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Aggregate Planning Based on Stochastic Demand DEA Model With an Application in Production Planning

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Abstract

Market demand and inventory are usually known in traditional DEA-based resource allocation method. However, market demand is always changing according to the market discipline. Thus, it is not rigorous to view market demand as constant. To cope with the uncertain demand, we further develop our mathematical model and impose the normality assumption for the stochastic product demands. In the end, a numerical example with hypothetical production data is used to illustrate the model.

Key words: Production planning; Resource allocation; Data envelopment analysis (DEA); Stochastic demand

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INTRODUCTION

Nowadays, production planning becomes a hot issue. Many factors should be considered in the production planning process, such as inventory, market demand, and raw material and so on. Organizations always pursue the maximum of the profit and the minimum of the cost which is discussed by many researches. Fisher et al. (2001) built a cost-based minimization model based on the assessment of ending inventory to deal with a production planning problem. Leung et al. (2003) also constructed a multi-objective model to cope with a centralized decision-making situation. The profit was maximized during the process.

During recent production planning researches, people usually assume that the market demand is known in the model. However, Demand is always changed according to the environment. Thus, the uncertainty was always neglected in the past researches. It is obviously not always a realistic assumption. Deterministic planning methods may produce unreasonable results when the effect of demand uncertainty is not captured. Without the description of demand uncertainty could lead to either loss of market share and unsatisfied customer demand or excessively high inventory holding costs (Petkov & Maranas, 1998). It may also happen that the data available to the decision makers can vary within some limits due to various reasons. For example, the demands are always stochastic in nature, due to market trends, seasonality, etc. One of the key sources of uncertainty in the production-planning system is demand uncertainty (Gupta & Maranas, 2003). Thus, it is necessary to consider the influence of demand uncertainty involved in the production process. To cope with such uncertainty, stochastic models including demand uncertainty are desirable in planning future activities. Bertrand and Rutten. (1999) investigated three different production-planning procedures that make use of recipe flexibility to cope with uncertainty in demand and supply. These three procedures were investigated via an experimental design of computer simulations of an elementary small scale model of the production planning situation. Gupta and Maranas (2000) proposed a two-stage, stochastic programming approach is proposed for incorporating demand uncertainty in multi-site midterm supply-chain planning problems. Sodhi and Tang (2009) extended the linear programming (LP) model of deterministic supply-chain planning to take demand uncertainty and cash flows into account for the medium term. Their survey could be a basis for making modeling/ solution choices in research and in practice to manage the risks pertaining to unmet demand, excess inventory, and cash liquidity when demand is uncertain. In the following, a general form and a specific form of stochastic scenario are presented, which extends our analyses in the previous section to incorporate the uncertainty

Since Data envelopment analysis (DEA) was first developed by Charnes, Cooper, and Rhodes (1978), it has been widely used on the measurement of the relative efficiency of Decision Making Units (DMUs) involving multiple-input and multiple-output. The DEA technique is a nonparametric method which does not need any prior information about quantifying or weighing qualitative factors. Du et al. (2010) addressed two DEA-planning ideas for arranging new input-output combinations in the next period given the forecasted demand changes. One is to optimize the average production efficiency within the whole organization measured by CCR efficiency, and the other is to maximize total output while simultaneously minimize the total inputs expended in the production process. However, many DEA models proposed by Charnes and other scholars were deterministic. DEA models and concepts are formulated in terms of the "P-Models" of Chance Constrained Programming, which is then modified to contact the "satisficing concepts" of H.A. Simon. (Cooper, Huang, & Li, 1996)

Through the summary of the past papers, we can find that the application of DEA models in the production planning becomes a hot issue. However, most of these researches are based on the known market demand. It is inappropriate to always assume that demand is known. The market demand may be changed by the market environment, seasonality and so on. Some demand may conform to certain statistical law. Thus, the novelty of the current paper lies in the combination of stochastic demand and DEA resource allocation model. We introduce the "service level" which helps the managers make plans with high service quality. While the managers choose high service level, they will pay more for the production cost.

The rest of the paper is organized as follows: In section 1, we propose a multi-period model and exhibit the associated assumptions and notations. Then a numerical example is given to illustrate the proposed approach in section 2. Finally, we conclude the paper in Section 3.

