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Multi-Plant Production and Transportation Planning Based on Data Envelopment Analysis

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Abstract

This paper proposes a methodology for developing a coordinated aggregate production plan for manufacturers producing multiple products at multiple plants simultaneously, in a centralized environment via data envelopment analysis (DEA).

Based on demand forecast of the planning horizon, the central decision maker (DM) specifies the optimal combination of input resources required by the optimal output targets for each plant to keep the supply and demand in balance, and the accompanying transportation trips and volumes among distribution centers (DCs) or warehouse facilities. In this paper, we focus on an integrated production-transportation problem since production and transportation are two fundamental ingredients in the whole operation chain. We deal with multiple products manufactured in multiple plants.

The proposed mixed integer DEA models minimize both production costs and transportation costs. The capacity constraint for each plant is enforced by using the production possibility set theory. Finally, we validate our models by a numerical example and sensitivity analysis.

Key words: Integrated production-transportation planning; Data envelopment analysis

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INTRODUCTION

We consider an integrated production and transportation planning problem: how to optimally determine aggregate production planning and transportation trips among distribution centers (DCs) and the corresponding transportation volumes, where production and transportation plan are considered simultaneously.

The production decisions concerns how to allocate input resources and set output targets among different production units, while the transportation decisions work out how to transport superfluous outputs for one DC to other under-supply DCs when all these DCs are accommodated by the corresponding production unit. We are interested in making an integral decision to minimize the aggregate costs including production costs, here mainly referring to the costs of input resources, and transportation costs to satisfy each DC's market demand.

In supply chain management, it concerns efficient policies related to purchasing raw materials from suppliers according to order or market forecast, transforming them into finished goods considering production capacity, and delivering them to end customer. Traditionally, the activities are optimized separately due to the intractability of large model. It is obvious that such pattern neglects the internal relation in the chain compared with optimizing these steps simultaneously since optimization of each step separately does not necessarily lead to the optimization of all steps in an integrated manner. That is especially true when we deal with multi-plant and multi-DC under a centralized environment, where mutual cooperation is permitted and often required as long as such decision is cost-efficient for each DC to meet its demand.

Consequently, the coordinated operations of the main stages will lead to remarkable cost reductions for the company. For example, in a research of Libbey-Owens-Ford Company (Martin et al., 1993), integrated approach saves nearly $2,000,000 compared with separated operations in annual cost. Another production-distribution study for Procter&Gamble (P&G) company (Camm et al., 1997) shows that integrated planning cuts down almost 20% of total cost. Integrated production and planning has become a new branch of supply chain management (Hugos, 2011; Papageorgiu, 2009).

Our model differs from previous works in the technique to characterize production function. We assume no *a priori* information on production technology. In particular, this paper introduces data envelopment analysis (DEA), a nonparametric method to describe production process, into integrated production-transportation problem, which is a different approach compared to the previous works in this field. There have been many papers covering the integrated production-transportation problem in a tactical level, some of which include the management of inventory especially in multi-period situations. However, most of them link the production process with *a priori* production relationship. For example Zuo et al., (1991), Barbarosoğlu and Özgür (1999), Jayaraman and Pirkul (2001), Jain and Palekar (2005), Kanyalkar and Adil (2007) etc. propose models with production capacity or capacity expansion as consistent constraints; Tuy et al., (1993), Hochbaum and Hong (1996), Tuy et al., (1996), Kuno and Utsunomiya (1997; 2000), etc. explicitly draw on exogenous production functions. In fact, such valuable *a priori* information is not always available, which reduces the applicability of their models.

DEA is the one of the best modeling tools for providing a satisfactory solution. By using "satisfactory solution", we imply that our model is based on limited information about production process that the decision maker (DM) could be able to secure. The characterization of functional dependency between inputs and outputs in a production process is not an easy undertaking in some applications. This becomes more severe when the dimensions of inputs and outputs increase as exemplifying the features of the modern manufacturing, which partially motivate the research of this paper. Besides, DEA technique helps to identify whether the production process is efficient or not.

The rest of the paper is organized as follows. Section 1 reviews the current literature on DEA-based production planning and integrated production-transportation problem. An integrated model of DEA-based production and transportation planning is proposed in section 2. An illustration of the model is given in section 3. Sensitivity analyses on the input's and transportation price's order of magnitude in section 4. Conclusions are drawn in the last section.

1. LITERATURE REVIEW

Our paper relates to two bodies of research: The literature on integrated production-transportation and the literature on production planning based on DEA.

