CSCanada
Management Science and Engineering
Vol. 10, No. 4, 2016, pp. 58-61
DOI:10.3968/9182

ISSN 1913-0341 [Print]
ISSN 1913-035X [Online]
www.cscanada.net
www.cscanada.org

Capacity Allocation Study Based on Lead Time

WEI Chao[a], *

[a]School of Business Administration, South China University of Technology, Guangzhou, China.
*Corresponding author.

Received 13 September 2016; accepted 9 December 2016
Published online 26 December 2016

Abstract

In this paper, we study a firm serving two kinds of products (services) which are fast lead time service and slow lead time service. The products (services) differ only in their delivery times and prices. We assume that there are two types of customers which are time-sensitive and price-sensitive customers in the market. We find that the optimal proportion of capacity allocation is influenced by the proportion of lead time sensitive customers, capacity and the total size of the market. An increase in the optimal proportion of capacity allocation can increase the capacity and the proportion of lead time sensitive customers, but decreases the total size of the market.

Key words: Capacity allocation; Lead time; Heterogeneous customer

Wei, C. (2016). Capacity Allocation Study Based on Lead Time. *Management Science and Engineering*, *10*(4), 58-61. Available from: URL: http://www.cscanada.net/index.php/mse/article/view/9182
DOI: http://dx.doi.org/10.3968/9182

INTRODUCTION

Since Stalk introducing "time-based competitor" concept in the late 80, the "lead time" not only becomes the focus of scholars, but also becomes an important factor affecting competitiveness (Stalk, 1988). For the past few decades, with the emphasis on lead-time competition of enterprises, more and more enterprises start to provide two kinds of products (services) (fast lead time service and slow lead time service), such as UPS, Fedex and so on. Based on this, how to allocate the limited capacity for different kinds of products (services) have already become the challenges which this kind of company is facing.

Lead-time competition has become the focus of research, in particular taking into account capacity constraints. Palaka et al. (1998) take into account the capacity constraints and study lead time and price decision under the assumption that the demands are sensitive to lead time. Based on the service level constraint, So and Song (1998) provide the optimal values of price and lead time through modelling a firm as model a firm as a single server queue. So (2000) extends So and Song (1998) to introduce competition mechanism. Ray and Jwekes (2004) assume that increased investment can reduce lead times and increase capacity, and build a model to study lead-time decision, where price itself is sensitive to lead time. Based on M/M/1 queuing systems, Teimoury et al. (2011) study the lead time, price and capacity optimization issues of MTO (make to order) company, and taking into account the time-sensitive and price-sensitive customers. Zhu (2015) consider a decentralized supply chain consisting of a supplier and a retailer and study lead time, price and capacity decisions when the demand is sensitive to price and lead time.

However, there are two deficiencies among the above papers. First, they consider capacity, but do not take into account capacity allocation, and the impact of lead time for capacity allocation. Second, they did not take into account heterogeneity of the customer, in fact, heterogeneous customer has become one of the most important factor that affect capacity allocation decision. When more time-sensitive consumers, companies which provide fast and slow lead time service allocate more capacity for the fast lead time service, such as SF- Express. When more price-sensitive consumers, companies which provide fast and slow lead time services allocate more capacity for the fast lead time service, such as YT- Express. This paper considers a company which provides fast and slow lead time service, and analyzes the optimal proportion of capacity allocation through modelling.

1. MODEL

In this paper, we consider a company which provides two kinds of products (services). One is faster, and the other is slower. Capacity and Total demand is assumed to be constant. The company pursue to maximize its profit through making capacity allocation decisions. And we classify customers into two groups: lead time sensitive (LS) customer and price sensitive (PS) customer. Let θ denote the proportion of lead time sensitive customers, and the proportion (1-θ) are price sensitive customers. Let λ denote the arrival rate of customers, so $\theta\lambda$ are the rate of lead time sensitive customers and $(1-\theta)\lambda$ are the rate of price sensitive customers. Note that is $0<\theta<1$. It has been shown in the literature that customer arrivals are modeled as a Poisson process (So & Song, 1998, 2000).

Whether an arrived customer places an order depends on price and lead time. To model the expected demand rate, we use a linear function

$$d_i = \omega - p_i - \beta L_i, i \in \{f, s\}$$

for each customer segment, where f represent the faster case and s represent the slower case. And the parameter ω represents maximum attainable demand (market potential) corresponding to zero price and zero lead time. The lead time sensitivity β measures the urgency degree that the customer wants to complete the product or service. With one unit time delay, the demand will reduce β units. The parameter p_i represents the price of product or service and L_i denote the lead time. Our objective is to maximize the expected net profit through capacity allocation between the two kinds of products (services). Note that is the demand must be positive. This means that $\omega-p_i-\beta L_i>0$.