1. THE MODEL

1.1 Notations

The notations in the model are defined as follows:

(1) x_{ij}^h and x_{sj}^h are historical input data, where x_{ij}^h represents i-th variable input and x_{sj}^h s-th non-variable input for j-th historical data. Specially x_s^h represents the s-th non-variable input since it stays the same in the predicting scenario, hence $x_s^h = x_{sj}^h$, $j = 1,2,\dots k$.

(2) y_{rj}^h indicates r-th output for j-th historical data.

(3) x_i^p is the i-th predicted variable input value of the company's p-th period in future.

(4) y_r^p is the r-th target output of the company's p-th period in future.

(5) I_r^p is the p-th time period inventory volume for r-th output, while I_r^0 represents the initial stock, which is a constant quantity.

(6) w_i^p indicates the unit cost for the i-th input in the p-th time period.

(7) q_r^p indicates the unit inventory cost for the r-th output in p-th time period.

(8) D_r^p represents the p-th period market demand for the r-th output, which is a known quantity when scheduling the production process for certain time period.

(9) i is the subscript for variable input.

(10) s is the subscript for non-variable input.

(11) p is the superscript for the forecasting time period.

(12) n is the total number of forecasting time period.

(13) r is the subscript for output.

(14) j is period for reference data, here signifying the subscript for processing units.

1.2 The Deterministic Model

Complying with the aforementioned notations, we firstly introduce a multi-period production planning model based on known market demand (Bi, Mao, & Ding, 2013):

$$\min \sum_{p=1}^{n}\sum_{i=1}^{m} x_i^p w_i^p \sum_{p=1}^{n} + q_r^p I_r^p \qquad (1)$$

$$
s.t.\left\{
\begin{array}{lll}
\left.\begin{array}{ll}
\sum \lambda_j^1 x_{ij}^h \le x_i^1 & (2)\\
\sum \lambda_j^1 x_{sj}^h = x_s^h & (3)\\
\sum \lambda_j^1 y_{rj}^h \ge y_r^1 & (4)\\
y_r^1 + I_r^0 - D_r^1 = I_r^1 & (5)
\end{array}\right\} & \text{the first time period} \\[2ex]
\left.\begin{array}{ll}
\sum \lambda_j^2 x_{ij}^h \le x_i^2 & (6)\\
\sum \lambda_j^2 x_{sj}^h = x_s^h & (7)\\
\sum \lambda_j^2 y_{rj}^h \ge y_r^2 & (8)\\
y_r^2 + I_r^1 - D_r^2 = I_r^2 & (9)
\end{array}\right\} & \text{the second time period} \\[2ex]
\left.\begin{array}{ll}
\sum \lambda_j^3 x_{ij}^h \le x_i^3 & (10)\\
\sum \lambda_j^3 x_{sj}^h = x_s^h & (11)\\
\sum \lambda_j^3 y_{rj}^h \ge y_r^3 & (12)\\
y_r^3 + I_r^2 - D_r^3 = I_r^3 & (13)
\end{array}\right\} & \text{the third time period} \\
\vdots & \\
\left.\begin{array}{ll}
\sum \lambda_j^n x_{ij}^h \le x_i^n & (14)\\
\sum \lambda_j^n x_{sj}^h = x_s^h & (15)\\
\sum \lambda_j^n y_{rj}^h \ge y_r^n & (16)\\
y_r^n + I_r^{n-1} - D_r^n = I_r^n & (17)
\end{array}\right\} & \text{the n-th time period} \\[2ex]
I_r^0 \text{ is a constant quantity,} & (18) \\
I_r^p, \lambda_j^p, x_i^p, y_r^p, w_i^p \ge 0, & (19) \\
i = 1,\dots,m,\ j = 1,\dots,k,\ p = 1,\dots,n,\ r = 1,\dots,l, & (20)
\end{array}\right.
$$