Dhaenens-Flipo and Finke (2001) deals with a multi-facility and multi-product planning problem, where production costs and transportation costs are regarded simultaneously. Simchi-Levi et al., (2004) gives a comprehensive review on the explicit production-distribution (EPD) problems. Various EPD problems are classified by three criteria: decision level, integration structure and problem parameters. In this paper we focus on the production-transportation problems, one class of great attention. Kanyalkar and Adil (2007) present a linear programming model to overcome the weaknesses of sequential planning approaches in a multi-site environment, where specific factors, are considered for a consumer goods enterprise. Alemany et al., (2010) proposes a mixed-integer linear programming (MILP) model under a centralized ceramic tile sector. The objective function is to maximize total net profit while the master planning is determined in multi-period and multi-item. Kopanos et al., (2012) develop a discrete/continuous-time MILP model in real-life semi-continuous food industries. They take alternative transportation modes, for example different kinds of trucks, into account.

First put forward by Charnes et al., (1978) as a nonparametric method for estimating the relative efficiency of a group of homogenous decision making units (DMUs), DEA now has been widely applied to the public sector. Recently DEA has been applied to make production planning. This approach bases on history inputs and outputs data. Golany (1988) first presents an interactive multi-objective linear programming procedure to help the central DM decide realistic performance goals. Beasley (2003) puts forward an approach to maximizing average DMU efficiency while simultaneously deciding for all DMUs more acceptable results. Korhonen and Syrjänen (2004), like Golany, suggest a method to maximize the total amount of outputs of all DMUs by a multi-objective linear programming to find the most preferred allocating plan. Du et al., (2010) recommend two planning ideas for arranging new input-output mix. One is to optimize the average production efficiency, and the other is to maximizing total outputs while simultaneously minimizing the total inputs. As far as we are aware, there is no DEA-based work regarding integrated production-transportation problem in the literature. Thus, the current paper suggests a new direction to address this problem.

2. THE MODEL

The problem we study can be graphically illustrated in Figure 1. There are several production plants, each of which is directly connected to a large-scale DC by a solid line. Each production plant has its own production plan. These solid lines indicate that all the goods are transported to the corresponding DC once finished. In addition, these DCs are inter-connected by the dotted lines, which indicate possible transportation trips. Here "possible" means transportation trips are needed depends on whether they are cost-effective. In Figure 1, each DC is surrounded by some people. This indicates the predicted market demand in next period. Note that the transportation volume depends on the production plan we make and how is the DC's market demand satisfied by the corresponding production plants. As a result, all production plants are connected together indirectly.

