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Sliding Mode Control of Magnetic Levitation Systems Using Hybrid Extended Kalman Filter

Enayatollah Taghavi Moghaddam[1,*]; Jabbar Ganji[1]

[1]Department of Electrical Engineering, Mahshahr Branch, Islamic Azad University, Mahshahr, Iran
[*]Corresponding author.
Email: enayattaghavi@yahoo.com

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Abstracts

This paper presents an approach to control a magnetic levitation system with uncertainty in the dynamics and the measurements. First, Sliding Mode Controller (SMC) is applied to the magnetic levitation system. Then, Hybrid Extended Kalman Filter (HEKF) is used to increase the robustness of the magnetic levitation system to uncertainties. The efficiency of such combined control method is verified by simulation results and performance parameters.

Key words: Magnetic levitation system; Sliding mode control; Hybrid extended kalman filter

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INTRODUCTION

Magnetic levitation systems are widely used in different fields of industrial applications such as high speed trains, so-called Maglev, frictionless bearings and magnetically suspended wind tunnels[1]-[4]. The magnetic levitation system is difficult to control since it is unstable in the open-loop form and the dynamics of the system is defined by a high order nonlinear equation. Therefore, the design of a highly efficient method to control the movable object of the system in order to be tracked in a desired position has a great deal of importance.

A lot of research has been done for controlling the magnetic levitation system in recent years. In[5]-[6], the feedback linearization method has been proposed to design a controller for magnetic levitation system. Due to the use of a simplified dynamic model, only the nominal parameters of the system were considered in the design procedure, which has resulted in some problems for stability, accuracy and robustness of controller. The problems arise from variations of the parameters due to environmental conditions or thermal drifts. Nonlinear controllers[7, 8], robust linear controllers such as H∞, optimal control and μ-synthesis[9, 10], control based on phase space[11], neural network methods[12, 13] and fuzzy control[14] are other proposed approaches to control magnetic levitation systems. The proposed dynamic models for the magnetic levitation system usually have some uncertainties due to simplified dynamic equation and its parameters. Moreover, the position measurements of the levitated object are noisy and it leads to some difficulties in the feedback control systems. SMC is a powerful nonlinear control method as it has low sensitivity to the plant parameter variations and disturbances. This property moderates requirement of exact modeling[15]. One of the first studies for applying SMC to the magnetic levitation system was done in[16], which compared sliding mode controller with the classical controllers. The main drawbacks of the traditional SMC are extreme control efforts and reduced performance, especially in regions of the operating space where the model is accurate. An important issue in the SMC design is estimating the magnitudes of modeling uncertainty such that the SMC gain will ensure stability. Buckner[17] proposed a neural network approach for estimating the uncertainty bounds and used it to control a magnetic levitation system in sliding mode. In[18], a sliding mode controller is designed, in which a neural network is used to estimate the uncertain system dynamics online.

In the SMC, the SMC gain depends on the uncertainty bounds and must be large enough to ensure closed-loop stability over the entire operating space. On the other hand, larger control gains increases chattering phenomenon. Hence, the SMC gain must be selected to compromise between the chattering and the robustness of the controller. In this paper, the combination of the SMC and HEKF (SMCHEKF) is proposed to control the magnetic levitation system. It is shown that this combined method increases the robustness of the magnetic levitation system to uncertainties of the dynamics and measurement system. As a result, the system performance and other drawbacks of the traditional control method are improved using this method.

The rest of the paper is organized as follows. Section 2 presents the dynamic model for the system. Design of a sliding mode controller for the magnetic levitation system is considered in Section 3. The uncertainties of the dynamics and the measurement system are modeled in section 4, and their effects in the performance of the controller are studied. In section 5, HEKF is used to increase the robustness of the system. In section 5, Simulation results are given to evaluate the performance of the SMCHEKF. Concluding remarks are discussed in section 6.

1. MAGNETIC LEVITATION DYNAMICS

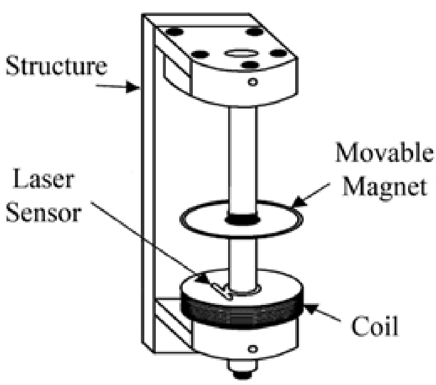

Figure 1
Experimental Set-up of Magnetic Levitation System

The physical setup of a typical magnetic levitation system is shown in Fig. 1[13]. The plant consists of a coil that produces a magnetic field, a magnetic levitated body (which is a permanent magnet and can be moved along a grounded glass rod), and a laser-based measurement system to measure the magnet position. The controller produces an appropriate direct current to suspend magnet in a desired position by supplying the coil. The magnet is suspended by a repulsive magnetic force when the coil is supplied. The dynamic equation of system can be written as:

