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Research on Cooperative Advertising Decisions in Dual-Channel Supply Chain Under Asymmetric Demand Information When Online Channel Implements Discount Promotion

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Abstract

This paper analyzes the both online-channel price discount and advertising decisions in a dual-channel supply chain involved one manufacturer and one retailer. A Stackelberg game dominated by the manufacturer is established. The influence of asymmetric demand information is analyzed. The study shows that retailer has a motivation to lie about the offline demand information and it always announces a higher advertising impact factor. To induce the retailer to reveal to true demand information, a franchise-fee contract is designed.

Key words: Cooperative advertising; Price discount; Dual channel; Demand information asymmetry

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INTRODUCTION

With the development of E-commerce, great changes have happened in marketing methods and manufactures' distribution channels. Nowadays, large numbers of manufactures have opened their own websites to sell products online, while through the offline retailing channel. Many manufactures have operated their market through this kind of online and offline dual channel, such as Samsung, IBM, Lenovo and Apple. Usually, online websites can only provide brief products information for consumers, but offline retailing channels can help consumers acquire full information about products. In order to improve online market share and obtain more revenue, manufactures often provide price discount on their own websites. This kind of discount promotion strategy helps the manufactures increase online consumers, but also hurts the offline consumer amount. As a result, retailers will get dissatisfied with this strategy. In this condition, manufactures can use cooperative advertising method, which is widely used in practical operation, to relieve the channel conflict. So, it is necessary for manufactures to keep a balance between the use of online discount strategy and cooperative advertising method. Furthermore, retailers actually have a better knowledge about demand information than manufactures, because retailers always have direct contact with consumers. Asymmetric demand information may exist between manufactures and retailers. Therefore, it is highly possible for retailers to lie about the demand information, and it is important to help manufactures prohibit retailers' lying behavior.

As a coordination method between manufacture and retailers, cooperative advertising has attracted lots of attention from world-wide researchers. Our paper focuses on the cooperative advertising problem in dual channel. Zhang et al. (2014) examine the effects of supply chains members' cooperative advertising and costs sharing behavior on dual channel coordination on condition that manufacturer opens online and retail channel at the same time. Results show that no matter what effect retailer's promotion has on brand image, when manufacturer pays part of retailer's advertising cost, the outcome of two members would be better than that in the decentralized channel, but worse than that in the centralized situation. Opening a new online channel besides the offline channel, Berger et al. (2006) examine integration decisions from a cooperative advertising perspective to determine the profitability of various integration strategies. Yan et al. (2006) obtain equilibrium pricing and co-op advertising policies under two different competitive scenarios: Bertrand and Stackelberg equilibrium. They also compare the profit gains under these two marketing games. Wang and Zhou (2009) analyze the pricing and advertising decision under different pricing schemes. The impact of cooperative advertising on the optimal decisions is investigated. Huang et al. (2011) study the influence of cooperative advertising strategy on channel supply chain pricing decision, the two-echelon supply chain system composed of one manufacture and one retailer was considered. Li et al. (2015) consider a dyadic supply chain consisting of a manufacturer and a traditional retailer. In addition, the effect of a fairness concern of the manufacturer is investigated. Chen et al. (2016) focus on the cooperative advertising in a dual-channel supply chain where price competition and advertising competition exist simultaneously between manufacturer's online channel and retailer's traditional channel.

Besides considering both price discount and cooperating in a dual channel, our paper also investigates the demand information asymmetry between manufacture and retailer. Özalp et al. (2006) study how to assure credible forecast information sharing between a supplier and a manufacturer. When the buyer has better knowledge about demand than supplier, Burnetas et al. (2007) investigates how a supplier can use a quantity discount schedule to influence the stocking decisions of a downstream buyer that faces a single period of stochastic demand. Gan et al. (2010) study a drop-shipping supply chain in which the retailer receives a customer's order and the supplier fills it. In such a chain, the supplier keeps inventory and bears inventory risks; the retailer focuses on marketing and customer acquisition, and forwards the orders to the supplier. Babich et al. (2012) solve a buyback contract design problem for a supplier who is working with a retailer who possesses private information about the demand distribution. When demand is uncertain and unobservable to the supplier, Heese et al. (2014) consider a supply chain with a supplier that sells to a retailer under a revenue-sharing arrangement. Yang et al. (2015) analyze the advertising decisions in a dual-channel supply chain involved one manufacturer and one retailer. The influence of asymmetric demand information and dual-channel on the cooperative advertising decisions is also analyzed.

Reviewing the above literature, we find that the most relevant paper to our paper is Yang et al. (2015). However, the do not consider the impact of online price discount on both retailer and manufacture's decisions. Meanwhile, they do not conduct further analysis on the effect of demand information.