The objective function in (1) minimizes the total costs. The first part of objective function represents input costs (the sum of products of unit cost for each input and the corresponding volume of input resource in every period) and the rest represents stock costs (the sum of products of unit inventory cost and the corresponding volume of inventory in every period). (2)-(17) are 4 groups of cognate constraints. Take the first period as an example. (2)-(4) are the company's capacity constraints: constraint (2) demonstrates the constraints for variable inputs; (3) for non-variable inputs; constraint (4) for outputs, which are all derived from the theory of production possibility set, one of the central elements in DEA field. Constraint (5) implies that the products of the firm must satisfy the market demand, where initial stock I^0_r and demand D^p_r are known quantities. The posterior constraints for period 2, period 3 until period n all conform to similar situations. To speak of the aforesaid assumption (1), constraints (3), (7), (11) and (15) are actually reduced to convex weights constraints ($\sum \lambda^p_j=1$). Therefore they represent the technology of variable return to scale in nature.

Here we illustrate the multi-period model of a specific situation for a company. Our model is to determine the production plan for the next three months and use the historical data of input-output of the six preceding months and the next months demand. When the total cost of the company represented by the objective function is minimized, we can obtain the rational resource allocation plan, output target and inventory storage. A month can also be replaced by a quarter, a year and so on.

1.3 Chance Constraints

In this section, we introduce the chance constraint to consider the impact of the environment variation. Since the proper representation of stochastic parameters is a key component of incorporating the consideration of uncertainty into the stochastic model and the demand is an essential stochastic variable in the production planning process. We will model the demand as normally distributed with a specified mean and standard deviation. The normality assumption is widely invoked in literature (S Nahmias; Steven Nahmias & Cheng, 1997; Wellons & Reklaitis, 1989) because it has the essential features of demand uncertainty and is convenient to use.

Let us introduce the chance constraint which is used to consider the demand uncertainty. We assume that demand variables are random variables, but the other variables remain deterministic as before. By introducing the stochastic variable $\tilde{D}^p_r(p=1,\dots n)$ to substitute for the deterministic variable D^p_r, the constraints including variable D^p_r can be transformed as:

$$P\{y^p_r + I^{p-1}_r - \tilde{D}^p_r \ge I^p_r\} \ge 1-\alpha_p, \qquad (21)$$

Here, P means "Probability" and the $\tilde{D}^p_r$ is used to identify the demand as a random variable with a known normality distribution and parameters for p-th period (Cooper, Huang, & Li, 1996). Other variables remain the same definition as before.

Since we can let I^p_r=0 and $y^p_r + I^{p-1}_r > \tilde{D}^p_r$ the constraint (21) is satisfied evidently. Thus it is reasonable to write:

$$P\{y^{p^*}_r + I^{p-1}_r - \tilde{D}^p_r \ge I^{p^*}_r\} = 1-\alpha_p^* \ge 1-\alpha_p, \qquad (22)$$

Here, "*" means the optimal value. Thus, the α^*_p means the possibility of achieving this optimal value. We should note that we must have $\alpha^*_p \le \alpha_p$, since 1-α_p is prescribed in the constraint (22) for period p as the chance allowed for stochastic demand. Thus, 1-α^*_p represents a service level. We can set the 1-α_p to make the constraint of service level.

The constraint (22) is intended mainly to explain the stochastic conception. Now we provide the "deterministic transformation" for the computation feasibility. We start by assuming $\tilde{D}^p_r$ is a random variable with a known normal distribution of $N(\mu_p, \sigma^2_p)$. The index p means μ_p and σ_p is defined for $\tilde{D}^p_r$ in p-th period. Thus, we can obtain:

$$P\{\tilde{D}^p_r \le y^p_r + I^{p-1}_r - I^p_r\} \ge 1-\alpha_p, \qquad (23)$$

$$P\{\frac{\tilde{D}^p_r - \mu_p}{\sigma_p} \le \frac{y^p_r + I^{p-1}_r - I^p_r - \mu_p}{\sigma_p}\} \ge 1-\alpha_p, \qquad (24)$$

Here, we introduce a new variable $\tilde{z}_p$, defined by:

$$\tilde{z}_p = \frac{\tilde{D}^p_r - \mu_p}{\sigma_p}, \; p = 1,\dots n, \qquad (25)$$