$$F_m - mg - c\dot{x}_r - F_L = m\ddot{x}_r \tag{1}$$

Where x_r denotes the distance between the coil and the magnet, m is the mass of the magnet, c is the friction constant, g is the gravitational constant, F_m is the magnetic force, and F_L is the external force disturbance. The magnetic force can be modeled as[19]:

$$F_m = \frac{u}{a(x_r + b)^N} \tag{2}$$

Where u is the control law. N, a and b can be determined by numerical modeling or experimental methods[18] (Typically $3<N<4.5$). These parameters can be estimated by constant values in the desired region of operation. However, because of the intrinsic nonlinearity of the magnetic fields, these constants will vary when the dynamics goes out of parameter determination region. Replacing (2) into (1), we get:

$$\ddot{x}_r = -\frac{c}{m}\dot{x}_r + \frac{u}{ma(x_r + b)^N} - g - \frac{F_L}{m} \tag{3}$$

Let the states of the system be chosen such that $x_1=x_r$, $x_2=\dot{x}_r$. The magnetic levitation dynamic can be rewritten as:

$$\ddot{x}_r = f(X;t) + G(X;t)U(t) + d(X;t) \tag{4}$$

Where:

$X = [x_1 x_2]^T$ & $d(X;t) = -g - \frac{F_L}{m}$

$G(X;t) = \frac{1}{ma(x_1 + b)^N}$ & $f(X;t) = -\frac{c}{m}x_2.$

$U(t)$ is the control law and X is the state vector. To separate the nominal system and the uncertainties (in which the external disturbance $F_L=0$[13]), the dynamics equation can be rewritten as:

$$\ddot{x}_r(t) = [f_n(X;t) + \Delta f] + [G_n(X;t) + \Delta G]U(t) + [d_n(X;t) + \Delta d] \tag{4}$$

$$\ddot{x}_r(t) = f_n(X;t) + G_n(X;t)U(t) + d_n(X;t) + L(X;t) \tag{5}$$

Where the index of n presents the nominal part of the equation terms and $L(X;t)$ is the lumped uncertainty:

$$L(X;t)=\Delta f+\Delta G\,U(t)+\Delta d \tag{6}$$

It is assumed that the bound of L is known as:

$$L(X;t)<K \tag{7}$$

2. CONTROLLER DESIGN

2.1 Sliding Mode Control

The design of the sliding mode controller consists of two stages. The first is to define a sliding surface in the state variable space to ensure good control performance. The second is to design a control law to reach the state of the system on the desired predefined surface and to maintain its position on it. Let $e=x_1-x_{ref}$ be the tracking error (the error between the desired position x_{ref} , and the true position x_1). The sliding surface is defined as[18]:

$$S = \dot{e}(t) + \lambda_1 e(t) + \lambda_2 \int_0 e(\tau)d\tau \tag{8}$$

Where λ_1 and λ_2 are positive constants. The globally asymptotic stability of (8) is guaranteed when the following control law is applied to the magnetic levitation system[15]:

$$U(t) = G_n(X;t)^{-1}[-f_n(X;t) - d_n(X;t) + \ddot{x}_{ref} - \lambda_1\dot{e}(t) - \lambda_2 e(t) - Ksgn\{S(t)\}] \tag{9}$$

where *sgn* is the signum function.

2.2 Stability Analysis

Lyapunov function candidate is defined as[15]:

$$V = \frac{1}{2}S^2 \tag{10}$$

Differentiating V with respect to time and using (5) and (8), we get:

$$\begin{aligned}\dot{V} &= S\dot{S} = S\big(\ddot{e}(t) + \lambda_1\dot{e}(t) + \lambda_2 e(t)\big)\\ &= S\big[f_n(X;t) + G_n(X;t)U(t) + d_n(X;t) + L(X;t) - \ddot{x}_{ref}(t) + \lambda_1\dot{e}(t) + \lambda_2 e(t)\big]\end{aligned} \tag{11}$$

Substituting control law from (9) into (11) results in the following:

$$\begin{aligned}\dot{V} &= S[f_n(X;t) + G_n(X;t)(G_n(X;t)^{-1}[-f_n(X;t) -\\ &\quad d_n(X;t) + \ddot{x}_{ref} - \lambda_1\dot{e}(t) - \lambda_2 e(t) - Ksgn\{S\}])\\ &\quad + d_n(X;t) + L(X;t) - \ddot{x}_{ref}(t) + \lambda_1\dot{e}(t) + \lambda_2 e(t)]\\ &= S(L(X;t) - Ksgn\{S\})\end{aligned} \tag{12}$$