1. BASIC MODEL

1.1 Model Description and Assumption

A supply chain comprised of a manufacture and a retailer is investigated in the basic Stackelberg game model. In this basic model, manufacture is the leader and retailer is the follower. The manufacture opens its online channel while sells products through an offline retailer. Assuming the retailer's sales represented by p_r, manufacture sets the wholesale price as w . Through providing a price discount on the online channel, manufacture's online price is p_e, $w < p_e \le p_r$. In this way, the price discount provided online is $\sigma=1-p_e / p_r$. Then, we see that the price discount σ must be confined to a closed interval, say $0 \le \sigma < 1-w/p_r$. Given p_r fixed, we can infer that higher price discount σ can lead to a lower online price. Meanwhile, define the value of parameter $b(b \ge 0)$ as the retailer's advertising effort on products. Like many former researches, the total advertising cost will be represented in a quadratic form $C(b)=b^2/2$. Manufacture shares a part of the total advertising cost with retailer, and the part ratio is $1-t$. So, the manufacture will afford the expense $(1-t)b^2/2$ and the retailer's shared cost will be $tb^2/2$.

Chen et al. (2016) set the demand model in the form of price discount effect multiplied the advertising effect. However, their model may be unreasonable when advertising effort is zero. Unlike Cheng et al.'s (2016) research and simplify the problem, our paper assumes the demand function in the form of linear model. We can get demand functions of the both online and offline channels:

$$D_e = s \cdot a + \theta_e \cdot \sigma + \gamma_e \cdot b, \tag{1}$$

$$D_r = (1-s) \cdot a - \theta_r \cdot \sigma + \gamma_r \cdot b. \tag{2}$$

From Formulas (1) and (2), total market size is a. When manufacture does not provide price discount and retailer does not invest in advertisement, online initial market share is $s \cdot a$ and the offline market share is $(1-s) \cdot a$. Providing the price discount, more consumers may be attracted by the products. So, the total market size will increase. Because some of the offline consumers are price-sensitive, part of the consumers will transfer from offline to online. When σ increases, it is reasonable to believe that online demand D_e increases and offline demand D_r decreases. θ_e and θ_r are σ's impact factor on online and offline demand, $\theta_e > \theta_r > 0$. Different from σ, advertising effort b has a positive impact on both offline and online demand, $\gamma_e > 0$ and $\gamma_r > 0$. As this paper focuses on the influence of price discount and cooperative advertising, the model is simplified in three aspects. The first is that we do not consider price's effect on demand. The sales price p_r and wholesale price w are given exogenously. The second is that we only consider the retailer's advertising behavior, but do not consider manufacture's own advertising effort. It is reasonable not to consider manufacture's advertising effort. Because the price discount promotion strategy has the same effect like advertisement to some extent. It is not necessary for manufacture to operate price discount and advertising simultaneously. Without loss of generality, the third assumption is that manufacturing cost of the product is zero. Revenue functions of the both manufacture and retailer are:

$$\pi_m = p_r \cdot (1-\sigma) \cdot (s \cdot a + \theta_e \cdot \sigma + \gamma_e \cdot b) + w \cdot [(1-s) \cdot a - \theta_r \cdot \sigma + \gamma_r \cdot b] - (1-t) \cdot b^2/2 \,, \tag{3}$$

$$\pi_r = (p_r - w) \cdot [(1-s) \cdot a - \theta_r \cdot \sigma + \gamma_r \cdot b] - tb^2/2 \,. \tag{4}$$

According to the preceding article, the manufacture acts as the leader, who first announces its price discount σ and advertisement sharing ratio 1–t to maximize its revenue. In response to σ and 1–t, the traditional retailer (the follower) updates its advertising effort b to maximize its revenue. Through the standard backward induction, we can easily derive the optimal decision of both retailer and manufacture in 2.2 under asymmetric information.

1.2 Decision Analysis

Given manufacture's decision of σ and 1–t, the retailer's advertising effort reaction is:

$$b = (p_r - w) \cdot \gamma_r / t \tag{5}$$

and the retailer's optimal revenue as a function of σ and b is:

$$\pi_r = (p_r - w) \cdot [(1-s) \cdot a - \theta_r \cdot \sigma + \gamma_r \cdot b] - b \cdot (p_r - w) \cdot \gamma_r / 2 \,. \tag{6}$$

From Formula (5), we can infer that b has a linear correlation with t. To solve the decision of t, we can instead solve the decision of b. Substituting Formula (5) into Formula (4) and simplifying, we get

$$\pi_m = p_r \cdot (1-\sigma) \cdot (s \cdot a + \theta_e \cdot \sigma + \gamma_e \cdot b) - w \cdot [(1-s) \cdot a - \theta_r \cdot \sigma + \gamma_r \cdot b] - b^2/2 - b \cdot (p_r - w) \cdot \gamma_r / 2 \,. \tag{7}$$