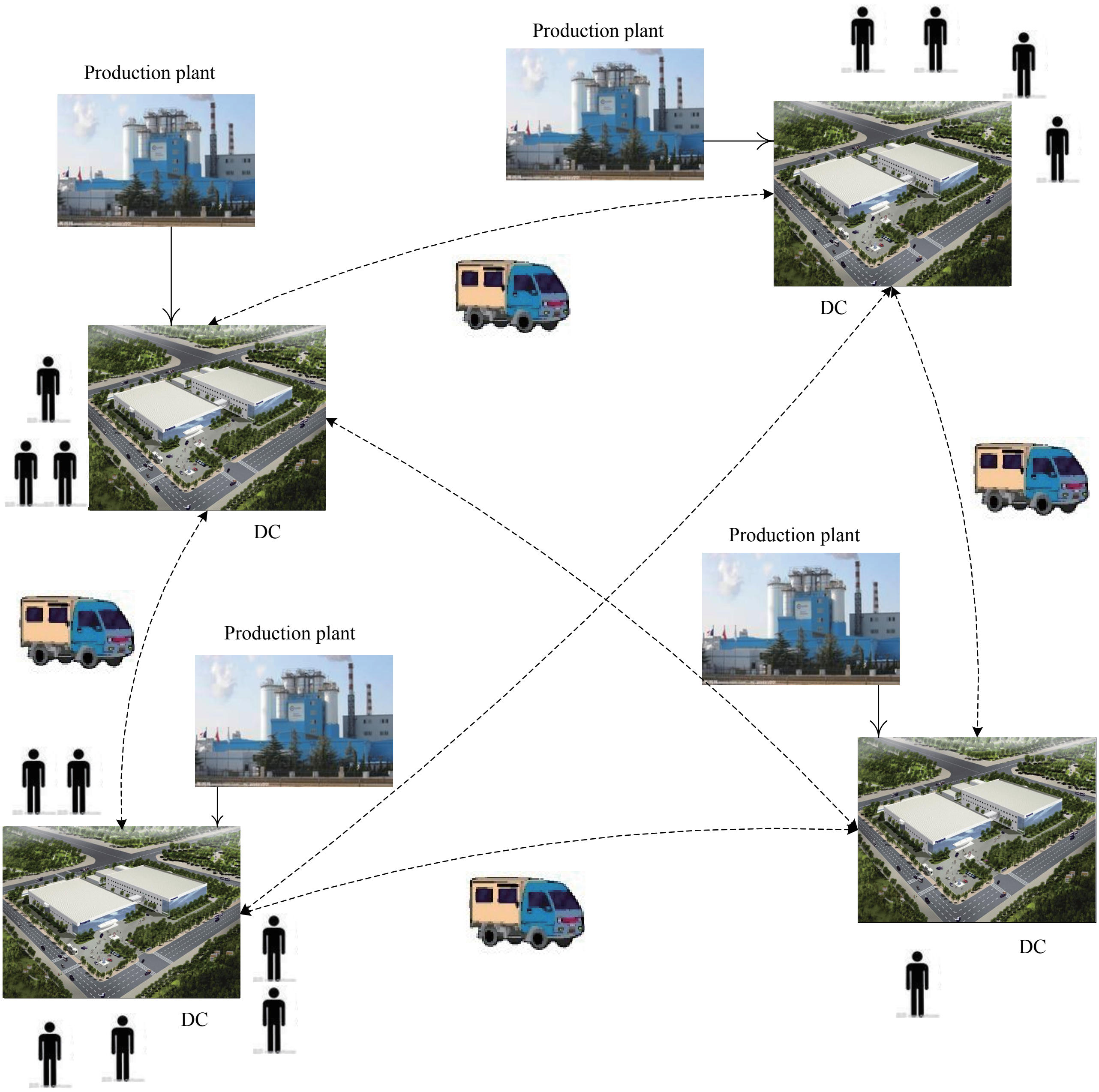

Figure 1
Graphical Representation of the Problem

Our objective is to minimize total costs, including production costs taking place in production plants and transportation costs among DCs, while the transportation costs between production plants and DCs are neglected to highlight the other two kinds of costs.

For modeling purpose, we assume there are *n* production plants (denoted as DMUj (j=1,…,n)). History inputs and outputs data are stated as x^h_{ij} (i=1,2,...m) and y^h_{ri} (r=1,2,...s) for i-th input and r-th output of j-th DMU respectively. Here the data are non-negative and the superscript h stands for history data. x_{ij} (i=1,2,...m) and y_{rj} (r=1,2,...$s$represent i-th input and target r-th output of j-th DMU for the next period. They, as a whole, are the production plans for all DMUs in our integrated production-transportation problem. We use the notation d_{rj} for the j-th DC's r-th market demand which can be predicted in advance, and $t^{(r)}_{jk}$ for the transportation volume of the r-th product from the j-th DC to the k-th DC. According to the problem description above, model (1) for the production-transportation planning is given.

$$\min \sum_{i=1}^{m} c_i \sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n}\sum_{k=1}^{n} e_{jk} \sum_{r=1}^{s} t_{jk}^{(r)}$$
$$s.t.\begin{cases} \sum_{p=1}^{n} \lambda_{pj} x_{ip}^{h} \le x_{ij}, i=1,2,...m, j=1,2,...n \\ \sum_{p=1}^{n} \lambda_{pj} y_{rp}^{h} \ge y_{rj}, r=1,2,...s, j=1,2,...n \\ \sum_{p=1}^{n} \lambda_{pj} = 1, \quad j=1,2,...n \\ y_{rj} - \sum_{k=1}^{n} t_{jk}^{(r)} + \sum_{k=1}^{n} t_{kj}^{(r)} \ge d_{rj}, \ r=1,2,...s, j=1,2,...n \\ \left(\sum_{k=1}^{n} t_{jk}^{(r)}\right)\left(\sum_{k=1}^{n} t_{kj}^{(r)}\right) = 0, r=1,2,...s, j=1,2,...n \\ x_{ij}, y_{rj}, \lambda_{pj} \ge 0, t_{jk}^{(r)} \ge 0, t_{jj}^{(r)} = 0, \\ k,j,p=1,2,...n, i=1,2,...m, r=1,2,...s \end{cases} \tag{1}$$