Considering the sign of the sliding surface, (8), and the bounding condition expressed in (7), the time derivative of the candidate Lyapunov function can be separated as:

1) $S < 0 \rightarrow sgn(S) = -1 \rightarrow L(X;t) - Ksgn\{S\} > 0$
$\dot{V} = S(L(X;t) - Ksgn\{S\}) < 0$

2) $S = 0 \rightarrow \dot{V} = 0$

3) $S > 0 \rightarrow sgn(S) = +1 \rightarrow L(X;t) - Ksgn\{S\} < 0$
$\dot{V} = S(L(X;t) - Ksgn\{S\}) < 0$

$$(1), (2), (3) \rightarrow \dot{V} \leq 0 \tag{13}$$

Thus, the reaching condition[15] is satisfied. Beside the asymptotic stability, the SMC guaranties that the state trajectory of the system reaches the sliding surface in a finite time and stays on it, with any initial condition. Moreover, SMC law provides the system dynamic with an invariance property to the uncertainty, once the system dynamics are controlled in the sliding mode. Control gain (K) is the trade-off parameter of control. The value of K depends on the uncertainty bounds and must be large enough, since the uncertainties (such as parameter variations and exact value of external load disturbance) are difficult to measure and appropriate reaching time is necessary. However, the un-modeled dynamics and the discontinuous control law result in undesirable oscillations with finite frequencies and amplitude in the control law, the so-called chattering. Larger control gains increase this phenomenon. Hence, K must be selected to compromise between the chattering and the robustness of the controller. Moreover, it is rather convenient to supply a smooth control law. Therefore, chattering reduction is very important.

2.3 Results of SMC

Two common methods in chattering reduction are Dynamic Sliding Mode Control[20] and Boundary Layer Control (BLC)[15]. In BLC method, the control law is interpolated in a boundary layer as:

$$B(t)=\{x|s(x, t)|\leq\varphi\},\ \varphi>0 \tag{14}$$

where φ is the boundary layer thickness. Realizing such interpolation, the sgn function in the control law should be replaced with saturation function defined as:

$$sat\left(\frac{x}{\varphi}\right) = \begin{cases} \dfrac{x}{\varphi} & if|x| \leq 1 \\ sgn\left(\dfrac{x}{\varphi}\right) & if|x| > 1 \end{cases} \tag{15}$$

The saturation function is a function with two criterions. Furthermore, it has discontinuities in its derivative that yield some problems in the realization of the extended kalman filter which is discussed in section IV. In the extended kalman filter, the Jacobian of the system (partial derivative of dynamic equations with respect to state variables) is needed. Thus, a trigonometric function tanh is proposed to replace with the *sat* function to reduce the chattering. In this case, we have a one-criterion, differentiable and smoother function. The replacement is performed as follows:

$$sat(x) \propto tanh\left(\frac{\pi}{4}x\right) \rightarrow sat\left(\frac{x}{\varphi}\right) \propto tanh\left(\frac{\frac{\pi}{4}x}{\varphi}\right) \tag{16}$$

The thickness of the boundary layer (φ) is 0.1. The system parameters are evaluated using a curve-fitting technique

on the basis of the variation in the control law to provide a magnetic force equal to the magnet weight. The resulting parameters are[18]:
m=0.121 kg, c = 2.69, a=1.65, b = 6.2
n=4, λ_1=10, λ_2=30, K=15