It is easily to know that π_m is respectively concave in σ and b. We use the two-stage optimization method to maximize the manufacture's revenue π_m, i.e., we first derive the optimal price discount σ for any given advertising effort b, then we determine the optimal advertising effort b to maximize π_m. The optimal price discount and advertising effort are:

$$\sigma^* = [\, w \cdot \theta_r - (p_r + w) \cdot \gamma_r \cdot \gamma_e \cdot p_r \cdot /2 + p_r^2 \cdot \gamma_e^2 - p_r \cdot (\theta_e - s \cdot a)] / (p_r^2 \cdot \gamma_e^2 - 2 \cdot p_r \cdot \theta_e) \,, \tag{8}$$

$$b^* = [-(p_r + w) \cdot \gamma_r \cdot \theta_e \cdot p_r - p_r \cdot \gamma_e \cdot w \cdot \theta_r - p_r^2 \cdot \gamma_e \cdot (\theta_e + s \cdot a)] / (p_r^2 \cdot \gamma_e^2 - 2 \cdot p_r \cdot \theta_e) \,. \tag{9}$$

To examine Formula (9), b^* is determined not to be negative. So, we can have

$$b^* = [-(p_r + w) \cdot \gamma_r \cdot \theta_e \cdot p_r - p_r \cdot \gamma_e \cdot w \cdot \theta_r - p_r^2 \cdot \gamma_e \cdot (\theta_e + s \cdot a)] / (p_r^2 \cdot \gamma_e^2 - 2 \cdot p_r \cdot \theta_e) \geq 0. \tag{10}$$

According Formula (10), we can know that

$$p_r^2 \cdot \gamma_e^2 - 2 \cdot p_r \cdot \theta_e < 0 \,. \tag{11}$$

While analyzing formula (7), the hessian matrix of $\pi_m(\sigma, b)$ is

$$H = \begin{bmatrix} -2 \cdot p_r \cdot \theta_e & -p_r \cdot \theta_e \\ -p_r \cdot \theta_e & -1 \end{bmatrix} \,.$$

Combine Formula (11), we can derive that $|H| = 2 \cdot p_r \cdot \theta_e - p_r^2 \cdot \gamma_e^2 > 0$. That is to say, $\pi_m(\sigma, b)$ is jointly concave in σ and b. Then we can get the theorem 1

Theorem 1

Under the symmetric demand information scenario, the optimal equilibrium advertising effort, price discount and cost sharing ratio for the retailer and manufacture are given by

$$b^* = [-(p_r + w) \cdot \gamma_r \cdot \theta_e \cdot p_r - p_r \cdot \gamma_e \cdot w \cdot \theta_r - p_r^2 \cdot \gamma_e \cdot (\theta_e + s \cdot a)] / (p_r^2 \cdot \gamma_e^2 - 2 \cdot p_r \cdot \theta_e) \,,$$

$$\sigma^* = [w \cdot \theta_r - (p_r + w) \cdot \gamma_r \cdot \gamma_e \cdot p_r \cdot /2 + p_r^2 \cdot \gamma_e^2 - p_r \cdot (\theta_e - s \cdot a)] / (p_r^2 \cdot \gamma_e^2 - 2 \cdot p_r \cdot \theta_e) \,,$$

$$t^* = (p_r - w) \cdot \gamma_r / b^* .$$

To examine the impact of s, γ_e and γ_r on the online retailer's optimal advertising decision, we take the first-order derivatives of b^* with respect to s, γ_e and γ_r. Then it is also interesting to examine the impact of s, θ_e and θ_r on the online manufacture's optimal decisions, we also take the first-order derivatives of σ* and t^* with respect to s, θ_e and θ_r.

We can obtain proposition 1.

Proposition 1

(i) $\partial b^*/\partial s > 0$; $\partial b^*/\partial \gamma_e > 0$; $\partial b^*/\partial \gamma_r > 0$

(ii) $\partial \sigma^*/\partial s < 0$; $\partial \sigma^*/\partial \theta_e < 0$; $\partial \sigma^*/\partial \theta_r < 0$

(iii) $\partial t^*/\partial s < 0$; $\partial t^*/\partial \gamma_e < 0$; $\partial t^*/\partial \theta_r < 0$

From proposition 1(i), we can get that: retailer should always increase the advertisement expense when the retailer's initial market share (1–s) is getting smaller. Meanwhile, once the advertising is easier to convert to demand in any distribution channel, it is more profitable for retailer to invest in more advertisement fee. From (ii) and (iii), we can propose that: manufacture should decrease their price discount σ* and increase cost sharing ratio 1–t^* when the online initial market share is getting larger. In contrast, it should decrease σ* when it becomes easier to attract online consumers through providing price discount. 1–t^* should be increased when it becomes easier to attract online consumers through advertising. In the end, if it is easier to attract consumers from offline to online by providing price discount, manufacture should decrease σ* and increase 1–t^*. For the manufacture, all changes of 1–t^* are contrary to the changes of σ*, because price discount is a competitive tool to get consumer from retailer, while the cost sharing ratio is an effective tool to coordinate conflict caused by competition.