$$\min \sum_{i=1}^{m} c_i \sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n}\sum_{k=1}^{n} e_{jk} \sum_{r=1}^{s} t_{jk}^{(r)}$$
$$s.t.\begin{cases} \sum_{p=1}^{n} \lambda_{pj} x_{ip}^{h} \le x_{ij}, i=1,2,...m, j=1,2,...n \\ \sum_{p=1}^{n} \lambda_{pj} y_{rp}^{h} \ge y_{rj}, r=1,2,...s, j=1,2,...n \\ y_{rj} - \sum_{k=1}^{n} t_{jk}^{(r)} + \sum_{k=1}^{n} t_{kj}^{(r)} \ge d_{rj}, \ r=1,2,...s, j=1,2,...n \\ \left(\sum_{k=1}^{n} t_{jk}^{(r)}\right)\left(\sum_{k=1}^{n} t_{kj}^{(r)}\right) = 0, r=1,2,...s, j=1,2,...n \\ x_{ij}, y_{rj}, \lambda_{pj} \ge 0, t_{jk}^{(r)} \ge 0, t_{jj}^{(r)} = 0, \\ k,j,p=1,2,...n, i=1,2,...m, r=1,2,...s \end{cases} \tag{2}$$

where c_i stands for the unit price of the i-th input, e_{jk} for the unit transportation price from the j-th DC to the k-th DC.

The objective function is the total costs. The first-group and second-group constraints represent that new input-output combination must be enveloped by the production possibility set (PPS). Various retruns-to-scale (VRS) assumption (Banker et al., 1984) is indicated by the third-group constraints. Note that if the convexity constraints are deleted, then we get the model concerning constant return-to-scale (CRS) assumption (Charnes et al., 1978), which corresponds to model (2). The fourth-group constraints in model (1) consider the outcome after transportation. They imply that the sum of each DC's volume of products which are equal to the products produced locally and the net volume of products of transportations are able to satisfy the corresponding market demand, where $\sum_{k=1}^{n} t_{jk}^{(r)}$ means the amount of r-th output to be transported from the j-th DC to other DC and $\sum_{k=1}^{n} t_{kj}^{(r)}$ implies the amount of r-th output to be transported from other DC to the j-th DC. The fifth-group constraints in model (1) impose restrictions on the DC to ensure this DC's state of transportation for a certain kind of output. That means the state only belong to one of the three states, namely inward freight, outward freight and neither inward nor outward freight. Here, inward freight indicates transporting this certain kind of output from other DC to the under-considering DC, outward freight on the contrary, and the last state indicates zero transportation. As a consequence, this will prevent redundant conveyance, avoiding the appearance of both inward and outward freight at the same time. Since model (1) and (2) are so much alike except the convexity constraints, in the following we mainly deal with model (1) while similar results about model (2) can be achieved in analogous way.

Note that the product of decision variables makes model (1) a non-linear programming. By introducing 0-1 variables, model (1) is equivalently transformed to the following model (3), where $\rho_{rj}^{(i)}$ are binary variables and M is a sufficiently large positive number. After the transformation, model (3), equivalent to model (1), is a MILP. Model (2) can be transformed to a similar model except the convexity constraints; we omit it for simplicity in the following paper.

$$\min \sum_{i=1}^{m} c_i \sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n}\sum_{k=1}^{n} e_{jk} \sum_{r=1}^{s} t_{jk}^{(r)}$$
$$s.t.\begin{cases} \sum_{p=1}^{n} \lambda_{pj} x_{ip}^{h} \le x_{ij}, i=1,2,...m, j=1,2,...n \\ \sum_{p=1}^{n} \lambda_{pj} y_{rp}^{h} \ge y_{rj}, r=1,2,...s, j=1,2,...n \\ \sum_{p=1}^{n} \lambda_{pj} = 1, \quad j=1,2,...n \\ y_{rj} - \sum_{k=1}^{n} t_{jk}^{(r)} + \sum_{k=1}^{n} t_{kj}^{(r)} \ge d_{rj}, \ r=1,2,...s, j=1,2,...n \\ \sum_{k=1}^{n} t_{jk}^{(r)} \le (1-\rho_{rj}^{(1)})\ M, r=1,2,...s, j=1,2,...n \\ \sum_{k=1}^{n} t_{kj}^{(r)} \le (1-\rho_{rj}^{(2)})\ M, r=1,2,...s, j=1,2,...n \\ \sum_{k=1}^{n} t_{jk}^{(r)} + \sum_{k=1}^{n} t_{kj}^{(r)} \le (1-\rho_{rj}^{(3)})\ M, r=1,2,...s, j=1,2,...n \\ \rho_{rj}^{(1)} + \rho_{rj}^{(2)} + \rho_{rj}^{(3)} = 1, r=1,2,...s, j=1,2,...n \\ x_{ij}, y_{rj}, \lambda_{pj} \ge 0, i=1,2,...m, j,p=1,2,...n, r=1,2,...s, \\ t_{jk}^{(r)} \ge 0, t_{jj}^{(r)} = 0 r=1,2,...s, k,j=1,2,...n \end{cases} \tag{3}$$