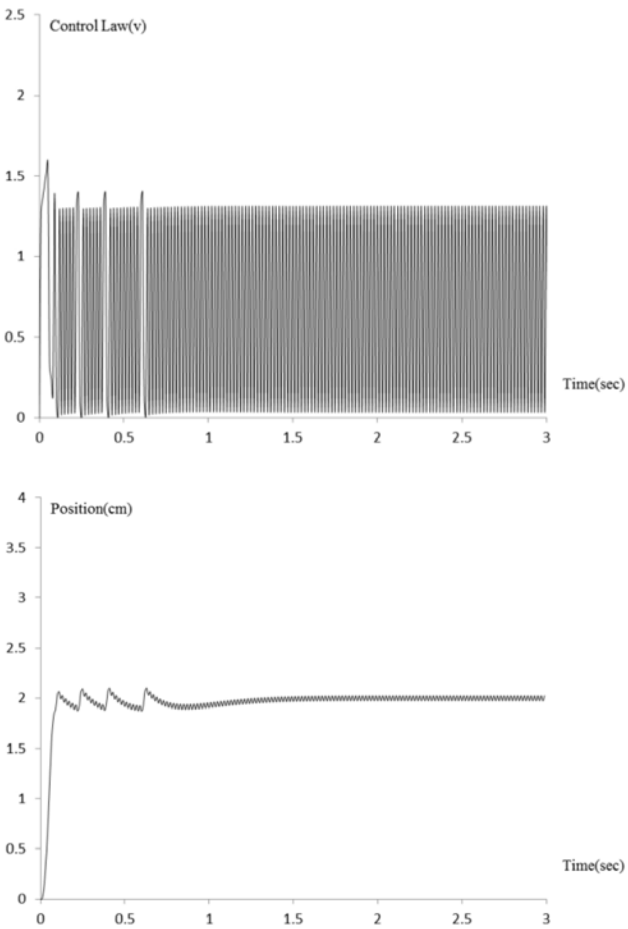

Figure 2
Control Law and the Magnet Position Using Classic Control Law (Using sgn Function)

K, λ_1 and λ_2 are selected to obtain the best performance of control, stability considerations and appropriate transient response. The control law and the magnet position using $sgn(S/0.1)$ and the $tanh(2.5\pi S)$, which is the smooth form of the $sat(S/0.1)$, are shown in Fig. 2 and Fig. 3, respectively. The desired position for the magnet is 2cm, i.e. x_{ref}=2cm. It can be seen from Fig. 3 that the chattering is eliminated effectively and the control law is smoothed using this method of BLC in comparison with Fig.2.

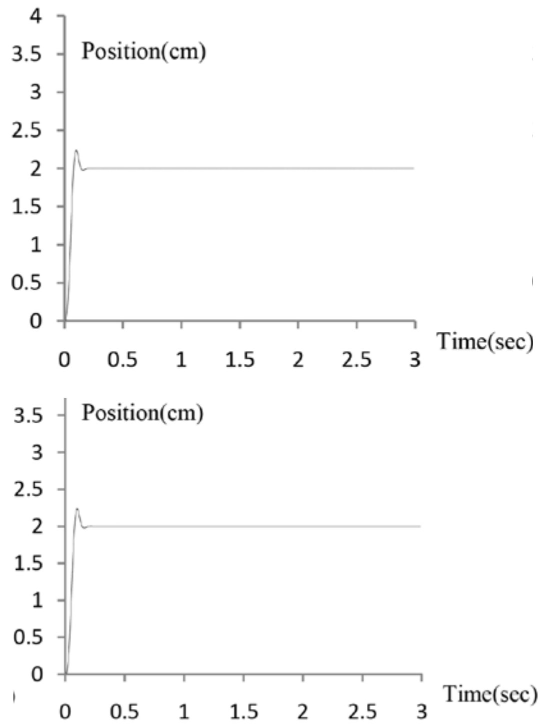

Figure 3
Control Law and the Magnet Position Using Proposed BLC Method (Using tanh Function)

3. EFFECT OF UNCERTAINTIES

3.1 Uncertainties Modeling

In practical feedback control systems, the output is sampled by a measurement system. Then, it is applied to the controller, i.e. the output of the measurement system is the input for controller. Most of the measurement systems have some inaccuracies. For example, in the sensor-based measurement systems, which are commonly used in the engineering systems, there are several measurement noise sources. Also, sensors have basic limitations related to the associated physical medium, and typically the output quality is decreased. Therefore, analytical measurement models typically contain some random measurement noises or uncertainties[21]. These uncertainties result in some random noise in the control law. Because the control law is designed due to exact output of system, but the output is measured by an inaccurate measurement system. Controller uncertainties lead to uncertainties in the system output. Modeling such uncertainties, a term should be added to the control law and the system output. So, the dynamics can be modified as:

$$\ddot{x}_r = -\frac{c}{\mathrm{m}}\dot{x}_r + \frac{u + \Delta u}{ma(x_r + b)^N} - g =$$
$$\frac{c}{m}x_2 + \frac{u}{ma(x_r + b)^N} - g + \frac{\Delta u}{ma(x_r + b)^N}$$
$$y = x_r + \Delta x_r \tag{17}$$

Where y is the output of the sensor-based measurement system, Δx and Δu are uncertainties of magnet position and control law, respectively. There are no certain mathematical expressions for values of Δx and Δu. The measurements must be interpreted as a random process. Central Limit Theorem formally demonstrates that under certain general conditions, sum of independent random variables with any distribution approaches a normal distribution[22]. So, we can model the uncertainties of the magnetic levitation system as random variables. Let:

$$\frac{\Delta u}{ma(x_r+b)^N}=\omega \ ,\Delta x_r=\nu \tag{18}$$

Where ω and ν are zero-mean, uncorrolated white noises. Rewrite (17) in a general form as:

$$\begin{aligned}\dot{X}&=f(X,u,\omega,t) \quad \omega\sim\mathcal{N}(0,\sigma_1)\\ y&=h(x_r,\nu,t) \qquad \nu\sim\mathcal{N}(0,\sigma_2)\end{aligned} \tag{19}$$

Where N and σ indicate normal distribution and standard deviation of random processes, respectively. The standard deviation of ω is evaluated according to the simulation results. The standard deviation of v depends on the accuracy of the measurements. Certainly, the standard deviation of the measurements depends on the measurement system's accuracy.

