space $\Omega_t \dot{=} M^4$ at the time $x^0 = ct = y_0^0$ and $x^3 = y_0^3$ in the Minkowski space-time M^4 . By imposing suitable boundary conditions for any one quantum field system considered as any one relativistic quantum field $j(x)$ fulfilled the Klein-Gordon equation, the total fields energy in any domain at the point (ct, \bar{x}) from the Minkowski space-time can be written as a sum of the energy of the “vacuum fluctuations” for

$$t \in (t_{-2(n-\frac{1}{2}j)}, t_{2(n-\frac{1}{2}j)}], \quad n = 0, 1, 2, \dots, \quad j = 0, 1, 2, \dots, 2n,$$

so that the additional causality condition $|\bar{y}_\perp| \leq R = (y^{02} - y^{32})^{1/2}$ is fulfilled and the ground

state of this concrete quantum field system must be conformed by those suitable additional causality and boundary conditions and so we can modelled the interaction of the concrete relativistic quantum field system to the external classical field by means of this suitable boundary value problem.

Our interest is concerned to the vacuum and especially the physical Casimir vacuum conformed by the suitable boundary value problem.

Nevertheless, the idea - that the vacuum is like a ground state of any one concrete relativistic quantum field system - is enormously fruitful for the biological systems from the point of view of the nanophysics, i.e. it is to consider the time’s arrow in the systems with a feedback. Moreover the Maxwell’s demon has an indefinite fully eigen time too, following on “allowed” world line in the Casimir world.

The obviously necessity to take in consideration the quantum field concepts by observation macroscopically objects present from infinity significant number of virtual particles and to be found by low temperatures is following from the elementary idea. Consider e.g. the obtained Casimir vacuum by reflections and hyperbolical turns at fixed times present from n stationary state level of the Casimir energy of “vacuum fluctuations” and occupied by $j = 2n-1$ virtual scalar bosons to be found in a volume V . In so one vacuum state every virtual scalar is surrounded closely from the neighbouring particles so that on his kind get a volume at every vacuum stationary energy level of the order

$$V/n \sim ((\bar{y}_\perp^2 + (y_{2(n-\frac{1}{2}j)}^3)^2)^{1/2})^3 = (y_{2(n-\frac{1}{2}j)}^3 + \text{const})^3, \quad (27)$$

$$\lim \bar{y}_\perp^2 / 2y_{2(n-\frac{1}{2}j)}^3 = \text{const} \in [0, 1] \text{ for}$$

$$\bar{y}_\perp^2 \rightarrow \infty, y_{2(n-\frac{1}{2}j)}^3 \rightarrow \infty, \text{ for } j = 0, n \rightarrow \infty$$

and the additional causality condition

$$|\bar{y}_\perp| < R = (y^{02} - y^{32})^{1/2}.$$

Also every virtual scalar particles at the state with the smallest energy possesses sufficient Casimir energy ω dependent from the distance between the mirrors and equally of the difference between dark energy and the dark matter

$$\omega = (E_d - m_d c^2),$$

and obtained by the “vacuum fluctuations” obtained elementary by the mass spectre of the fundamental matter scalars. By this “fluctuations” it is proportional to the minus third power of the distance $y^3_0 = (vt_0)^3$ between the mirrors at a given time $t = t_0$

$$\omega \sim (2m)^{-1} ((\bar{q}_\perp^2 + k^2)^{1/2})^3 \sim (2m)^{-1} (y^3_0 + \text{const})^{-3} \sim (2m)^{-1} (n/V)^{-3}, \tag{28}$$

for $q^3 = \pm k^3$, and $\bar{q}_\perp^2 \rightarrow 0, \bar{k}_\perp^2 \rightarrow 0, j = 0, n \rightarrow \infty$.

Moreover the distance between the ground state and the first excited level of the single see massless scalar particle will be of the same order that is for the Casimir energy ω too. It follows that if the temperature of the vacuum state of the relativistic sea quantum field system is less then the some one critical temperature T_c of the order of the temperatures of the “Einstein condensation” then in the Casimir vacuum state there are not the excited one particle states. Furthermore the temperature is not from significances for Casimir force which is the cause for expression of massless scalar Goldstones bosons.