To simplify, we use the following notation through the text:

Parameters:

M: Capacity cost parameter

μ: Service rate of a production queue serving customers

γ: Proportion of the capacity which used to provide the faster

θ: Proportion of lead time sensitive customers among all arrived customers

λ: Customer mean arrival rate

d: Expected demand generated by the quoted price and quoted lead time

p_i: Price in the option intended for type products

L_i: Lead time in the option intended for type products

ω: Maximum attainable demand (market potential) corresponding to zero price and zero lead time

α: Desired lead time reliability predetermined by managers

β: Lead time sensitivity of demand

In addition, In this paper, similar to So and Song(1998), we have the following lead time reliability(service level) constraint:

$$e^{-(\mu_i-d_i)L_i} \le 1-\alpha\ ,$$

where denotes the desired lead time reliability predetermined by managers. And here this paper assumes that the company has a same standard of the desired lead time reliability (α) no matter of what kinds of products. In addition, in order to facilitate the writing, we use superscripts to denote the product strategy. For example, p_f denotes the price of the product adopting fast lead time, p_s denotes the price of the product adopting slow lead time.

2. CAPACITY ALLOCATION

In this section, We study a company which provides slow lead time service and fast lead time at the same time. Hence, the LS customer will choose to buy the fast lead time service products. The PS customer will make a choice to buy the slow lead time service products. So the expected demand rate can be written as $d_f=\omega-p_f-\beta L_f$, $d_s=\omega-p_s-\beta L_s$ for each queue. The marketing and production decisions are considered simultaneously. And the firm seeks to maximize profit. Its goal can be written as:

max $\pi=(p_f-c_f)\theta\lambda+(p_s-c_s)(1-\theta)\lambda-M(\gamma\mu)-M(1-\gamma)\mu$.

Subject to

$$e^{-(\gamma\mu-\theta\lambda)L_f} \le 1-\alpha, e^{-((1-\gamma)\mu-(1-\theta)\lambda)L_s} \le 1-\alpha, \quad (1)$$

$$\theta\lambda \le \gamma\mu, (1-\theta)\lambda \le (1-\gamma)\mu, \quad (2)$$

$$\theta\lambda \le \omega - p_f - \beta L_f, (1-\theta)\lambda \le \omega - p_s - \beta L_s, \quad (3)$$

$$c_f \le p_f, c_s \le p_s, \quad (4)$$

$$0 < L_f < L_s, 0 < p_s < p_f\ . \quad (5)$$

Constraint (1) are the lead time reliability constraints for the lead time sensitive customers and the price sensitive customers, respectively. Constraint (2) is the stability condition that the ability must be more than the demand. Constraint (3) denote that the mean demand rate served by the firm does not exceed the demand generated by the quoted price and lead time. Constraint (4) ensure that the price must be higher than the variable cost. Constraint (5) require that the lead time is shorter and thus the price is higher in the express service option than in the regular service option.

As the previous studies (So & Song, 1998; Zhao et al., 2012), the lead time reliability constraint in the each model should be binding at optimality. So the optimal lead time for each queue is

$$L_f^* = \frac{k}{\gamma\mu-\theta\lambda}; L_s^* = \frac{k}{(1-\gamma)\mu-(1-\theta)\lambda}\ . \quad (6)$$

From the constraint (3), we can see that the optimal price for each queue is

$$p_f^* = \omega - \theta\lambda - \beta L_f^*; p_s^* = \omega - (1-\theta)\lambda - \beta L_s^*. \quad (7)$$

Combining (6) and (7) we can express the price again as:

$$p_f^* = \omega - \theta\lambda - \frac{k\beta}{\gamma\mu - \theta\lambda}; p_s^* = \omega - (1-\theta)\lambda - \frac{k\beta}{(1-\gamma)\mu - (1-\theta)\lambda} \quad . \tag{8}$$

Since, $0 < L_f^* < L_s^*, 0 < p_s^* < p_f^*$, so

. Thus, we can a range about capacity allocation coefficient as follows:

Proposition 1.

i. When $0 \le \theta \le \frac{1}{2}$, $1 - \frac{(1-\theta)\lambda}{\mu} \ge \gamma > \frac{1}{2} - \frac{(1-2\theta)\lambda}{2\mu}$.

ii. When $1 \ge \theta > \frac{1}{2}$, $1 - \frac{(1-\theta)\lambda}{\mu} \ge \gamma > \frac{(-1+2\theta)\lambda((-1+2\theta)\lambda+\mu)-2k\beta+\sqrt{4k^2\beta^2+(1-2\theta)^2\lambda^2(\lambda-\mu)^2}}{2(-1+2\theta)\lambda\mu}$.