3.2 Results of Uncertainties Effect

It is assumed that the magnet position is measured by a sensor which RMS error is 0.1cm. Also, the following values are chosen as:

σ_1=0.005, σ_2=0.1

Two test cases have been examined to demonstrate the effect of uncertainties in the efficiency of the controller. In the first case, the measurement system is considered as an ideal system, i.e. ν=0. The simulation results of the control law and the position of magnet are presented in Fig. 4. Despite the fact that the tracking is done due to SMC robustness property, it can be seen from the figure that the accuracy is degraded in comparison with ideal dynamics and measurements (shown in Fig. 3). Larger control gains can improve the degraded tracking performance, but the chattering will be increased significantly. In the second case, both uncertainties of the dynamics and the measurements are taken into account at the same time, i.e. v, $\omega \neq 0$. Then the system is simulated. Certainly, the second case is more reasonable from a practical point of view. Simulation results are shown in Fig. 5. It can be seen from the figure that the system has large oscillations. Thus, it is impossible for the magnet to reach the desired position, even though larger control gains are used. So, the SMC has no performance in this case, because SMC is designed based on precise state of dynamics, i.e. the uncertainty of the measurements is not considered. To solve around this problem, HEKF is applied to the magnetic levitation system.

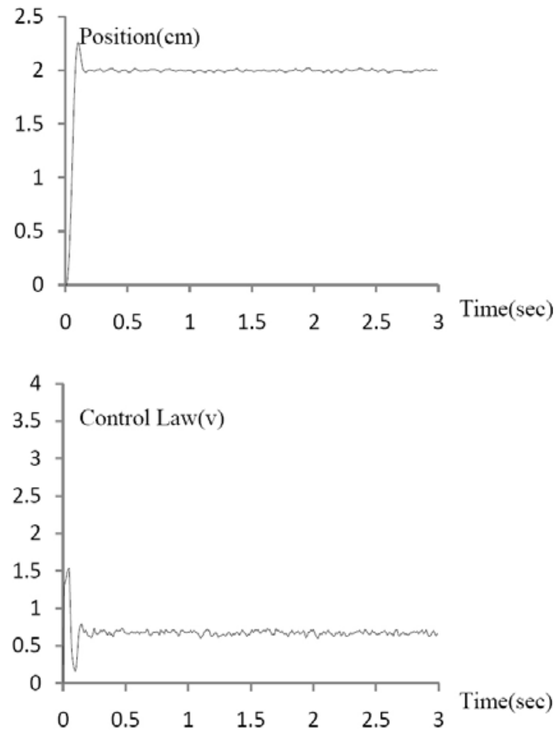

Figure 4
Control Law and the Magnet Position in the Case of Noisy Dynamic and Precise Measurements

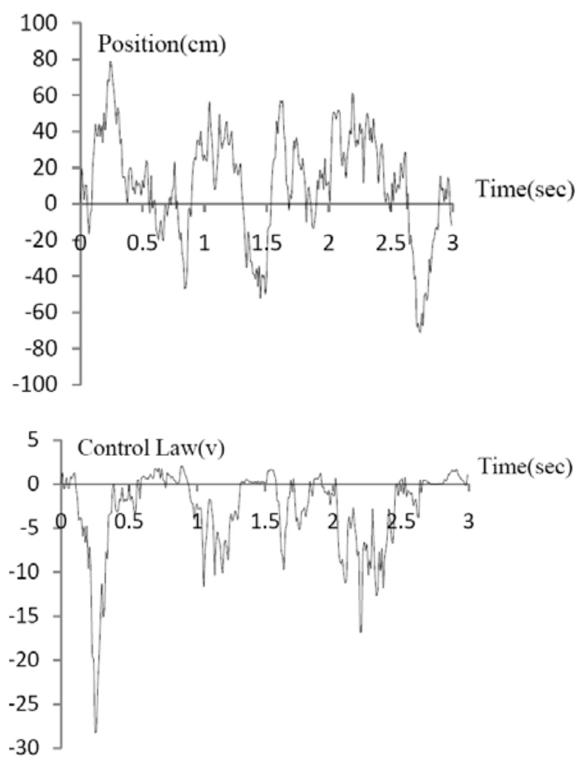

Figure 5
Control Law and the Magnet Position in the Case of Noisy Dynamic and Noisy Measurements Dynamic and Noisy Measurements