Now we consider the solvability of model (3). To begin with, the specific definition of PPS in DEA is introduced. The general form for PPS is expressed as $PPS=\{(x,y){:}x$ *can produce* $y\}$ .To maintain the uniformity in notation, $X^h=(x_{ij}^h)_{m\times n}$ and $Y^h=(y_{rj}^h)_{s\times n}$ are the history inputs matrix and outputs matrix respectively, while $x_{\bullet j}=(x_{1j},x_{2j},...x_{mj})^T$ and $y_{\bullet j}=(y_{1j},y_{2j},...y_{mj})^T$ stand for the possible inputs and outputs for the j-th DMU based on the PPS determined by history data, where the superscript T represents transposition. Then different PPSs in DEA deriving from different returns-to-scale technologies are as follows where $\lambda_{\bullet j}=(\lambda_{1j},\lambda_{2j},...\lambda_{mj})^T$:

$$PPS_{CRS-j} = \left\{(x_{\bullet j}, y_{\bullet j}) : Y^h\lambda_{\bullet j} \ge y_{\bullet j}, X^h\lambda_{\bullet j} \le x_{\bullet j}, \lambda_{pj} \ge 0, p=1,2,...n\right\} \tag{4}$$

$$PPS_{VRS-j} = \left\{(x_{\bullet j}, y_{\bullet j}) : Y^h\lambda_{\bullet j} \ge y_{\bullet j}, X^h\lambda_{\bullet j} \le x_{\bullet j}, \sum_{p=1}^{n}\lambda_{pj} = 1, \lambda_{pj} \ge 0, p=1,2,...n\right\} \tag{5}$$

Both (4) and (5) correspond to the PPS of the j-th DMU. Overall PPS in VRS is as follows, $P(x_1, x_2, ..., x_m) =$

$\{(y_1,...,y_s)\,|\,\sum_{j=1}^{n}(x_{\bullet j},y_{\bullet j})=(x,y),(x_{\bullet j},y_{\bullet j})\in PPS_{VRS-j}\}$, where $y=(y_1,y_2,...y_s)$ and $x=(x_1,x_2,...x_s)$. The overall PPS is defined as the Minkowski sum of n PPS's. Therefore we will get the following sufficient and necessary condition for solvability.

Proposition 1: model (3) in the setting of VRS is solvable if and only if $(\sum_{j=1}^{n}d_{1j},...,\sum_{j=1}^{n}d_{sj})\in P=\{(y_1,...,y_s)\,|\,\sum_{j=1}^{n}\lambda_j y_{rj}^h\geq y_r/n,\sum_{j=1}^{n}\lambda_j=1\}$, where P is the overall producible output set for the central DM in VRS. In particular, model (2) is always solvable in the setting of CRS no matter what the exact value of d_{rj} is.

Proof. See appendix

Proposition 1 gives a glimpse of the solvability of model (3). Besides it is beneficial to collect the influences of different PPSs, i.e., the influences of two return-to-scale technologies. Based on (4), (5) and the constraints in model (2) and (3), the following is evident: $obj(VRS)\geq obj(CRS)$ where $obj(\bullet)$is the optimal objective value corresponding to different technologies. Different technologies impact on the construction of PPS, leading to different objective values when model (2) and (3) are based on the same history data and the same cost parameters.

One significant characteristic of DEA-based production planning is that it always provides Pareto non-dominated plan, i.e., efficient plan. This can be found but not limited to the literatures, such as Lozano and Villa (2004), Du et al., (2010), and Amirteimoori and Kordrostami (2011). Similarly, the same thing holds true when it comes to this paper. Hence, we give the following property.

Property 2: The production plan from model (3) will guarantee that the new efficiency scores of all DMUs will reach one when estimated by the original PPS (model (2) owns the same result).

Proof. See appendix

It can be seen that the production plan is Pareto optimal (i.e., the production cost cannot be reduced if no output is sacrificed), or it contradicts that the production plan attains the minimum of (3). Therefore it should be rated as efficient by DEA models.