2. DECISION ANALYSIS AND CONTRACT DESIGN UNDER ASYMMETRIC DEMAND INFORMATION

Because retailer always has direct contact with offline consumers while manufacture does not. There is a high probability for retailer not to announce the demand information for its own revenue. This section, a dual-channel Stackelberg model will be considered. In this model, we assume the demand information is retailer's private information which is unknown to manufacture. According to (Lei et al., 2015), retailer has already known its demand information θ_r and γ_r before making decisions.

But manufacture does not know effects on offline demand caused by price discount and advertising effort. Apart from this, we assume that s, a, θ_r, γ_e are common knowledge between manufacture and retailer.

2.1 Retailer's Lying Behavior

Due to the asymmetric demand information, retailer is possible to lie about the impact factors θ_r and γ_r. In 3.1, we will investigate whether the retailer has the motivation to lie about the demand information. Assume the impact factors retailer announces to retailer is θ_r' and γ_r', according to Formulas (6), (8), and (9), the retailer's revenue, price discount and advertising effort will be:

$$\pi_r(\theta_r',\gamma_r')=(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\theta_r',\gamma_r')+\gamma_r\cdot b(\theta_r',\gamma_r')] -b(\theta_r',\gamma_r')\cdot(p_r-w)\cdot\gamma_r'/2,$$

$$\sigma(\theta_r',\gamma_r')=[w\cdot\theta_r'+(p_r+w)\cdot\gamma_r'\cdot\gamma_e\cdot p_r\cdot/2+p_r^2\cdot\gamma_e^2-p_r\cdot(\theta_e-s\cdot a)] /(p_r^2\cdot\gamma_e^2-2\cdot p_r\cdot\theta_e),$$

$$b(\theta_r',\gamma_r')=[-(p_r+w)\cdot\gamma_r'\cdot\theta_e\cdot p_r-p_r\cdot\gamma_e\cdot w\cdot\theta_r'-p_r^2\cdot\gamma_e\cdot(\theta_e+s\cdot a)] /(p_r^2\cdot\gamma_e^2-2\cdot p_r\cdot\theta_e).$$

Given θ_r', taking the second-derivative of $\pi_r(\theta_r',\gamma_r')$ with respect to γ_r', we get:

$$\partial^2\pi_r(\theta_r',\gamma_r')/\partial\gamma_r'^2=(p_r-w)\cdot(p_r+w)\cdot p_r\cdot\theta_e/(p_r^2\cdot\gamma_e^2-2\cdot p_r\cdot\theta_e) <0.$$

So, given θ_r', the optimal decision of γ_r' is:

$$\gamma_r'^*=\gamma_r+\gamma_e\cdot[p_r\cdot(\theta_e-\theta_r+s\cdot a)+w\cdot(\theta_r'-\theta_r)] /[2\cdot(p_r+w)\cdot\theta_e], \tag{12}$$

given the γ_r', take a first-derivative of $\pi_r(\theta_r',\gamma_r')$ with respect to θ_r'. We can get:

$$\partial\pi_r(\theta_r',\gamma_r')/\partial\theta_r'=w\cdot(\gamma_r'\cdot\gamma_e\cdot p_r\cdot/2-\gamma_r\cdot\gamma_e\cdot p_r-\theta_r) /(p_r^2\cdot\gamma_e^2-2\cdot p_r\cdot\theta_e). \tag{13}$$

Combine Formulas (12) and (13), we derive theorem 2:

Theorem 2

(i) if $\gamma_r'(\theta_r'=0)\cdot\gamma_e\cdot p_r\cdot/2-\gamma_r\cdot\gamma_e\cdot p_r-\theta_r<0$

$\theta_r'^*=0$, $\gamma_r'^*=\gamma_r+\gamma_e\cdot[p_r\cdot(\theta_e-\theta_r+s\cdot a)-w\cdot\theta_r)]/[2\cdot(p_r+w)\cdot\theta_e]$.