4. SMCHEKF

4.1 Hybrid Extended Kalman Filter

Kalman filter is a mathematical tool which acts as an estimator for what is called Linear Quadratic problem, which is the problem of estimating instant states of a linear dynamic system perturbed by white noise[23]. Extensions of the kalman are needed for using kalman filter in the nonlinear systems. A kalman filter that linearizes about current estimated state is called an extended kalman filter. The EKF is selected due to following reasons:

- It has a state space basis and uses all of the prior information about the internal model construction.
- It considers both the process and the measurement noise simultaneously.
- The estimation is statistically optimum with respect to any quadratic function of estimation error.

Real industrial systems have continuous-time dynamics and a discrete-time measurement system. So, we have a continuous process, which is sampled at a predefined rate. This is the most common situation in practice. HEKF considers such systems. For a nonlinear system with the following state space model:

$$\begin{aligned} \dot{x} &= f(x,u,\omega,t) \qquad \omega(t)\sim\mathcal{N}(0,Q) \\ y_k &= h_k(x_k,v_k) \qquad v_k\sim\mathcal{N}(0,R_k) \end{aligned} \tag{20}$$

Where f and h are nonlinear functions, $\omega(t)$ is a continuous-time white noise with covariance Q, v_k is a discrete-time white noise sequence with covariance R_k. Between sampling intervals, measurements have infinite covariance (R=∞)[24]. So, HEKF time update equations are formulated as:

$$\dot{\hat{x}} = f(\hat{x},u,\omega_0,t) \tag{21}$$

$$\dot{P} = AP + PA^T + LQL^T \tag{22}$$

Where ω_0 is the nominal process noise, that is, ω_0=0 since the $\omega(t)$ is a zero-mean white noise. Pis the error covariance matrix of estimation. $\hat{x}$ is the state estimate. u is the control input. The Jacobians are:

$$A = \frac{\partial f}{\partial x}\bigg|_{\hat{x}}, \qquad L = \frac{\partial f}{\partial \omega}\bigg|_{\hat{x}} \tag{23}$$

After solving the equations (20) and (21), at each measurement instant, Kalman gain, estimation, and the error covariance are updated as [24]:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1} \tag{24}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k[y_k - h_k(\hat{x}_k^-, v_0, t_k)] \tag{25}$$

$$P_k^+ = (I - K_k H_k)P_k^-(I - K_k H_k)^T + K_k M_k R_k M_k^T K_k^T \tag{26}$$

$$H_k = \frac{\partial h_k}{\partial x}\bigg|_{\hat{x}} \; M_k = \frac{\partial h_k}{\partial v}\bigg|_{\hat{x}}$$

Superscripts+ and - denote posteriori estimate and priori estimate, respectively. v_0 is the nominal measurement noise value, that is, v_0=0 since v_k is a discrete-time zero-mean white noise.

4.2 Results of SMCHEKF

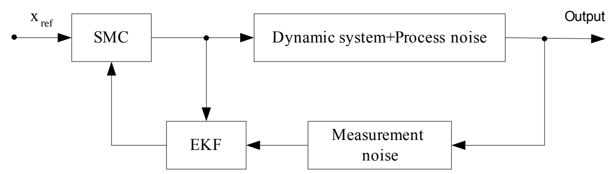

Figure6
Block Diagram Scheme of the Proposed Control Method (SMCHEKF)