3. EMPIRICAL ILLUSTRATION AND THE SENSITIVITY ANALYSIS

To better illustrate the proposed models in this paper, a set of production data consisting of ten DMUs with two inputs and two outputs is employed, which is extracted from Lozano and Villa (2004). This data are treated as history production mix. We use hypothetical data for unit transportation price among different DCs and unit price for each input. To move a step further, we perform sensitivity analysis to the orders of the magnitude of the unit transportation price and unit price for each input to explore their influence on production schedule. Here we assume that unit price for each input is in the same order of magnitude, while the unit transportation price is also in the same order of magnitude which is not necessary the same as that for unit input price. The sensitivity analysis focuses on the ratio of order of magnitude of input price to that of for transportation.

We first generate two unit prices for input at random; then again randomly generate the unit transportation price among the 10 DCs in the same magnitude. Then the transportation price matrix is magnified and minified in equal proportion to obtain different matrixes corresponding to different magnitudes. With regard to market demand, we suppose that $\sum_{j=1}^{n}d_{rj}=\sum_{j=1}^{n}y_{rj}^h(r=1,2,...s)$ both in VRS and CRS, namely the market demand for next time period equals to the history output. In the short run, a mature market can be seen in a comparatively stable condition, which means the fluctuation in market demand is quite small. Hence, it is understandable to treat the market demand as unchanged. Besides in practice, consensus is reached about the above idea among most of the business entities, because it helps business entities to make a rough production planning at the very beginning. This is also a particular situation in **Proposition** 1, which will be get when setting $\lambda_j=1/n(\,j=1,...,n)$ and $\sum_{j=1}^{n}\lambda_j y_{rj}^h=y_r/n$ in VRS, also a particular situation in CRS. The history data for each output is regarded as a known permutation; then a random arrangement is obtained based on it. Thus the market demand for each output of each DC is achieved while the requirement in **Proposition** 1 is satisfied. The unit price for each input, unit transportation price among each DC, and predicted market demand for next period are listed in **Appendix**. The sensitivity analysis is discussed in 9 scenes, that is magnitudes of the ratio of transportation price to input price are 10000, 1000, 100, 10, 1, 0.1, 0.01, 0.001, 0.0001 respectively.

The optimal objective value of model (3) is given in Table 1. Table 1 shows that as the magnitudes of ratio of transportation price to input price change from 10000 to 0.0001 the optimal objective value in both PPS declines. As the transportation price decreases, it is cost-effective to transport superfluous output for certain DC to DCs whose outputs are not enough. If every DMU choose to meet its DC's market demand alone, advantages of scale production are abandoned. In particular, the minimum cost in CCR is always smaller than that in BCC. This has to do with the different PPSs corresponding to different returns to scale. When the history production data is determined, PPS in CCR is more efficient than that in BCC. According to Property 2, the efficiency score of production schedule will reach 1; then the schedule will lie on the production frontier. Hence, schedule in CCR more efficient than that in BCC, which leads to less costs in CCR compared to BCC.

Table 1
The Optimal Value of Model (3) in 9 Scenes

	SCENE 1	SCENE 2	SCENE 3	SCENE 4	SCENE 5	SCENE 6	SCENE 7	SCENE 8	SCENE 9
CCR	64.602 86	64.602 86	64.602 86	61.6381	58.225 38	57.672 83	57.5567	57.545 09	57.543 97
BCC	230.5334	92.807 4	75.978 13	68.587 52	65.096 83	64.5082	64.449 34	64.443 45	64.442 87

Table 2
Input Schedule for Next Period in CCR

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
SCENE1	10.857	8	9.6	8.75	9.6	3.2	4	10.5	6	6
	13.143	8	9.6	3.75	9.6	3.2	3.333	4.5	7.5	5
SCENE2	10.857	8	9.6	8.75	9.6	3.2	4	10.5	6	6
	13.143	8	9.6	3.75	9.6	3.2	3.333	4.5	7.5	5
SCENE3	10.857	8	9.6	8.75	9.6	3.2	4	10.5	6	6
	13.143	8	9.6	3.75	9.6	3.2	3.333	4.5	7.5	5
SCENE4	10.8	8	7	6	9.6	1.2	3.2	8.75	6	6
	18	8	6.5	10	9.6	2	3.2	3.75	7.5	5
SCENE5	14.667	7.933	8	6	9.6	0	1.067	2.133	6	6
	24.444	7.9	6.667	10	9.6	0	1.067	3.556	7.5	5
SCENE6	6.8	6	14	2	4	2	12	7.2	6	6
	9.667	5	11.667	1.667	3.333	1.667	10	12	5	5
SCENE7	6.8	6	4	10	4	4	12	7.2	6	6
	9.667	5	3.333	8.333	3.333	3.333	10	12	5	5
SCENE8	6.8	6	14	2	4	2	12	7.2	6	6
	9.667	5	11.667	1.667	3.333	1.667	10	12	5	5
SCENE9	10	10	4	10	12	2	4	2	6	6
	15.833	8.333	3.333	8.333	10	1.667	3.333	1.667	7.5	5