For the Schrödinger wave functional Ψ_φ , defined on the involutes operator field Banach algebra the impulse wave functional equation is given for the Hamiltonian H and impulse operator Q defined in anyone functional Hilbert space with indefinite metric by the equations

$$-i\hbar\partial_t\Psi_\varphi(\alpha_{\kappa^j}, t) = H(\pi, \varphi)\Psi_\varphi(\alpha_{\kappa^j}, t), \text{ for } t \in (t_{-2(n-1/2j)}, t_{2(n-1/2j)}], \tag{29}$$

$$\Psi^+(\alpha_{\kappa^j}, t_{-2(n-1/2j)} + 0) = Q(\pi^+, \varphi)\Psi_\varphi(\alpha_{\kappa^j}, t_{-2(n-1/2j)}),$$

for $t = t_{-2(n-1/2j)} = t_{2(n-1/2(j+1))}, n = 0, 1, 2, \dots, j = 0, 1, 2, \dots, 2n,$ (30)
 where

$$\Psi^+(\alpha_{\kappa^j}, t_{-2(n-1/2j)} + 0) = \delta_{\alpha_{\kappa^j}} \Psi^+(\alpha_{\kappa^j}, t_{-2(n-1/2j)} + 0) f^{\pm}_{\kappa^j}$$

and

$$Q(\pi^+, \varphi) = -i\hbar(\partial_t + \pi^+ \delta_\varphi)$$

is also the impulse Schrödinger operator valued functional for the Hamiltonian operator valued functional $H(\pi, \varphi)$ fulfilling the additional causality and boundary conditions with the field variation given by $\delta_\varphi = \delta/\delta\varphi$, So the Banach field eigen vector φ describe a quantum vacuum fluctuations in the Minkowski space-time where the impulse operator valued functional $Q(\pi^+, \varphi)$ is the Casimir energy operator of the “zero fluctuations” obtained the Casimir energy of the relativistic local quantum scalar field system in the statement of the equally times and may be the so not clearly.

3. CONCLUSION

The supposition7 that by the absence of the attraction between the scalar particles the ground state will be total a stationary state in which all scalars “are condensate” in so one state with impulse $\bar{k}_\perp \rightarrow 0, \bar{q}_\perp \rightarrow 0$ and taking in to account the small attraction by the action at the large distance of the Casimir force in the manifold of the material points between the two mirrors the so called virtual virility scalars in vacuum state also it lead to so one stationary state of the quantum scalar field system in which then in the referent mass system on the mirror at the rest (a map) by the single scalars appear the mixture of the see excited states with impulse $k^3 \gg q^3 \neq 0$.

Yet of this way it can be understand the existence of the supper symmetry by the fundamental “matter” fields. The super symmetric partner scalar particles the so called scalarino of the massless Fermi scalar non local fields with a half spin are obtained by the non local wave function $\psi(\kappa x)$. It is also possibly to be obtained the non local interactions at the large distance by the virtual massless scalar fields vector states obtained by the so called

non local field operators defined in the Hilbert functional space with indefinite metric and appearing by the expansion on the light cone even for local crossing by $\kappa, \kappa' \rightarrow 0$

$$:\psi(\kappa x)\psi(\kappa' x): = [dq_\kappa dq_{\kappa'} \exp[iq_\kappa \kappa x + iq_{\kappa'} \kappa' x]:\psi(q_\kappa)\psi(q_{\kappa'}):.$$

Then also it is understandable for the interacting fields by the summation of the so called minimal local interaction in the global sense

$$\begin{aligned} :\bar{\psi}(\kappa x)\exp[\int_{\kappa'}^{\kappa} dj(\tilde{\tau x})]\psi(\kappa' x): &= :\bar{\psi}(\kappa x)\exp[\int_{\kappa'}^{\kappa} du(\tilde{\tau x})]\psi(\kappa' x): = \\ &:\bar{\psi}(\kappa x)\exp[\int_{\kappa'}^{\kappa} d\tilde{\tau x}^\mu \partial_\mu \tilde{\varphi}(\tilde{\tau x})]\psi(\kappa' x): = \\ &:\bar{\psi}(\kappa x)\exp[-ie \int_{\kappa'}^{\kappa} d\tilde{\tau x}^\mu A_\mu(\tilde{\tau x})]\psi(\kappa' x): \end{aligned}$$