This implies that need to satisfy this range, like the proposition 1, otherwise the company will use a single lead time model. And then substituting the optimal price and the optimal lead time into the profit function, we can get the optimal profit.

$$\pi^* = \theta\lambda\left(a - \theta\lambda + \frac{k\beta}{\theta\lambda - \gamma\mu} - c_f\right) + (1-\theta)\lambda\left(a + (-1+\theta)\lambda + \frac{k\beta}{\lambda - \theta\lambda + (-1+\gamma)\mu} - c_s\right) - M\mu \ . \tag{9}$$

First-Order Conditions (FOC) provide Equation (9) for γ. To show uniqueness, note that

$$\frac{\partial \pi^*}{\partial \gamma} = k\beta\lambda\mu\left(\frac{\theta}{(\theta\lambda - \gamma\mu)^2} - \frac{1-\theta}{\left((-1+\theta)\lambda + \mu - \gamma\mu\right)^2}\right). \tag{10}$$

And concavity of π^* for can be shown from the second-order conditions:

$$\frac{\partial^2 \pi^*}{\partial \gamma^2} = 2k\beta\lambda\mu^2\left(-\frac{\theta}{(\gamma\mu - \theta\lambda)^3} - \frac{1-\theta}{(\mu - \gamma\mu - (1-\theta)\lambda)^3}\right) < 0 \ .$$

So, accord to the optimization theory, we can see that there are at least one γ, which can make π^* achieve the maximize value. Therefore, we can make the Equation (10) equal to 0. Hence, the optimal capacity allocation coefficient γ is

$$\gamma_1^* = \frac{\theta\left(\mu - 2\lambda(1-\theta)\right) + (\mu - \lambda)\sqrt{(1-\theta)\theta}}{(2\theta - 1)\mu}, \tag{11}$$

$$\gamma_2^* = \frac{\theta\left(\mu - 2\lambda(1-\theta)\right) - (\mu - \lambda)\sqrt{(1-\theta)\theta}}{(2\theta - 1)\mu}. \tag{12}$$

According to the proposition 1 and combining (11) and (12), we can see, when $\frac{1}{2} < \theta \le 1$, γ_2^* is in the feasible scale of γ but γ_1^* is not. At the same time, γ_2^* and γ_1^* is not in the feasible scale of γ when $0 < \theta \le \frac{1}{2}$. Thus, we can reach the theory 1 as below.

Theory 1 *If and only if more than half of the customers in the market prefer to choose the fastest lead time service ($\frac{1}{2} < \theta \le 1$), the company, which pursues maximize the profits, will have a chance to provide Mix-lead time strategy. And when the company takes Mix-lead time strategy (using fast lead time service and slow lead time service), the optimal percentage of capacity*

$$\gamma = \frac{\theta\left(\mu - 2\lambda(1-\theta)\right) - (\mu - \lambda)\sqrt{(1-\theta)\theta}}{(2\theta - 1)\mu}.$$

Theory 1 states when the number of customers, which prefer to choose the fastest lead time service, is more than half the total market share, enterprises will have a chance to provide two kinds of products (services), which is that the company provide fast lead time service and slow lead time service. And then the optimal proportion of capacity allocation γ is $\frac{\theta\left(\mu - 2\lambda(1-\theta)\right) - (\mu - \lambda)\sqrt{(1-\theta)\theta}}{(2\theta - 1)\mu}$.

First-Order Conditions (FOC) provide Equation (12) for. To show uniqueness, note that

$$\frac{\partial \gamma_2^*}{\partial \theta} = \frac{-16\theta^3\lambda + 8\theta^4\lambda + \sqrt{-(-1+\theta)\theta}(\lambda - \mu) + 2\theta^2(6\lambda - \mu) + 2\theta(-2\lambda + \mu)}{2(1-2\theta)^2(-1+\theta)\theta\mu}. \tag{13}$$

Second-order condtitions for θ provided by Equation (12) can be shown:

$$\frac{\partial^2 \gamma_2^*}{\partial \theta^2} = \frac{\left(-1 + 4(-1+\theta)\theta\left(-3 + 4\sqrt{-(-1+\theta)\theta}\right)\right)(\lambda - \mu)}{4(-(-1+\theta)\theta)^{3/2}(-1+2\theta)^3\mu} \ . \tag{14}$$

And because $\theta \epsilon (\frac{1}{2}, 1]$, we can see $\frac{\partial^2 \gamma_2^*}{\partial \theta^2} > 0$ and $\frac{\partial \gamma_2^*}{\partial \theta} > 0$ easily. Therefore, as θ increases, the optimal capacity allocation will also increase. This states the number of the customer who chooses the faster lead time service in the market can influence the optimal capacity allocation. The more this kind of customer, enterprises will allocate more ability in the fastest lead time service.

