The Random Storage Model under the Condition of Limited-space Storehouse

MODÈLE DE STOCKAGE ALÉATOIRE SOUS LA LIMITATION DE LA CAPACITÉ D'EMMAGASINAGE

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Abstract: Aiming at the storage problem of stores, this paper set up storage model to reduce cost and improve benefit under the condition of limited-space storehouse. According to model, we calculated and obtained the ordering point where the general cost is the lowest. At the end of the paper, the model was proved by an example.
Key words: storage model, optimum ordering point, expected value, Lagrange multiplier

Résumé: Pour résoudre le problème de stockage des magasins dans la condition de la capacité d'emmagasinage limité, le présent article établit un modèle de stockage visant à réduire le coût et à augmenter les profits, calcule le point de commande optimale où la perte atteint le minimum, et en donne des exemples.
Mots-Clés: modèle de stockage, point de commande, valeur espérée, Multiplicateur de Lagrange

1. INTRODUCTION

Factory should regularly order all kinds of raw materials, and the merchants should purchase various commodities in batches. Whether raw materials or products, there is a problem of storage. Less storage, it would be unable to meet the demand, and affect the profits; too much storage, the costs will be high. Thus, storage management is the way to reduce costs and improve the economic benefits.

In the solution of the random storage, as the time of delivery is a random variable, it is a programming storage model with the random parameter. To figure out this kind of problem, literature 5 has built up an optimization model whose objective function is mathematical expectation counting by average cost; literature 6 solves it by the establishment of the nonlinear integer programming model. This paper combines the models of literature 5 and 6, fetches the mathematical expectation counting by the random variable, and then transforms from a random programming to a certain mathematic programming, accordingly builds up an expectation model. This model is simpler and more effective. In this paper, we suppose that probability density function for variable of the time of delivery is known. Base on the total fees is sum by order fees, storage fees and charge shortage, we establish a minimum cost expectation model using the minimum expectation fees to fix on the optimal order point L^*.

2. UP BUILD AND SOLUTION FOR MODEL

A shopping mall sells a commodity. Let merchandise sales rate be a Constant——r; Let each Order charges be a constant c_1 which is unrelated with the number and variety of goods; when using their own warehouse for storage. Let the storage fees for commodity unit be c_2; For the limit of the storage, let the storage fees

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for leased warehouse for commodity unit be c_3, and $c_2 \le c_3$; We allow the lack of the goods, and let the loss of selling for the lack be c_4; Let the arrival day be X which is random; Let the maximum capacity be Q_0, Every time after arrival we add q to the value of the fixed value Q, and $Q_0 < Q$; Whenever q falls to L, we order some more.

2.1 Model assumptions

1st. Operating of the shopping mall is continues.

2nd. If we need to order some commodity, it is sufficient.

3rd. Probability and the probability density function of the time of delivery is known.

2.2 Model foundation

Delivery time is random variable and probability density function is known. This planning which contains random parameters is random planning. For different management objectives and technical requirements, the methods of stochastic programming problem in the random variables vary. The more common approach is: After getting the mathematical expectation of the random variable, we transform stochastic programming to be a mathematical programming. Under this restriction of the expectation, we name the model which let the probability expectations be optimizing expectations model. In this paper, we just build up a minimum cost expectations model. Then we can minimize the fees to confirm the best point L^*.

Probability X (Delivery time) is $f(x)$, $f(x)=\int_0^{\frac{L}{r}} p(x)dx$, $p(x)$ is X's Probability density function.

Using the equality: total fees= order fees+ storage fees+ lack fees, we can get:

$$C(L)=\sum_{x=x_1}^{\frac{L}{r}}[c_1 f(x)+\frac{1}{2}c_2(Q_0+L-xr)(\frac{Q_0-L}{r}+x)f(x)+\frac{1}{2}c_3(Q-Q_0)\frac{Q-Q_0}{r}f(x)]$$
$$+\sum_{x=\frac{L}{r}}^{\infty}[c_1 f(x)+\frac{1}{2}c_2(Q_0+L-xr)(\frac{Q_0-L}{r}+x)f(x)+\frac{1}{2}c_3(Q-Q_0)\frac{Q-Q_0}{r}f(x)+c_4(xr-L)f(x)]$$

Also L^* just is L.

