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The Ruled Surface Nature of Hyperbolic Paraboloid and Its Applications

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Abstract
In geometry, the curved surface formed by the movement of a straight line is called a ruled surface, and the hyperbolic paraboloid is a typical ruled surface. In this study, by analyzing the image and properties of the hyperbolic paraboloid, the related geometric properties are listed. Its ruled surface nature was proved by constructing a straight generatrix on the curved surface, and the related theorem of straight generatrix was obtained. Due to the advantages of ruled surface nature, the hyperbolic paraboloid has a wide range of applications in daily life, including construction, machinery, and water conservancy, etc.
Key words: Hyperbolic paraboloid; Ruled surface; Application

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INTRODUCTION

Analytic geometry has been promoting the development of mathematics; moreover, it has entered a new development stage in both content and method. The phenomenon of straight line contained in curved surface, curve formed by straight line does not seem to exist, but this kind of curved surface--ruled surface indeed exists in geometry. In life, for instance, if you swing a stick at will, the movement trajectory plane of the stick will contain straight lines, and such a curved surface is made up of straight lines. Ruled surface is an important concept in non-Euclidean geometry. Its application in real life is very common, and there are mathematical models of it in architecture, hydraulic engineering and daily life.

Definition 1 (Lü, & Xu, 2001): the curved surface generated by a family of rectilinear motion is called ruled surface in geometry, and the straight lines in motion are called rectilinear generators.

Definition 2 (Wang & Fu, 1999): in rectangular coordinate system, the curved surface represented by the equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ is called a hyperbolic paraboloid, in which, a,b is any positive constant.

Rectilinearity of hyperbolic paraboloid (Zhang, et al, 1995):

Property 1: Hyperbolic paraboloid is a ruled surface, which is a curved surface generated by rectilinear motion.

Property 2: Any two rectilinear generators of the same family are in the different planes.

Property 3: Any rectilinear generator will intersect all straight-line generators of another family.

Property 4: For any point on a hyperbolic paraboloid, one rectilinear generator in any one of the two families will pass through the point.

1. THE RULED SURFACE NATURE OF HYPERBOLIC PARABOLOID

Theorem (Lü & Xu, 2001): The hyperbolic paraboloid is a ruled surface, and there are two straight generatrices that form the hyperbolic paraboloid.

Proof The equation of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ (1);

By decomposing the factor,it is obtained that

$$(\frac{x}{a}-\frac{y}{b})(\frac{x}{a}-\frac{y}{b})=2z \quad .$$

Therefore, the determinant $\begin{vmatrix} \frac{x}{a}+\frac{y}{b} & 2 \\ z & \frac{x}{a}-\frac{y}{b} \end{vmatrix}=0$ can be equations

$\begin{cases} \left(\frac{x}{a}+\frac{y}{b}\right)\lambda_1+2\lambda_2=0 \\ z\lambda_1+\left(\frac{x}{a}-\frac{y}{b}\right)\lambda_2=0 \end{cases}$ has non-zero solutions, that is, there are non-zero λ_1,λ_2 such that $\begin{cases} \left(\frac{x}{a}+\frac{y}{b}\right)\lambda_1=-2\lambda_2 \\ z\lambda_1=-\left(\frac{x}{a}-\frac{y}{b}\right)\lambda_2 \end{cases}$.

Since λ_1, λ_2 are non-zero, λ_1 can be the denominator, and then the above formula can be changed to

$$\begin{cases} \frac{x}{a}+\frac{y}{b}=2\left(-\frac{\lambda_2}{\lambda_1}\right) \\ z=\left(\frac{x}{a}-\frac{y}{b}\right)\left(-\frac{\lambda_2}{\lambda_1}\right) \end{cases} .$$

Suppose $-\frac{\lambda_2}{\lambda_1}=\lambda$, then the above formula can be changed to $\begin{cases} \frac{x}{a}+\frac{y}{b}=2\lambda \\ z=\left(\frac{x}{a}-\frac{y}{b}\right)\lambda \end{cases}$ where $\lambda\in R$. (2)