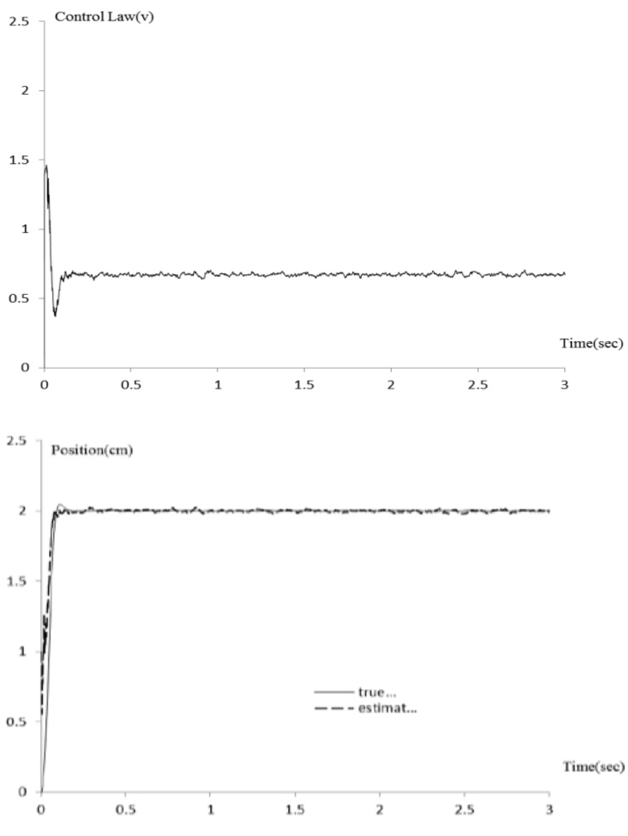

Figure 7
Control Law and the Magnet Position of the SMCHEKF

The block diagram of the SMCHEKF is shown in Fig. 6. As it is illustrated, the noisy measurements of the magnet position are estimated. Then sliding mode controller is applied to get the state of the system to track the desired state x_{ref}. The magnet position and the control law of the proposed control method (SMCHEKF) are shown in Fig. 7. SMCHEKF reaches to desired position after a time less than 0.5s. The sample rate of measurements is 1ms. Simulations show efficiency of the method for controlling magnetic levitation system. Another test has been done to confirm the performance of SMCHEKF. The sum of squared error is defined as[17]:

$$SSTE = \sum_{k=1}^{n} (e(kT))^2 \tag{27}$$

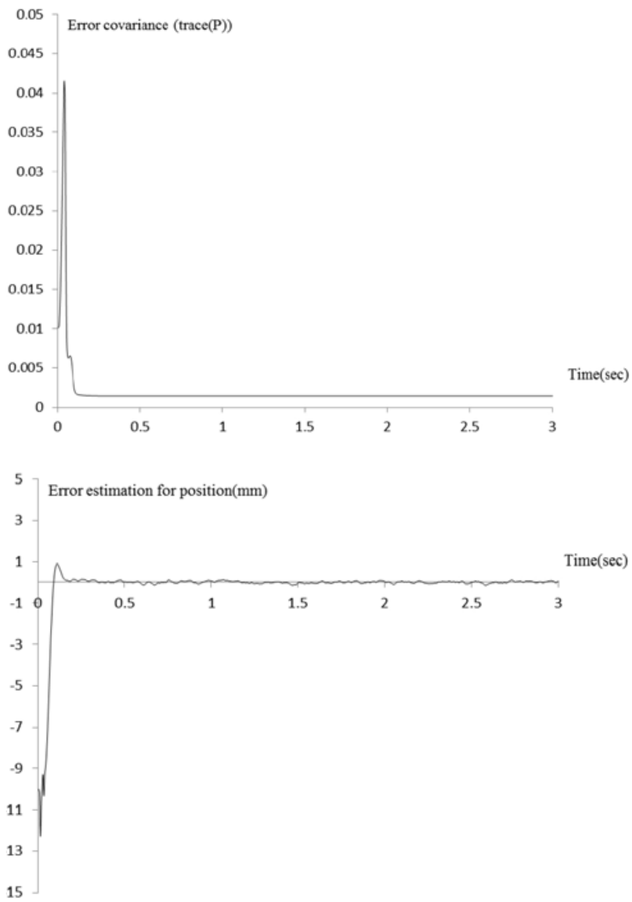

Figure 8
Error Covariance (Trace of Matrix P) and Error Estimation of SMCHEKF

where $e(kT)$ presents the tracking error, $t=KT$ is calculated from 0 to 10 seconds, and $T=2.65$ *ms* is the sample rate. The value of SSTE is 15.387 mm^2 which is a suitable Estimation error and Estimation error covariance (trace of the matrix P) for SMCHEKF is presented in Fig. 8. As time progresses, more measurements are processed, and the error covariance decreases that eventually reaches the steady state. Root Mean Square (RMS) of tracking error and estimation of HEKF in the first 10 seconds of simulation are 1.02 mm and 0.91 mm, respectively. These results, altogether, confirm the proposed SMCHEKF's significant performance.

5. CONCLUSION

This paper considers the control problem of magnetic levitation systems that includes uncertainty in both the dynamics and the measurements. A sliding mode controller is used to set the levitated object in a reference (desired) position. Then, it is shown that sliding mode control cannot perform the tracking when there are uncertainties in dynamics and measurements simultaneously. Therefore, uncertainties were modeled as white noises. HEKF is proposed to estimate the system state by noisy measurements. Then, the estimated position of magnet is applied to the sliding mode controller and the performance of SMCHEKF is verified successfully.

REFERENCES

[1] Rote , D. M., & Cai , Y. (2002). Review of Dynamic Stability of Repulsive-Force Maglev Suspension System. *IEEE Trans. Magn., 38*(2), 1383–1390.

[2] Hajjaji, A. E., & Ouladsine, M. (2001). Modeling and Nonlinear Control of Magnetic Levitation Systems. *IEEE Trans. Ind. Electron., 48*(4), 831–838.