2.3 Model solution

1st. Let random values of Delivery time in order of size:

$x_1,x_2,x_3,\cdots,x_i,x_{i+1},\cdots,x_m$ · $x_i<x_{i+1}, x_{i+1}-x_i=\Delta x_i \ne 0(i=1,2,\cdots,m)$

2nd. Get L from $x_1r,x_2r,\cdots,x_mr$. While L=x_ir, mark it as L_i

$\Delta L_i^*=L_{i+1}^*-L_i^*=x_{i+1}r-x_ir=\Delta x_ir\ne 0(i=1,2,\cdots,m)$

3rd. Solve the L for the minimum of $C(L)$

Because $C(L_i)=\sum_{x=x_1}^{\frac{L_i}{r}}[c_1 f(x)+\frac{1}{2}c_2(Q_0+L_i-xr)(\frac{Q_0-L_i}{r}+x)f(x)+\frac{1}{2}c_3(Q-Q_0)\frac{Q-Q_0}{r}f(x)]$

$+\sum_{x=\frac{L_i}{r}}^{\infty}[c_1 f(x)+\frac{1}{2}c_2(Q_0+L_i-xr)(\frac{Q_0-L_i}{r}+x)f(x)+\frac{1}{2}c_3(Q-Q_0)\frac{Q-Q_0}{r}f(x)+c_4(xr-L_i)f(x)]$ is L, so L_i should meet the following inequality:

① $C(L_{i+1})-C(L_i)\ge 0$, ② $C(L_i)-C(L_{i-1})\ge 0$

Definition $\Delta C(L_i)=C(L_{i+1})-C(L_i)$, $\Delta C(L_{i-1})=C(L_i)-C(L_{i-1})$

We can export: $\Delta C(L_i) = \frac{1}{2}c_2 \Delta L_i \sum_{x=x_1}^{\frac{L_i}{r}} f(x) + (\frac{1}{2}c_2 - c_4)\Delta L_i \sum_{x=\frac{L_i}{r}}^{\infty} f(x)$

$$= \frac{1}{2}c_2 \Delta L_i \sum_{x=x_1}^{\frac{L_i}{r}} f(x) + (\frac{1}{2}c_2 - c_4)\Delta L_i (1 - \sum_{x=x_1}^{\frac{L_i}{r}} f(x)) = (\frac{1}{2}c_2 - c_4)\Delta L_i + (c_2 + c_4)\Delta L_i \sum_{x=x_1}^{\frac{L_i}{r}} f(x) \geq 0$$

For $\Delta L_i \neq 0$, then $(\frac{1}{2}c_2 - c_4) + (c_2 + c_4)\sum_{x=x_1}^{\frac{L_i}{r}} f(x) \geq 0$,

We have $\sum_{x=x_1}^{\frac{L_i}{r}} f(x) \geq \frac{c_4 - \frac{1}{2}c_2}{c_2 + c_4} = N$.

We can also export: $\sum_{x=x_1}^{\frac{L_{i-1}}{r}} f(x) \leq N$.

Then using the two inequalities, we get $\sum_{x=x_1}^{\frac{L_{i-1}}{r}} f(x) < N = \frac{c_4 - \frac{1}{2}c_2}{c_2 + c_4} \leq \sum_{x=x_1}^{\frac{L_i}{r}} f(x)$.