Table 3
Input Schedule for Next Period in BCC

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
SCENE1	13	8	12	6	12	6	6	11.333	6	6
	8	8	10	10	10	10	10	7.333	10	10
SCENE2	13	8	12	6	12	6	6	11.333	6	6
	8	8	10	10	10	10	10	7.333	10	10
SCENE3	13	8	8	6	8	6	6	11.333	6	6
	8	8	8	10	8	10	10	7.333	10	10
SCENE4	6	8	6	6	8	6	6	8.667	6	6
	10	8	10	10	8	10	10	8.667	10	10
SCENE5	6	7	6	6	8	6	6	6	6	6
	10	9	10	10	8	10	10	10	10	10
SCENE6	6	7	6	6	8	6	6	6	6	6
	10	9	10	10	8	10	10	10	10	10
SCENE7	6	7	6	6	8	6	6	6	6	6
	10	9	10	10	8	10	10	10	10	10
SCENE8	6	7	6	6	8	6	6	6	6	6
	10	9	10	10	8	10	10	10	10	10
SCENE9	6	7	6	6	8	6	6	6	6	6
	10	9	10	10	8	10	10	10	10	10

Production plan for next period is given in Table 2 and Table 3 for 9 scenes. Table 2 exhibits that the input schedule for scene1 to scene3 is the same, since the transportation price is larger than input price. Therefore the optimal plan for scene1 to scene3 is meeting every DC's market demand independently. Scene3, scene4, scene5, scene6 and scene7 have different input schedules because of magnitudes of transportation price. Scene6, scene8 have the same input schedule, which is a result of the less change in transportation price compared to the constant input price. Even though the input schedule in scene9 is different from that in scene8, the minimum cost in Table 1 changes little. The difference between them is caused by the change of transportation price. Table 3 comes to similar conclusion. Scene1, scene2 have the same schedule, while scene5, scene6, scene7, scene8, and scene9 also have the same input schedule. Schedules among scene3, scene4 and scene5 are different. And the related transportation plans are given in Table 4, where t(i,j,k) stands for the *i*-th output transportation amount from DC_j to DC_k.

Table 4
Transportation Plans for Model (3)

	CCR	BCC
SCENE1	No transportation	t(1,9,1)=1
SCENE2	No transportation	t(1,9,1)=1
SCENE3	No transportation	t(1,8,1)=1,t(2,6,3)=1,t(2,7,5)=1
SCENE4	t(1,1,8)=1, t(1,5,7)=0.8, t(2,1,6)=1.4	t(1,6,1)=3, t(2,4,3)=2, t(2,6,3)=1, t(2,7,5)=1,t(2,8,1)=1
SCENE5	t(1,1,8)=4.222, t(1,3,6)=1, t(1,5,7)=1.6, t(2,1,6)=2, t(2,1,7)=1.333, t(2,8,2)=0.067	t(1,2,8)=1, t(1,6,1)=3, t(2,4,3)=2, t(2,6,3)=1, t(2,7,5)=1, t(2,8,1)=1, t(2,8,2)=1
SCENE6	t(1,3,4)=4, t(1,7,1)=3, t(1,7,9)=1, t(2,3,6)=1, t(2,7,5)=4, t(2,8,1)=0.6, t(2,8,2)=2	t(1,2,8)=1, t(1,6,1)=3, t(2,4,3)=2, t(2,6,3)=1, t(2,7,5)=1, t(2,8,1)=1, t(2,8,2)=1
SCENE7	t(1,6,3)=1, t(1,7,1)=3, t(1,7,9)=1, t(2,4,3)=4, t(2,7,5)=4, t(2,8,1)=0.6, t(2,8,2)=2	t(1,2,8)=1, t(1,6,1)=3, t(2,4,3)=2, t(2,6,3)=1, t(2,7,5)=1, t(2,8,1)=1, t(2,8,2)=1
SCENE8	t(1,6,3)=1, t(1,7,1)=3, t(1,7,9)=1, t(2,4,3)=4, t(2,7,5)=4, t(2,8,1)=0.6, t(2,8,2)=2	t(1,2,8)=1, t(1,6,1)=3, t(2,4,3)=2, t(2,6,3)=1, t(2,7,5)=1, t(2,8,1)=1, t(2,8,2)=1
SCENE9	t(1,2,3)=1, t(1,2,8)=1, t(1,5,8)=4, t(2,1,6)=1, t(2,4,3)=4	t(1,2,8)=1, t(1,6,1)=3, t(2,4,3)=2, t(2,6,3)=1, t(2,7,5)=1, t(2,8,1)=1, t(2,8,2)=1