We can see that $\tilde{z}_p$ follows the standard normal distribution which means $\tilde{z}_p$ to has unit variance and zero mean. Then we can introduce the direct substitution in (25) for each period.

$$P\{\tilde{z}_p \le \frac{y^p_r + I^{p-1}_r - I^p_r - \mu_p}{\sigma_p}\} \ge 1-\alpha_p, \qquad (26)$$

Thus, we can use the property of $\tilde{z}_p$ with standard normal distribution to transform (26) as:

$$\frac{y^p_r + I^{p-1}_r - I^p_r - \mu_p}{\sigma_p} \ge \Phi^{-1}(1-\alpha_p), \qquad (27)$$

Where, Φ means the standard normal distribution function, so Φ^{-1} is its inverse function which is so-called fractile function.

Through above transformation, we can transform the stochastic problem into the deterministic problem. μ_p and μ_p can be obtained by the market statistic or other data statistic.

2. EMPIRICAL ILLUSTRATION

Since it is hard to obtain the data from company, we use hypothetical production to validate rationality of our model.

2.1 Data

We generally assume that a company has 3 kinds of inputs. One non-variable input: capital and two variable inputs: the monthly staff number and the monthly total amount of raw material. One output is the number of products. The hypothetical historical data for six months are shown in Table 1. Table 2 exhibits the market demand in the next three months (D^p_r), the unit cost for input resources (w^p_1 and w^p_2), and unit cost for inventory (q^p_r). The initial stock I^0_1=1800.

According to the notations in Section 2, the index is i=2， s=1， p=3， r=1， j=6. Here, we view the six processing units in the sequential months as DMUs. The DMUs are compared with each other to form a possible production set. The plan of next 3 months is forecasted by the production set.

Table 1
Hypothetical Historical Data

(DMU$_j$)	x_{1j} (staff number/people)	x_{2j}(amount of raw material/ton)	x_{3j}(fixed input/million dollar)	y_j(product number/dozen)
1	55	57,200	100	4,800
2	60	62,400	100	5,000
3	58	59,200	100	4,900
4	54	55,080	100	4,500
5	50	53,000	100	4,400
6	50	54,000	100	4,450

Table 2
Demand For Period 1, 2, 3 and Other Parameters

Month(p)	D^p_1(dozen)	w^p_1(dollar/people)	w^p_2(dollar/ton)	q^p_1(dollar/dozen)
1	5,150	1	2.5	0.4
2	5,200	2	2.5	0.4
3	5,300	0.8	2	0.5

2.2 Results of Deterministic Method

Table 3 exhibits the solution of the problem which is solved by the deterministic model. We can see a general trend from month 1 to month 3. Inputs and outputs are gradually increased while the inventories decreased as time flow. The stock volume for month 3 eventually goes to 0 and the total cost is 386782.8 dollars.

Table 3
Forecasting Results

Month (p)	x^p_1(staff number/people)	x^p_2(amount of raw material/ton)	y^p_1(product number/dozen)	I^p_1(product number/dozen)
1	50	53,000	4,400	1,050
2	53	55,625	4,650	500
3	55	57,200	4,800	0
min		386782.5(dollar)		

2.3 Results of Stochastic Method

To consider the problem of market variation, we add the chance constraints. And the main aspect of the market variation is the demand uncertainty. We may not know the accurate amount of demand, but the probability distribution can be calculated by mathematical statistics. We assume that the stochastic variable of demand follows the normal distribution, thus the mean value and standard deviation is shown in Table 4. To compare the results with the deterministic method, we let the μ equal to the demand value in Table 2, and other data are the same in section 2.1.

The solution of the stochastic model is shown in Table 5 and Table 6. Table 5 exhibits the results with α_p=0.9, p=1,2,3. which means a low service level. When we compare Table 5 with Table 3, it is interesting to find that the total cost is decreased. The direct reason is the decrease of the input of raw material in month 2. Actually, the results reflect that the company sacrifices their quality of service to save the total cost when they face the market variation. The company chooses to cut down their input of raw material in month 2 which leads to a high possibility of not satisfying the demand in this period. Some customer will even be angry about not getting their product.