As well as the probability of delivery time is $f(x_1), f(x_2), f(x_3), \cdots, f(x_m), \sum_{i=1}^{m} f(x_i) = 1$, we know

$$f(x_1) + f(x_2) + \cdots + f(x_i) \geq N \qquad (1)$$

Choosing the minimum value x_i and depending on $L = x_i r$, we can gain the best delivery time L^*. So best order point optimal model is: $L^* = \min\{x_i \mid f(x_1) + f(x_2) + \cdots + f(x_i) \geq N\} r$

3. APPLICATIONS

Some real data follows from a shopping mall:

Commodity: Kangshifu Instant noodles

r =12 box/day ; c_1 =10 Yuan ; c_2 =0.01 Yuan/box. day ; c_3 =0.02 Yuan/box. day ; c_4 =0.95 Yuan/box. day ; Q_0 =40 box ; Q =60box.

We get 36 data which is the continuous time after delivery:

3 3 7 1 2 3 3 0 3 4 6 3 1 4 3 3 2 5 2 3 2 5 3 2 3 3 0 3 4 3 1 4 5 4 3 1 .

Then we work out best order point L^*

We simulate the data, and get a chart

Let probability density function is

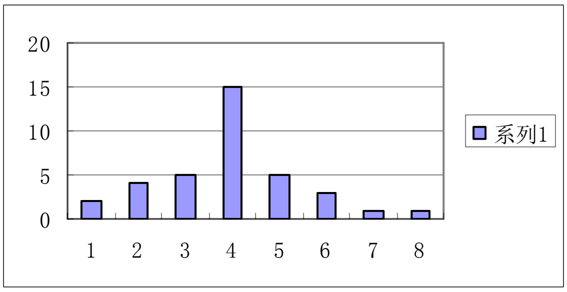

Chart 1

In chart 1, the abscissa values 1,2,3,4,5,6,7,8 means the arrival time 0,1,2,3,4,5,6,7 . We can use the computer to fit the normal distribution of X.

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u)^2}{2\sigma^2}}$$

Use maximum likelihood estimate ways, we can figure out $\begin{cases} u = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \\ \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \end{cases}$. Then we use the data of the arrival time to estimate the mean and variance:

u=2.972222 ≈ 3, σ=2.313492 ≈ 2.3

Then we know the probability of the delivery time is $f(x) = \int_0^x \frac{1}{\sqrt{2\pi}2.3} e^{-\frac{(t-3)^2}{2 \cdot 2.3^2}} dt$. The standardization on it is $F(x) = f(\frac{x-u}{\sigma}) = \frac{1}{\sqrt{2\pi}} \int_0^{\frac{x-u}{\sigma}} e^{-\frac{t^2}{2}} dt$.

Due to the formula (1): $f(x_1) + f(x_2) + \cdots + f(x_i) \geq N$, then we have $\frac{1}{\sqrt{2\pi}} \sum_{j=1}^{i} \int_0^{\frac{x_j-u}{\sigma}} e^{-\frac{t^2}{2}} dt \geq N$, and $x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4, x_6 = 5, x_7 = 6, x_8 = 7$.

Base on question 1, we can work out the minimum value $x_i = 4$.

Base on $N = \frac{c_4 - \frac{1}{2}c_2}{c_2 + c_4}$, we can figure out N=0.857143.

We know from $\min\{x_i \mid f(x_1) + f(x_2) + \cdots + f(x_i) \geq N\}$ that x_i =4, so L^* =48

List 1 Commodity

x	0	1	2	3	4	5	6	7
probability	0	0.1	0.2743	0.4207	0.758	0.9032	0.97725	
The cumulative value	0.02275	0.12275	0.39705	0.81775	1.57575	2.47895	3.4562	3.4562

4. TAGS

Storage model allowing shortage of lack of storage spreads out solving storage management decisions. During the actual operation of inventory management, limited storage capacity and shortages may arise, and the lack of shortage is unavoidable. Lack of shortage can cause descending of optimal production so that reduce the assembly costs. Thus, storage model allowing shortage of lack of storage is full of practical value.

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