Then, let $\varepsilon>0$ be arbitrary, it follows that

$$|\rho|_T \le \varepsilon + M\sum_{k=1}^{m} l_k |\rho|_T$$

which together with (4) and the arbitrariness of ε imply that

$$|\rho|_T \to 0.$$

Thus, h is continuous.

Accordingly, applying Lemma 2.4 we conclude that $\mathcal{F}$ has at least one fixed point in $\mathcal{D}$, which is a mild solution of (1). The proof is complete.

3. AN EXAMPLE OF EXISTENCE RESULT

Take X=$L^2(0,\pi)$ and denote its norm by $\|\cdot\|$ and inner product by$(\cdot,\cdot)$ to illustrate our abstract results, let us consider the system of partial differential inclusion in the form

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} - \dfrac{\partial^2 u(t,x)}{\partial x^2} \in F(t,x,u(t,x),u_t(x)), \quad (t,x)\in[0,1]\times[0,\pi], t\ne t_k, \\ u(t_k^+,x)-u(t_k,x) = \dfrac{1}{pm}\sin(x(t_k,x)), \quad t_k = \dfrac{k}{m+1}, k=1,...,m, \\ u(t,0)=x(t,\pi)=0, \quad t\in[0,1], \\ u(t,x)=\varphi(t,x), \quad t\in[-h,0], x\in[0,\pi], \end{cases} \tag{10}$$

where $p>1$, $F(t,x,u,v)=[f_1(t,x,u,v), f_2(t,x,u,v)]$ for each $(t,x,u,v)\in[0,1]\times[0,\pi]\times R\times PC_0$.

Let $A:D(A)\subset X\to X$ be operator defined by$A\omega=\dfrac{\partial^2\omega}{\partial x^2}$ with domain $D(A)=\{\omega\in X:\omega,\omega'$are absolutely continuous,$\omega''\in X$ and $\omega(0)=\omega(\pi)=0\}$.

It is known that A has a discrete spectrum and the eigenvalues are$-n^2, n\in N$,with the corresponding normalized eigenvectors $\varpi_n(x)=\sqrt{\dfrac{2}{\pi}}\sin(nx), 0\le x\le\pi.$ Moreover, A generates a compact, analytic semigroup$\{T(t)\}_{t\ge 0}$on X:

$$T(t)\omega=\sum_{n=1}^{\infty} e^{-n^2 t}(\omega,\varpi_n)\varpi_n, \quad \|T(t)\|_{L(X)}\le e^{-t}\le 1 \quad \text{for all } t\ge 0$$

(see Henry, 1981). According to the compactness of $T(t)$ for $t>0$, one can verify that $T(t)$ is uniform operator topology continuous for $t>0$.

To treat the system (10), we assume that the functions f_i:$[0,1]\times[0,\pi]\times R\times PC_0\to R(i=1,2)$ satisfy

(F_1) $f_1(t,x,u,v)\le f_2(t,x,u,v)$ for each $(t,x,u,v)\in[0,1]\times[0,\pi]\times R\times PC_0$,

(F_2) f_1 is *l.s.c.*and f_2 is *u.s.c.*;

(F_3) there exist functions $\eta_1,\eta_2\in L^\infty(R^+,R^+)$ such that

$$|f_i(t,x,u,v)|\le \eta_1(t)(|u|+|v|_0)+\eta_2(t);$$

Then one can verify (see Chen, 2013; Vrabie, 2012) that the multi-valued function F:$[0,1]\times X\times PC_0\to 2^X$ defined as

$$F(t,u,v)=\{y\in X: y(x)\in[f_1(t,x,u(x),v), f_2(t,x,u(x),v)] \ a.e. \text{in}[0,\pi]\}$$

satisfies assumptions (H_1)-(H_2) (with $\eta(t)=\sqrt{\pi}\max\{\eta_1(t),\eta_2(t)\}$ in (H_2)).