Accord to the same way, we can see the first-order conditions provide Equation (12) for μ and λ.

$$\frac{\partial \gamma_2^*}{\partial \mu} = \frac{(2(1-\theta)\theta + \sqrt{(1-\theta)\theta})\lambda}{(2\theta - 1)\mu^2}, \frac{\partial \gamma_2^*}{\partial \lambda} = \frac{-2(1-\theta)\theta - \sqrt{(1-\theta)\theta}}{(2\theta - 1)\mu}. \tag{15}$$

And because $\theta \epsilon (\frac{1}{2}, 1]$, it is clear that $\frac{\partial \gamma_2^*}{\partial \mu} > 0$ and $\frac{\partial \gamma_2^*}{\partial \lambda} < 0$. So the theory 2 can be concluded as follows:

Theory 2 *When the company provide two kinds of products (services) (fast lead time service and slow lead time service), the optimal proportion of capacity allocation has the following properties:*

i. An increase in the proportion of lead time sensitive customers θ can results in an increase in γ.

ii. An increase in the capacity μ can results in an increase in γ.

iii. An increase in the total size of the market λ can results in a decrease in γ.

Hence, Theory 2 states when the number of lead time sensitive customers (LSC) more than half of the total market share, enterprises will have a chance to adopt the fast lead time service. Otherwise, the enterprises will not adopt fast lead time service. With the increase in the proportion of lead time sensitive customers θ, the ability assigned to the fast lead time service will be higher. With the enhancement of enterprise capabilities, the ability assigned to the fast lead time service will be higher. With the increase in the total size of the market, the ability assigned to the fast lead time service will be less. This shows that when the market is not yet fully developed, companies will adopt the slow lead time service. This corresponds to the reality. The 80's of the last century in China, because the market is not fully covered, enterprises mainly adopt slow lead time service. And with the development of the market in China, today's enterprises begin to adopt the fast lead time service.

CONCLUSION AND FUTURE WORK

In this paper, we study a firm with providing two kinds of products (services), the one is faster, and the other is slower. We provide analytical models and give the optimal capacity allocation. We consider heterogeneous customer. And we also find that when the proportion of lead time sensitive customers was more than half of the customers in the total market the company would have a chance to provide two kinds of products (services). Otherwise, the company will provide single product (service).

By building a model, we give the optimal capacity allocation. After analysis, we find that the optimal allocation of capacity under the influence of the proportion of lead time sensitive customers, enterprise capabilities and the total size of the market. The optimal capacity allocation is increasing in the proportion of lead time sensitive customers and capabilities, but decreasing in the total size of the market . And we give a formula about the optimal capacity allocation based on capacity, market share and the proportion of lead time sensitive customers, in order to give company inspiration which provides two kinds of products (services).

Our model has some limitations that can suggest future research. First, object of this paper is a company which provides two kinds of products (services). And then we can continue to study a company's product decisions, which is how to make the choice between single lead time strategy and mix-lead time strategy. Then, we can also consider the competitive scenario.

REFERENCES

Palaka, K., Erlebacher, S., & Kropp, D. H. (1998). Lead-time setting, capacity utilization, and pricing decisions under lead-time dependent demand. *IIE Transactions, 30*(2), 151-163.

Ray, S., & Jewkes, E. M. (2004). Customer lead time management when both demand and price are lead time sensitive. *European Journal of Operational Research, 153*(3), 769-781.

So, K. C., & Song, J. (1998). Price, delivery time guarantees and capacity selection. *European Journal of Operational Research, 111*(1), 28-49.

So, K. C. (2000). Price and time competition for service delivery. *Manufacturing & Service Operations Management, 2*(4), 392.

Stalk, Jr, G (1988). Time-the next source of competitive advo nioge. *Harvard Business Review, 7*(8), 41-52.

Teimoury, E., et al. (2011). Price, delivery time, and capacity decisions in an M/M/1 make-to-order/service system with segmented market. *The International Journal of Advanced Manufacturing Technology, 57*(1-4), 235-244.

Zhao, X., Stecke, K. E., & Prasad, A. (2012). Lead time and price quotation mode selection: Uniform or differentiated. *Production and Operations Management, 21*(1), 177-193.

Zhu, S. X. (2015). Integration of capacity, pricing, and lead-time decisions in a decentralized supply chain. *International Journal of Production Economics, 164, 1*4-23.