[3] Ono, M., Koga, S., & Ohtsuki, H. (2002). Japan's Superconducting Maglev Train. *IEEE Instrum. Meas. Mag., 5*(1), 9–15.

[4] Allaire , P., & Sinha, A. (1998). *Robust Sliding Mode Control of a Planar Rigid Rotor System on Magnetic Bearings* (pp. 577–586). Proc. 6th International Symposium on Magnetic Bearings, (Massachusetts).

[5] Trumper, D. L., Olson, M., & Subrahmanyan, P. K. (1997). Linearizing Control of Magnetic Suspension Systems. *IEEE Transactions on Control Systems Technology*, (4), 427–438.

[6] Barie, W., & Chiasson, J. (1996). Linear and Nonlinear State-Space Controllers for Magnetic Levitation. *International Journal of Systems Science, 27*(11), 1153–1163.

[7] Huang, C. M., Yen, J. Y., & Chen, M.-S. (2000). Adaptive Nonlinear Control of Repulsive Maglev Suspension Systems. *Control Engineering Practice, 8*(5), 1357–1367.

[8] Yang , Z. J., & Tateish, M. (1998). *Robust nonlinear control of a magnetic levitation system via back step-ping approach*. Proc. 37th SICE Annual Conference (SICE'98) (pp.1063–1066). Chiba, Japan.

[9] Fujita, M., Matsumura, F., & Uchida, K. (1990). *Experiments on the H∞ disturbance attenuation control of a magnetic suspension system. Proc*. 29th IEEE Conference on Decision and Control (pp. 2773–2778). Hawaii.

[10] Fujita, M., Namerikawa, T., Matsumura, F., & Uchida, K. (1995). μ-synthesis of an Electromagnetic Suspension System. *IEEE Trans. Automatic Control, 40*(3), 530–536.

[11] Zhao, F., Loh, S. C., & May, J. A.(1999). Phase-Space Nonlinear Control Toolbox: The Maglev Experience, 5th International Hybrid Systems. Workshop (P. Antsaklis, W. Kohn, M. Lemmon, A. Nerode, and S. Sastry, eds.).

Lecture Notes in Computer Science, 1567, 429–444

[12] Chen, C. H. (2009). Nonlinear System Control Using Adaptive Neural Fuzzy Networks Based on a Modified Differential Evolution. *IEEE Trans on, Systems, MAN, and Cybernetics-Part C: Applications and Reviews, 39*(4), 459-473.

[13] Lin, F. J., Teng, L. T., & Shieh, P. H. (2005). Hybrid controller with recurrent neural network for magnetic levitation system. *IEEE Trans. Magn., 41*(7), 2260–2269.

[14] Li, T. H. S., Kuo, C.L., & Guo, N.R. (2007). Design of an EP-Based Fuzzy Sliding Mode Control for a Magnetic Ball Suspension System. *Chaos, Solutions and Fractals, 33*, 1523-1531.

[15] Slotine , J.J., & Li, W. (1990). *Applied Nonlinear Control*. New Jersey: Prentice Hall.

[16] Cho, D., Kato, Y., & Spilman, D. (1993). Sliding Mode and Classical Controllers in Magnetic Levitation Systems. *IEEE Control Systems Magazine, 13*(1), 42–48.

[17] Buckner, G. D. (2001). Intelligent Bounds on Modeling Uncertainties: Applications to Sliding Mode Control of a Magnetic Levitation System. *IEEE International Conference on Systems, Man, and Cybernetics, 1*, 81–86.

[18] Lin, F. j., & Teng, L. T. (2007). Intelligent Sliding Mode Control Using RBFN for Magnetic Levitation System. *IEEE Trans on Industrial Electronics, 54*(3), 1752-1762.

[19] Moon, F. C. (1994). S*uperconducting Levitation: Applications to Bearings and Magnetic Transportation*. New York: Wiley.

[20] Al-muthairi, N. F., & Zribi, M. (2004). Sliding Mode Control of a Magnetic Levitation System. *Mathmatical Problems in Engineering, 2*, 93-107.

[21] Welch,G., & Bishop, G.(2001). *An introduction to the Kalman filter. Course 8*. In Proceedings of the ACM SIGGRAPH Conference, Los Angeles.

[22] Papoulis, A. (1991). *Probability, Random Variables and Stochastic Processes* (Third Edition). Singapore: McGraw-hill.

[23] Grewal, S., & Andrews, P. (2001). *Kalman Filtering Theory and Practice Using Matlab*, (2nd Edition). Wiley.

[24] Simon, D. (2006). *Optimal State Estimation Kalman, H∞, and Nonlinear Approaches*. New Jersey: Wiley.