Define

$$I_k(u(t_k))(x)=\frac{1}{pm}\sin(u(t_k,x)).$$

It is clear that

$$\|I_k(y)\|\le\frac{\sqrt{\pi}}{pm}, \quad k=1,...,m, y\in X,$$

$$\|I_k(y_1)-I_k(y_2)\|\le\frac{1}{pm}\|y_1-y_2\|, \quad k=1,...,m, y_1,y_2\in X.$$

These yield that the hypotheses(H_4)-(H_5) are satisfied.

Assume that F satisfies (H_3) with$\int_0^1\mu(s)ds<\dfrac{p-1}{4p}$, then all the conditions in Theorem 3.1 are satisfied. Hence, the system (10) has at least one mild solution.

REFERENCES

Ahmed, N. U. (2006). Measure solutions for impulsive evolution equations with measurable vector fields. *J. Math. Anal. Appl., 319,* 74-93.

Benchohra, M., Gatsori, E. P., Henderson, J., & Ntouyas, S. K. (2003). Nondensely defined evolution impulsive differential inclusions with nonlocal conditions. *J. Math. Anal. Appl., 286,* 307-325.

Benchohra, M., Henderson, J., & Ntouyas, S. K. (2006). *Impulsive differential equations and inclusions* (Vol.2). New York: Hindawi Publishing Corporation.

Bothe, D. (1998). Multi-valued perturbations of m-accretive differential inclusions. *Israel J. Math., 108,* 109-138.

Cardinali, T., & Rubbioni, P. (2008). Impulsive semilinear differential inclusions: Topological structure of the solution set and solutions on non-compact domains. *Nonlinear Anal., 69*(1), 73-84.

Chen, D. H., Wang, R. N., & Zhou, Y. (2013). Nonlinear evolution inclusions: Topological characteriza- tions of solution sets and applications. *J. Funct. Anal., 265,* 2039-2073.

Chuong, N. M., & Ke, T. D. (2012). Generalized Cauchy problems involving nonlocal and impulsive conditions. *J. Evol. Equ., 12,* 367-392.

Djebali, S., Gorniewicz, L., & Ouahab, A. (2011). Topological structure of solution sets for impulsive differential inclusions in Fréchet spaces. *Nonlinear Anal., 74,* 2141-2169.

Fečkan, M., Zhou, Y., & Wang, J. R. (2012). On the concept and existence of solution for impulsive fractional differential equations. *Commun. Nonlinear Sci. Numer. Simul., 17,* 3050-3060.

Gabor, G., & Grudzka, A. (2012). Structure of the solution set to impulsive functional differential inclusions on the half-line. *Nonlinear Differ. Equ. Appl., 19,* 609-627.

Henry, D. (1981). *Geometric theory of semilinear parabolic equations.* Springer, Berlin.

Kamenskii, M., Obukhovskii, V., & Zecca, P. (2001). *Condensing multi-valued maps and semilinear differential inclusions in banach spaces, de gruyter series in nonlinear analysis and applications* (Vol.7). Walter de Gruyter, Berlin, New York.

Lakshmikantham, V., Bainov, D. D., & Simeonov, P. S. (1989). *Theory of impulsive differential equations.* Singapore: World Scientific Pub Co Inc.

O'Regan, D., & Precup, R. (2001). Existence criteria for integral equations in Banach spaces. *J. Inequal. Appl., 6,* 77-97.

Obukhovskii, V., & Yao, J. C. (2010). On impulsive functional differential inclusions with Hille-Yosida operators in Banach spaces. *Nonlinear Anal., 73,* 1715-1728.

Samoilenko, A. M., & Perestyuk, N. A. (1995). *Impulsive differential equations, world scientific.* Singapore.

Vrabie, I. I. (2012). Existence in the large for nonlinear delay evolution inclusions with nonlocal initial conditions. *J. Funct. Anal., 262,* 1363-1391.

Wang, R. N., & Zhu, P. X. (2013). Non-autonomous evolution inclusions with nonlocal history conditions: Global integral solutions. *Nonlinear Anal., 85,* 180-191.

Wang, R. N., & Ma, Q. H. (2015). Some new results for multi-valued fractional evolution equations. *Appl. Math. Comput, 257,* 285-294.