(ii) if $\gamma_r'(\theta_r'=\theta_e)\cdot\gamma_e\cdot p_r\cdot/2-\gamma_r\cdot\gamma_e\cdot p_r-\theta_r>0$

$$\theta_r'^*=\theta_e,\ \gamma_r'^*=\gamma_r+\gamma_e\cdot[p_r\cdot(\theta_e-\theta_r+s\cdot a)+w\cdot(\theta_e-\theta_r)] /[2\cdot(p_r+w)\cdot\theta_e].$$

(iii) if $\gamma_r'(\theta_r'=0)\cdot\gamma_e\cdot p_r\cdot/2-\gamma_r\cdot\gamma_e\cdot p_r-\theta_r<0<\gamma_r'(\theta_r'=\theta_e)\cdot\gamma_e\cdot p_r\cdot/2-\gamma_r\cdot\gamma_e\cdot p_r-\theta_r$

$$\theta_r'^*=[2\cdot(p_r+w)\cdot\theta_e\cdot(\gamma_r\cdot\gamma_e\cdot p_r+2\cdot\theta_r)-\gamma_e\cdot p_r \cdot(\theta_e-\theta_r+s\cdot a)+w\cdot\theta_r]/(\gamma_e\cdot w),\ \gamma_r'^* =2\cdot(\gamma_r\cdot\gamma_e\cdot p_r+\theta_r)/(p_r\cdot\gamma_e).$$

From theorem 2, we can know that the retailer may announce different impact factors of price discount and advertising effort based on different conditions. So, we can obtain proposition 2.

Proposition 2

Retailer has a motivation to lie about the impact factors of price discount and advertising effort.

(i) When retailer lies a low $\theta_r'(\theta_r'=0)$, if the corresponding $\gamma_r'(\theta_r'=0)$ is high enough, the optimal decision of θ_r' should acquire a very low value ($\theta_r'=0$). The optimal decision of γ_r' has following characters:

$$\partial\gamma_r'^*/\partial s>0;\ \partial\gamma_r'^*/\partial a>0;$$
$$\partial\gamma_r'^*/\partial\gamma_e>0;\ \partial\gamma_r'^*/\partial w<0;$$
$$\partial\gamma_r'^*/\partial\theta_r<0.$$

(ii) When retailer lies a high $\theta_r'(\theta_r'=\theta_e)$, if the corresponding $\gamma_r'(\theta_r'=\theta_e)$ is low enough, the optimal decision of θ_r' should acquire a very high value ($\theta_r'=\theta_e$). The optimal decision of γ_r' has the same characters in (i).

(iii) Except the above condition, retailer may lie a moderate $\theta_r'(0<\theta_r'^*<\theta_e)$. The optimal decision of γ_r' has following characters:

$$\partial\gamma_r'^*/\partial\theta_r>0,\ \partial\gamma_r'^*/\partial p_r>0,\ \partial\gamma_r'^*/\partial\gamma_e<0.$$

Also, we should point out that retailer will always lie a higher impact factor of advertising effort than the real one ($\gamma_r'>\gamma_r$). Furthermore, it is easy to prove that retailer's lying behavior may cause revenue loss of manufacture ($\pi_m(\theta_r,\gamma_r)^*>\pi_m(\theta_r'^*,\gamma_r'^*)$).

According to proposition 2(i), the retailer's lying behavior contains three different conditions. When retailer's optimal announcement of price discount impact factor $\theta_r'^*$ is rather low $\theta_r'^*=0$ or high $\theta_r'^*=\theta_e$, the optimal announcement of advertising effort impact factor $\gamma_r'^*$ has the same characters. When the total market size a, manufacture's initial online market share s and offline advertising effort γ_e increase, retailer may lie higher $\gamma_r'^*$. However, when wholesale price w and offline price discount impact factor θ_r increase, the retailer may lie lower $\gamma_r'^*$. On other conditions, retailer may lie a moderate $\theta_r'^*$. The advertising effort impact factor decision $\gamma_r'^*$ will have an opposite character from (i), (ii).

2.2 Contract Description

In proposition 2, we have proposed that retailer will lie about the demand information to obtain higher revenue. This lying behavior will hurt manufacture's revenue. So, it is necessary for manufacture to take some action to deal with this condition. From the perspective of manufacture, effective strategies like contract designing should be conducted to prohibit retailer from lying. In this part, an optimal contract menu is obtained to induce retailer to real demand information.