for the gauge vector potential i.e. $A'_\mu(\tilde{\tau x}) = ie(A_\mu(\tilde{\tau x}) + \partial_\mu \tilde{\varphi}(\tilde{\tau x}))$ where $\partial_\mu = \partial/\partial x_\mu$ and by the condition $A'_\mu(\tilde{\tau x}) = 0$ e.g. $F'_{\mu\nu} = 0$ by the super symmetries considerations is to be taken under account the following condition for the quantum scalar field for $\kappa, \kappa' \rightarrow 0$

$$\lim_{\kappa, \kappa' \rightarrow 0} \langle 0 | \bar{\psi}(\kappa x)\psi(\kappa' x) | 0 \rangle = \langle 0 | \varphi | 0 \rangle = \varphi = \text{const.}$$

In 1946 the shift for scalar field $j(x) = \text{const} + u(x)$ i.e. $dj(x) = du(x)$ has been given at the first by N. N. Bogolubov in the theory of microscopically super fluidity.

ACKNOWLEDGMENTS

The author takes pleasure in acknowledging the help and encouragement given to him by akad. Ts. D. Tsvetkov, Institute for Cryobiology and Food Technology - Sofia and V. A. Matveev - Dubna.

REFERENCES

- Belaus, A. M., & Tsvetkov, Ts. D. (1985). *Nauchnie ocnovi tehnology sublimatsionova konservaniya* (p.208). Kiev, Naukova Dumka, (Ru).
- Bogolubov, N. N., Logunov, A. A., Oksak, A. I., Todorov, I. T. (1987). *Obshie prinzipi kvantovoi teorii polya* (p.615). Kiev, Naukova Dumka, (Ru).
- Bogolubov, P. N., Matveev, V. A., Petrov, G., Robaschik, D. (1976). Causality properties of form factors, *Rept. Math. Phys*, 10, 195-202.
- Bordag, M., L., Kaschlunh, G. P., Robaschik, D. (1983). On nonlocal light-cone expansion in QCD. *Yd. Fiz.*, 37(1), 193 - 201.
- Bordag, M., Petrov, G., Robaschik, D. (1984). Calculation of the casimir effect for scalar fields with the simplest non-stationary boundary conditions. *Yd. Fiz.*, 39, 1315-1320.
- Casimir, H. B. G. (1948). On the attraction between two perfectly conducting plates. *Proc. Kon. Ned. Akad. Wet.*, 60, 793.
- Fortin, J. F., Grinstein, B., & Stergiou, A. (2011). Scale without conformal invariance: An example. arXiv:1106.2540.