When we choose a high service level with α_p=0.1, p=1,2,3, the total cost is raised to 387, 792 which is listed in Table 6. We can notice that the input 54, 56028 in month 2 in Table 6 are higher than that in Table 3. The company uses more inputs to produce more products to obtain a high possibility to satisfy the demand in month 2. Although the total cost is high, this choice offers a high quality service level.

The different option of the 1-α(service level) gives the manager of the company a trade-off between service level and the total cost. A trade-off curve helps the manger to make the decision is shown in Figure 1. We can see that the total cost increase as the service level is improved. The increased fee is produced by the high level of service quality. It is interesting to see that the total cost equals to the results in Table 2 if α=0.5.We call this point a Neutral Point. At this Neutral Point, the total cost is neither higher nor lower than the total cost. It just equals to the value in Table 2 which means we don't pay for the promotion of service quality or save the money from the bad service quality. Different companies have different strategies. For some military companies, they have strict request of their product demand. If they cannot supply sufficient product in such period, it will result in huge lose. Thus, the military companies should choose a low value of α to ensure the supply of the product such as bullets, guns, etc.. For some other companies, they may more careful about their cost. These companies can choose a relatively low service level to save money.

Table 4
Mean Value and Standard Deviation

Period(p)	μ_p(mean value)	σ_p(standard deviation)
1	5,150	10
2	5,200	10
3	5,300	10

Table 5
Forecasting Results With α_p=0.9, p=1, 2, 3

Month(P)	x^p_1(staff Number/People)	x^p_2(amount of raw material/ton)	y^p_1(product number/dozen)	I^p_1(product number/dozen)
1	50	53,000	4,400	1,063
2	53	55,222	4,612	487
3	55	57,200	4,800	0
min		385,775 (dollar)		

Table 6
Forecasting Results With α_p=0.1, p=1, 2, 3

Month(p)	x^p_1(staff number/people)	x^p_2(amount of raw material/ton)	y^p_1(product number/dozen)	I^p_1(product number/dozen)
1	50	53,000	4,400	1,037
2	54	56,028	4,688	513
3	55	57,200	4,800	0
min		38,7792 (dollar)		

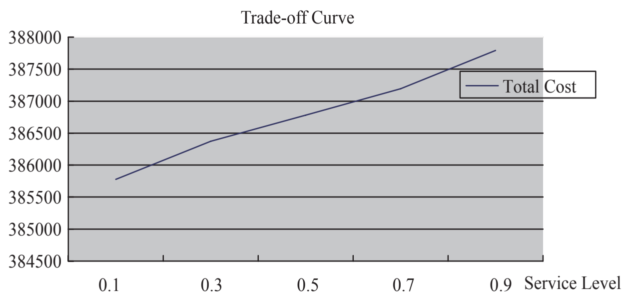

Figure 1
Trade-Off Curve

CONCLUSION

DEA has been widely used to various performance evaluation situations especially for non-profit entities since DEA does not require a priori information regarding concrete operating process. Moreover, it is important to schedule the production planning reasonably. Even though many scholars do a lot of researchers on this subject, most of these researchers are based on the deterministic methodologies. Thus, they cannot combine stochastic demand and production planning together.

A DEA-based multi-period production planning model was constructed in this paper. This approach further promotes the development of production planning, especially in the direction of stochastic approach. A hypothetical numerical example was applied to illustrate this model. The results indicate that it is possible to obtain a detailed plan for input resource allocation and inventory policy when minimizing the enterprise's costs after the market demand is satisfied. Moreover, a stochastic model is given to adapt the variation of the market in which demand is defined as stochastic variable. In the trade-off curve, we conclude that if a company wants a high level of service level, it should pay more money to produce more products to satisfy the customer.

Finally, our paper points out a feasible direction for non-profit organizations in the multi-input and multi-output. For example, how to arrange patients in diverse departments to receive high quality medical treatment and simultaneously keep the hospital operating in highly efficient status is now in the spotlight. The manager could use the trade-off curve to make the decision when considering the service level and the efficiency simultaneously.

Future research can consider different statistical hypothesis for the market demand, for example, *t*-distribution, and *f*-distribution and so on. Although each of the above situations will bring a number of obstacles for researchers, they are the ones which offer possibilities and future research directions to promote further progress in this area.

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