The contract menu should consider both individual rational (IR) constraint and incentive compatibility (IC) constraint. The individual rational constraint means that retailer may acquire a higher revenue if it accepts the revenue-sharing than not. The incentive compatibility constraint means that retailer will be induced to share the true demand information. Though the manufacture does not know retailer's demand information θ_r and γ_r, it resorts to a prior belief and considers them continuous random variables with values in $[\underline{\theta_r},\overline{\theta_r}]$ and $[\underline{\gamma_r},\overline{\gamma_r}]$ with c.d.f. $F_{\theta r}(\cdot)$, $F_{\gamma r}(\cdot)$ and p.d.f. $f_{\theta r}(\cdot)$, $f_{\gamma r}(\cdot)$. θ_r and γ_r are independent from each other. The timing of our model is as follows: (i) The retailer knows the demand information while manufacture knows the demand distribution; (ii) The manufacture designs the solutions set as $\{\sigma(\theta_r,\gamma_r), t(\theta_r,\gamma_r), L(\theta_r,\gamma_r)\}$, $\sigma(\theta_r,\gamma_r)$ is price discount, $t(\theta_r,\gamma_r)$ is advertisement sharing ratio, $L(\theta_r,\gamma_r)$ is the franchise fee from the retailer to the manufacturer; (iii) The retailer is induce to tell true demand information and to make advertising effort b. The θ_r and γ_r retailer announces are θ_r' and γ_r'.

The manufacture's revenue function:

$$\pi_m(\sigma(\theta_r',\gamma_r'),t(\theta_r',\gamma_r'),L(\theta_r',\gamma_r'))=p_r\cdot(1-\sigma(\theta_r',\gamma_r'))\cdot(s\cdot a+\theta_e\cdot\sigma(\theta_r',\gamma_r')+\gamma_e\cdot b(\theta_r,\gamma_r))+w\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\theta_r',\gamma_r') +\gamma_r\cdot b(\theta_r,\gamma_r)]-(1-t(\theta_r',\gamma_r'))\cdot b(\theta_r,\gamma_r)^2/2+L(\theta_r',\gamma_r'). \quad (14)$$

The retailer's revenue function:

$$\pi_r(\sigma(\theta_r',\gamma_r'),t(\theta_r',\gamma_r'),L(\theta_r',\gamma_r'),\theta_r,\gamma_r)=(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\theta_r',\gamma_r')+\gamma_r\cdot b(\theta_r,\gamma_r)]-t(\theta_r',\gamma_r')\cdot b(\theta_r,\gamma_r)^2/2+L(\theta_r',\gamma_r') \quad (15)$$

According to incentive compatibility (IC) constraint, retailer will be induced to share the true demand information:

$$\pi_r(\sigma(\theta_r,\gamma_r),t(\theta_r,\gamma_r),L(\theta_r,\gamma_r),\theta_r,\gamma_r)\geq\pi_r(\sigma(\theta_r',\gamma_r'),t(\theta_r',\gamma_r'),L(\theta_r',\gamma_r'),\theta_r,\gamma_r). \quad (16)$$

According to individual rational (IR) constraint, retailer may acquire a higher revenue if it accepts the franchise-fee contract than not:

$$\pi_r(\sigma(\theta_r,\gamma_r),t(\theta_r,\gamma_r),L(\theta_r,\gamma_r),\theta_r,\gamma_r)\geq\pi_{\min}^r. \quad (17)$$

Take retailer's optimal decision of b into retailer's revenue function, we can get:

$$\pi_r(\sigma(\theta_r',\gamma_r'),t(\theta_r',\gamma_r'),L(\theta_r',\gamma_r'),\theta_r,\gamma_r)=(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\theta_r',\gamma_r')+(p_r-w)\cdot\gamma_r^2/(2\cdot t(\theta_r',\gamma_r'))]-L(\theta_r',\gamma_r')$$

given γ_r, there should be

$$\partial\pi_r(\theta_r,\gamma_r)/\partial\theta_r=\partial\pi_r(\sigma(\theta_r',\gamma_r'),t(\theta_r',\gamma_r'),L(\theta_r',\gamma_r'),\theta_r,\gamma_r)/\partial\theta_r|_{\theta r'=\theta r,\gamma r'=\gamma r}=-(p_r-w)\cdot\sigma(\theta_r,\gamma_r);$$

given θ_r, there should be

$$\partial\pi_r(\theta_r,\gamma_r)/\partial\gamma_r=\partial\pi_r(\sigma(\theta_r',\gamma_r'),t(\theta_r',\gamma_r'),L(\theta_r',\gamma_r'),\theta_r,\gamma_r)/\partial\gamma_r|_{\theta r'=\theta r,\gamma r'=\gamma r}=(p_r-w)^2\cdot\gamma_r/t(\theta_r,\gamma_r).$$