If $M(x_0,y_0,z_0)$ is a point on the hyperbolic paraboloid, it satisfies equation (1), which is equivalent to that $M(x_0,y_0,z_0)$ satisfies equation (2). That is, the point M is a point on the straight generatrix set (2). Therefore, the hyperbolic paraboloid is composed of the straight generatrix set (2). In the same way, the other straight generatrix set can be obtained as

$$\begin{cases} \frac{x}{a}-\frac{y}{b}=2\mu \\ z=\left(\frac{x}{a}+\frac{y}{b}\right)\mu \end{cases} ,$$

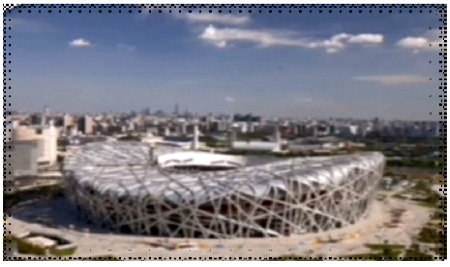
where $\mu\in R$ .The above steps prove that the hyperbolic paraboloid is a ruled surface with two straight generatrices.

2. REAL-LIFE APPLICATIONS OF THE HYPERBOLIC PARABOLOID

A hyperbolic paraboloid is a typical quadratic ruled surface and is often used in the design of construction and mechanical parts. Utilizing the nature of the straight generatrices, a building with a curved shape can be constructed by using straight components as the skeleton. Sturdy buildings can be built by laying steel bars along the straight generatrices to fix the intersection of the two sets of straight generatrices. The ideal property makes it widely used in life.

2.1 Beijing National Stadium

Beijing National Stadium (Bird's Nest) is located in the southern part of the central area of Beijing Olympic Park. It was the main stadium for the 2008 Beijing Olympic Games. The project covers a total area of 21 hectares, with approximately 91,000 audience seats in the venue. The opening and closing ceremonies of the Olympic and Paralympic Games, track and field matches, and football match finals have all been held here. After the Olympics, it became a public venue for people in Beijing to do sports activities and enjoy entertainment and became a landmark sports building and an Olympic heritage. The shape of the stadium is like a nest and cradle that breeds life, and it embodies the hope of mankind for the future. Of the architecture design, the roof of the Bird's Nest is part of a hyperbolic paraboloid.



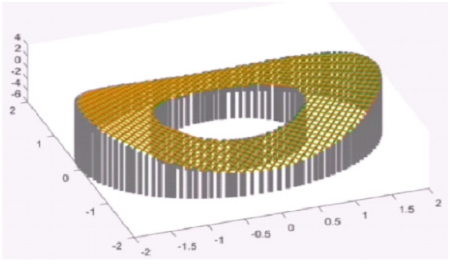


Figure 1
Beijing National Stadium

2.2 Xinghai Concert Hall of Guangzhou

The Xinghai Concert Hall of Guangzhou is located in the Xinghai Academy of Music, and large-scale music and art exhibitions are often held in the concert hall. The roof of Xinghai Concert Hall is designed as a standard hyperbolic paraboloid. Rooves of this type have excellent performance in water draining when it rains. Rainwater can flow down smoothly along the parabola of the roof. The hyperbolic parabolic structure also saves a large quantity of building materials. It is the most noteworthy that the parabola on the top has a unique and graceful arc, just like a swan spreading its wings. People are able to appreciate the aesthetics of the architecture while enjoying the music.