- Geyer, B., Petrov, G., & Robaschik, D. (1977). Kausalitaetseigenschaften von formfaktoren und kinematische bedingungen. *Wiss. Z. Karl-Marx-Univ., Leipzig, Math.-Naturwiss. R.*, 26, 101-122 (DE).
- Goodwin, B. C. (1963). Temporal organisation in cells, (A dynamik theory of cellular control processes). *Academic Press London and New York*, 251.
- Dodonov, V. V. (2001). Nonstationary casimir effect and analytical solutions for quantum fields in cavities with moving boundaries. Retrieved from http://arxiv.org/PS_cache/quant-ph/pdf/0106/0106081v1.pdf
- Kaempffer, F. A. (1965). Concepts in quantum mechanics (pp.392). New York and London: Academic Press.
- Mitter, H., & Robaschik, D. (1999). *Eur. Phys. J. B.*, 13, 335-340.
- Petrov, G. (1974, May). The causality of the form factors of the virtually compton scattering amplitude. *I. Scientific Symposium. of Bulg. Students, Berlin*, 10-12, (DE).
- Petrov, G. (1978). Kausalität von formfaktoren, kinematische bedingungen und folgerungen für das Scaling-Verhalten im Nichtvorwärtsfall der tiefinelastischen Lepton-Hadron-Streuung. Dissertation Zur Promotion A, Karl-Marx-Universität, Leipzig.
- Petrov, G. (1978). Restrictions on the light-cone singularity imposed by causality. *Karl-Marx-Univ., Leipzig*, L. 856/78 III/8/1767.
- Petrov, G. (1985). Calculation of casimir effect for the electromagnetic field with the non-stationary boundary conditions. *Bulg. J. Phys.*, 12(4), 355-363.
- Petrov, G. (1989). Casimir relativistic effect for massless scalar field with uniformly moving mirrors. *Rev. Roum. Phys.*, 34(5), 453 - 460.
- Petrov, G. (1992). Impulsive moving mirror model in a schrödinger picture with impulse effect in a banach space. *Dubna, JINR E2-92-272*.
- Petrov, G. (1992). Impulsive moving mirror model and impulse differential equations in Banach space. *Dubna, JINR E2-92-276*.
- Petrov, G. (2010). Non equilibrium thermodynamics by the concrete quantum field system with a impulse effect in the nanophysics. *Agriculture and Biology Journal of North America* ISSN Print: 2151-7517, ISSN Online: 2151-7525.
- Tsvetkov, Ts. D., & Petrov, G. (2004). Non equilibrium thermodynamic and fields effects in the cellular cryobiology and anhydrobiology. *Bulgarian Journal of Agricultural Science*, 10, 527-538.
- Tsvetkov, Ts. D., Petrov, G., & Tsvetkova, P. (2005). Non equilibrium thermodynamics by the concrete quantum field systems in the cellular cryobiology an anhydrobiology. *Bulgarian Journal of Agricultural Science*, 6, 387-403.
- Tsvetkov, Ts. D., Petrov, G., & Tsvetkova, P. (2005). Non equilibrium thermodynamics by the vacuum waves in the Banach space for the see scalar's systems in the cellular cryobiology an anhydrobiology. *Bulgarian Journal of Agricultural Science*, 6, 645-6602.
- Tsvetkov, Ts. D., Petrov, G., & Tsvetkova, P. (2006). Non equilibrium thermodynamics for the interactions of the see quantum scalar's system with the classical bio fields in the cellular cryobiology and anhydrobiology. *Bulgarian Journal of Agricultural Science*, 12, 621 - 628.

- Tsvetkov, Ts. D., Petrov, G., & Tsvetkova, P. (2007). Non equilibrium thermodynamics of the cryobiology and quantum field theoretical methods in the temporal organization in cells. *Bulgarian Journal of Agricultural Science*, 13, 379-386.
- Tsvetkov, Ts. D., Petrov, G., & Tsvetkova, P. (2007). Causality implies the Lorenz group for cryobiological "freezing-drying" by the vacuum sublimation of living cells. *Bulgarian Journal of Agricultural Science*, 13, 627-634.
- Tsvetkov, Ts. D., Petrov, G., & Tsvetkova, P. (2008). Non equilibrium thermodynamics by vacuum energy momentum tensor of interacting quantum fields and living cells. *Bulgarian Journal of Agricultural Science*, 14, 351-356.
- Tsvetkov, Ts. D., Petrov, G., & Tsvetkova, P. (2009). Causality properties of the energy-momentum tensor of the living cells by the low temperatures. *Bulgarian Journal of Agricultural Science*, 15, 487-493.
- Tsvetkov, Ts. D., Petrov, G., & Hadzhy, A. (2011). Theoretical field analysis of the concrete quantum field system with AN impulse effect in the elementary living cells. *Bulgarian Journal of Agricultural Science*, 17, 1-10.
- Tsvetkov, Ts. D., Petrov, G., & Hadzhy, A. (2012). Boundary conditions on the quantum scalar field system with a fluctuation's impulse operator of the vacuum state in living cells. *Bulgarian Journal of Agricultural Science*, 18, 147-152.
- Tsvetkov, Ts. D., & Petrov, G. (2012). The scalar particles in Casimir vacuum state of the relativistic quantum field system in living cells. *Bulgarian Journal of Agricultural Science*, 18, 819-826.
- Tsvetkov, Ts. D., & Petrov, G. (2013). Casimir vacuum state of the relativistic quantum scalar field system in living cells. *Bulgarian Journal of Agricultural Science*, 19(6), 387-396.
- Tsvetkov, Ts. D., & Petrov, G. (2013). Scale without conformal invariants in the non local relativistic quantum field systems in living cells. *Bulgarian Journal of Agricultural Science*, 19(6), 1171-1182.
- Zwetkow, Zw. D. (1985). *Vakuumgefrieretrocknung* (p.245). VEB Fachbuchverlag Leipzig.