The retailer's problem can be converted into:

$$\max_{\sigma,t,L}\int_{\underline{\gamma_r}}^{\overline{\gamma_r}}\int_{\underline{\theta_r}}^{\overline{\theta_r}}\pi_m(\sigma(\theta_r',\gamma_r'),t(\theta_r',\gamma_r'),L(\theta_r',\gamma_r'))\cdot f_{\theta_r}(\theta_r)\cdot f_{\gamma_r}(\gamma_r)\mathrm{d}\theta_r\mathrm{d}\gamma_r, \quad (18)$$

$$\text{s.t. } (p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\theta_r,\gamma_r)+(p_r-w)\cdot\gamma_r^2/(2\cdot t(\theta_r,\gamma_r))]-L(\theta_r,\gamma_r) \geq(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\theta_r',\gamma_r')+(p_r-w)\cdot\gamma_r^2/(2\cdot t(\theta_r',\gamma_r'))]-L(\theta_r',\gamma_r'), \quad (19)$$

$$(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\theta_r,\gamma_r)+(p_r-w)\cdot\gamma_r^2/(2\cdot t(\theta_r,\gamma_r))]-L(\theta_r,\gamma_r)\geq\pi_{\min}^r. \quad (20)$$

Lemma 1

According to the two IC and IR constraints, it can be easily derived that: $\sigma(\theta_r,\gamma_r)$ is only decreasing in θ_r; $t(\theta_r,\gamma_r)$ is only decreasing in γ_r.

From Lemma1, we can know that when both the information of θ_r and γ_r are asymmetric. It is hard to derive the optimal contract to maximize manufacture's revenue, while inducing retailer to tell the true information.

2.3 Contract Design

In this part, we consider a special case of the initial problem. We assume that impact of price discount is known to both manufacture and retailer. That is to say, only the γ_r is private information of retailer's information. Furthermore, we design the optimal franchise-fee contract menu $\{\sigma(\theta_r,\gamma_r),t(\theta_r,\gamma_r),L(\theta_r,\gamma_r)\}$ to solve the problem in (18)-(20). We can get:

$$\text{(i)}\quad \pi_r(\gamma_r)=\int_{\underline{\gamma_r}}^{\gamma_r}\frac{(p_r-w)^2\cdot y}{t(\theta_r,\gamma_r)}\mathrm{d}y+\pi_{\mathrm{m}}^r \quad (21)$$

$$\text{(ii)}\quad L(\gamma_r)=(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\gamma_r)+\frac{(p_r-w)\cdot\gamma_r^2}{2\cdot t(\gamma_r)}]-\int_{\underline{\gamma_r}}^{\gamma_r}\frac{(p_r-w)^2\cdot y}{t(\gamma_r)}\mathrm{d}y-\pi_{\min}^r. \quad (22)$$

Manufacture's expected revenue will be:

$$E(\pi_m)=\int_{\underline{\gamma_r}}^{\overline{\gamma_r}}\left\{\begin{array}{l} p_r\cdot(1-\sigma(\gamma_r))\cdot(s\cdot a+\theta_e\cdot\sigma(\gamma_r)+\gamma_e\cdot\dfrac{(p_r-w)\cdot\gamma_r}{t(\gamma_r)}) \\ +w\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\gamma_r)+\gamma_r\cdot\dfrac{(p_r-w)\cdot\gamma_r}{t(\gamma_r)}] \\ -\dfrac{1}{2}\cdot(1-t(\gamma_r))\cdot[\dfrac{(p_r-w)\cdot\gamma_r}{t(\gamma_r)}]^2+(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma(\gamma_r) \\ +\dfrac{(p_r-w)\cdot\gamma_r^2}{2\cdot t(\gamma_r)}]-\displaystyle\int_{\underline{\gamma_r}}^{\gamma_r}\dfrac{(p_r-w)^2\cdot y}{t(\gamma_r)}\mathrm{d}y \end{array}\right\}\cdot f_{\gamma_r}(\gamma_r)\mathrm{d}\gamma_r-\pi_{\min}^r.$$

Theorem 2

$$\sigma^f(\gamma_r)=1-[a\cdot s\cdot\gamma_r+\theta_e\cdot\gamma_r+\theta_r\cdot\gamma_r+(p_r\cdot\gamma_r^2+p_r\cdot\underline{\gamma}_r^2 +w\cdot\gamma_r^2-w\cdot\underline{\gamma}_r^2)\cdot\gamma_e/2]\,/[\gamma_r\cdot(2\cdot\theta_e-p_r\cdot\gamma_e^2)], \quad (23)$$

$$t^f(\gamma_r)=(p_r-w)\cdot\gamma_r^2\cdot(2\cdot\theta_e-p_r\cdot\gamma_e^2)\,/[p_r\cdot(\theta_e\cdot\gamma_r^2+\theta_e\cdot\underline{\gamma}_r^2 +\theta_e\cdot\gamma_r\cdot\gamma_e+\theta_r\cdot\gamma_r\cdot\gamma_e+a\cdot s\cdot\gamma_r\cdot\gamma_e)+w\cdot\theta_e\cdot(\gamma_r^2-\underline{\gamma}_r^2)], \quad (24)$$

$$L^f(\gamma_r)=(p_r-w)\cdot[(1-s)\cdot a-\theta_r\cdot\sigma^f(\gamma_r)+(p_r-w)\cdot\underline{\gamma}_r^2 /(2\cdot t^f(\gamma_r))]-\pi_{\min}^r. \quad (25)$$

See (24), we can find $t^f(\gamma_r)$ is decreasing in γ_r, which fulfils the requirements of IC and IR constraints. Furthermore, we can obtain proposition 3.