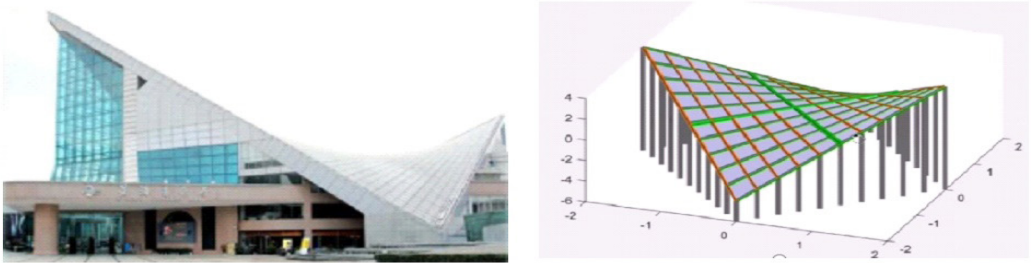


Figure 2
Guangzhou Xinghai Concert Hall

2.3 The Chapel of Palmyra in Cuernavaca and Los Manantiales

They are both works of engineer Felix Candela. Felix Candela is an excellent structural engineer and architect. He designed many hyperbolic parabolic roofs, and these buildings are very smart and beautiful. The design of the Chapel of Palmyra in Cuernavaca adopted a hyperbolic paraboloid, and that of Los Manantiales adopted four hyperbolic parabolas, as the most well-known piece of work by Felix Candela. In addition to these two pieces, he also designed and built buildings with hyperbolic parabolas including St Vicente de Paul Chapel, Church of La Milagrosa, and Iglesia de Santa Mónica, which amaze us with how much Felix Candela love such structure.

Figure 3
The Chapel of Palmyra in Cuernavaca

Figure 4
Los Manantiales

2.4 Lee Valley Velopark of London 2012 Summer Olympics

The 2012 London Olympics was held in the UK, and the Lee Valley Velopark also adopted a hyperbolic parabolic structure. Looking down from the sky, the Velopark is like a huge potato chip falling in the Olympic venue. The design of the Velopark provides the audience a good view so that they can better watch the exciting games. Meanwhile, it also increases the audience capacity of the Velopark and allows more people to share the joy of victory on the spot.

Figure 5
Lee Valley Velopark of London 2012 Summer Olympics

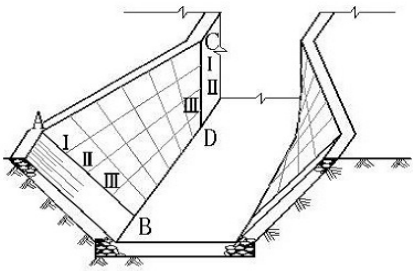


Figure 6
Drain connection

2.5 Connection of Drains

Hyperbolic paraboloids are also used in water conservancy projects. The side of the connection between the bottom of the channel and the gate is of such structure. The reason is that the slope of the straight generatrix of this structure is constantly changing, so it can ensure that the water is gentle when passing through the channel, preserving its stability.

2.6 Potato Chips, Leisure Seats

Hyperbolic paraboloids are also very common in our daily lives. The potato chips we like have a hyperbolic paraboloid shape. This shape is readily designed by the factory when potato chips are processed. Why this shape? We know that potato chips are very thin and crisp, and the shape of the hyperbolic paraboloid makes the potato chips not easy to be crushed during the transportation, ensuring the integrity of the potato chips, and thus ensuring the taste for us. There are also some creative new leisure seats designed into a hyperbolic parabolic shape. The seat surface of this kind of chairs is made up of several straight rattan rods, which are staggered to form a hyperbolic parabolic mesh surface, and they are aesthetically simple, structurally strong, not only allows people to lie down and sit comfortably but also brings people the enjoyment of beauty.

CONCLUSION

Hyperbolic paraboloid has its special properties in geometry, which is a curved surface generated by rectilinear motion. Any two rectilinear generators of the same family are in different planes; any of its straight lines intersect all straight lines of another family; for any point on the hyperbolic paraboloid, there is one rectilinear generator in the two families of rectilinear generators passing through the point, and it is just because of these characteristics that make it widely used in real life.

REFERENCES

Lü, L., & Xu, Z. Z. (2001). *Analytic geometry.* Beijing: Higher Education Press.

Wang, J. G., & Fu, R. R. (1999). *Spatial analytic geometry*. Beijing: Beijing Normal University Press.

Zhang, Z. J., et al. (1995). Application and teaching of ruled surface. *Journal of Hebei University of Coal and Architecture*, (4), 1-2.