Proposition 3

(i) $\partial\sigma^f/\partial s<0$; $\partial\sigma^f/\partial\theta_e>0$; $\partial\sigma^f/\partial\theta_r<0$

(ii) $\partial t^f/\partial s<0$; $\partial t^f/\partial\theta_e<0$; $\partial t^f/\partial\theta_r<0$

Proposition 3(i)(ii) is nearly the same as proposition 2, we will not explain it in details. In addition, from proposition 3(iii), we can derive that when the manufacture's initial online market grows up, manufacture should acquire a larger franchise fee from retailer. When it becomes easier to attract offline consumers to online, manufacture may also acquire a larger franchise fee.

CONCLUSION

In this paper, we have investigated a research on online price discount in an online and offline dual channel. Under the symmetric demand information scenario, once the advertising is easier to convert to demand in any distribution channel, it is more profitable for retailer to invest in more advertisement fee. We also find price discount is a competitive tool to get consumer from retailer, while the cost sharing ratio is an effective tool to coordinate conflict caused by competition. Under the asymmetric demand information scenario, we have derived that retailer has a motivation to lie about the impact factors of price discount and advertising effort. In different situations, retailer will adjust their lying about demand information in three different ways.

Furthermore, retailer will always lie a higher impact factor of advertising effort than the real one. And the retailer's lying behavior may cause revenue loss of manufacture. To help manufacture deal with the problem, we design a franchise-fee contract. However, when both the information of price discount impact and advertising effort impact are asymmetric. In contract menu, optimal price discount is only decreasing in offline price-discount impact factors; retailer's advertisement sharing ratio is only decreasing in offline advertising effort impact factors. It is hard to derive the optimal contract to maximize manufacture's revenue, while inducing retailer to tell the true information. Finally, we consider a special case of the initial problem. We assume that only the impact of price discount is known to both manufacture and retailer. Optimal franchise-fee contract can be obtained.

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APPENDIX

The proof of Lemma 1:

Considering $\pi_r(\theta_r,\gamma_r)$'s first-order derivatives of θ_r and γ_r, we can get:

$$
\begin{aligned}
\pi_r(\sigma(\hat{\theta}_r,\hat{\gamma}_r),t(\hat{\theta}_r,\hat{\gamma}_r),\theta_r,\gamma_r) &= \pi_r(\sigma(\hat{\theta}_r,\hat{\gamma}_r),t(\hat{\theta}_r,\hat{\gamma}_r),\hat{\theta}_r,\hat{\gamma}_r) + \int_{\theta_r}^{\hat{\theta}_r}(p_r - w)\cdot\sigma(\hat{\theta}_r,\hat{\gamma}_r)\mathrm{d}x + \int_{\hat{\gamma}_r}^{\gamma r}\frac{(p_r - w)^2\cdot y}{t(\hat{\theta}_r,\hat{\gamma}_r)}\mathrm{d}y \\
&= \pi_r(\sigma(\theta_r,\hat{\gamma}_r),t(\theta_r,\hat{\gamma}_r),\theta_r,\hat{\gamma}_r) + (p_r - w)\cdot\int_{\theta_r}^{\hat{\theta}_r}[\sigma(\hat{\theta}_r,\hat{\gamma}_r) - \sigma(\theta_r,\gamma_r)]\mathrm{d}x + \int_{\hat{\gamma}_r}^{\gamma r}\frac{(p_r - w)^2\cdot y}{t(\hat{\theta}_r,\hat{\gamma}_r)}\mathrm{d}y \\
&= \pi_r(\sigma(\theta_r,\gamma),t(\theta_r,\gamma),\theta_r,\gamma_r) + (p_r - w)\cdot\int_{\theta_r}^{\hat{\theta}_r}[\sigma(\hat{\theta}_r,\hat{\gamma}_r) - \sigma(\theta_r,\gamma_r)]\mathrm{d}x + (p_r - w)^2\cdot\int_{\hat{\gamma}}^{\gamma}\left[\frac{y}{t(\hat{\theta}_r,\hat{\gamma}_r)} - \frac{y}{t(\theta_r,\gamma_r)}\right]\mathrm{d}y\,.
\end{aligned}
$$

From this equation, Lemma 1 can be obtained.