Table 4 shows the detailed arrangements about the transportation plan. Allowing for the unit transportation price matrix in Appendix, the top 5 lowest unit transportation price are between DC_5 and DC_7, DC_1 and DC_8, DC_3 and DC_6, DC_3 and DC_4, and DC_1 and DC_6 respectively. Take the transportation plan in CCR as an example. When the transportation price is too high in comparison with input price, (i.e. in scene1, scene2, and scene3), there is no transportation trips among all the DCs. When the transportation trips first appear in scene4, all of them belong to the top 5 lowest unit transportation price. In scene5, 4 trips belong to the top 5, accounting for 80% trips. As transportation price declines, trips which are outside of the top 5 come out. Similar conclusions can be found in BCC. Note that transportation trips first appear in scene1 in BCC although transportation price is relatively high. This is because the strict requirements regarding variable returns to scale, which makes transportation necessary to meet market demand if the DMU can't produce enough outputs under current production technology. Excluding this particular situation, it is worth mentioning that in all the two kinds of PPSs, transportation trips in scene1, scene2, scene3, and scene4 all belong to the top 5. What's more, the amount related with such trips is relatively larger than other trips. This makes sense since the objective is to minimize total costs both in production and in transportation.

CONCLUSIONS

This paper presents a series of models to deal with integrated production-transportation problem based on DEA. It offers an alternative approach to investigate the production process aiming at efficiently organizing inputs for given output targets based on empirical PPS formed by history data. Since DEA deals with production planning without *a priori* information about the production relationship, the proposed method has comparative advantage as compared with the extant researches related to integrated production-transportation planning.

When the production planning is under consideration, how to transport output surpluses in one DC to other DCs which have insufficient outputs should be considered at the same time. Our proposed transportation scheme consists of transportation trips among DCs and the transportation volumes. The solvability is also considered in this paper. A numerical example is used to illustrate the feasibility of the proposed models and sensitivity analysis is made to better reflect the result.

To sum up, this paper helps the DM to determine an efficient and attainable plan both for production and transportation for the planning horizon. In the meantime, the aggregate costs are minimized. In this paper, the inventory is ignored since we deal with a single period. Besides, there might be other specific conditions corresponding to certain application scenario that are ignored either. Since we just provide general models which are amenable to further customization for a particular case, further research is still warranted, such as taking inventory condition into account, stochastic factors in demand predicted, and expanding the time horizon.

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APPENDIX: PROOFS OF PROPOSITIONS AND DATA

Proof of Proposition 1

First, we consider the solvability in VRS. To prove the sufficiency, we need to find a solution when $(\sum_{j=1}^{n} d_{1j},...,\sum_{j=1}^{n} d_{sj})$ belongs to P. In this condition, there exists a group of confirmable λ_j corresponding to this $(\sum_{j=1}^{n} d_{1j},...,\sum_{j=1}^{n} d_{sj})$. Then we might as well assume $\lambda_{Pj}^*=\lambda_P,P=1,2,...n$ for the j-th DMU, which indicates that all the DMUs follow the same production planning. Therefore we have $\sum_{j=1}^{n} d_{rj} = y_r \le n\sum_{j=1}^{n} \lambda_j y_{rj}^h = n\sum_{p=1}^{n} \lambda_{pj}^* y_{rp}^h$. Obviously, market demand is reached. Hence, $\sum_{p=1}^{n} \lambda_{pj}^* x_{ip}^h$, $\sum_{p=1}^{n} \lambda_{pj}^* y_{rp}^h$ and λ_{Pj}^* is a feasible solution to (3) with other decision variables equal to 0. Hence, the solution set to (3) is non-empty, which guarantees the solvability of (3) when it is a MILP programming. Then there comes the necessity. If $(\sum_{j=1}^{n} d_{1j},...,\sum_{j=1}^{n} d_{sj}) \notin P$, still we get a solution to (3). Assuming λ_{Pj}^*, P, j=1,2,...n is part of the solution with $\sum_{p=1}^{n} \lambda_{Pj}^*=1, j=1,2,...n$. Under this circumstance, if we define $\sum_{j=1}^{n} \lambda_{pj}^* \big/ n = \lambda_P^*$, p=1,2,...n then $\sum_{p=1}^{n} \lambda_p^* = 1$ is obvious. Hence, we get $\sum_{p=1}^{n} \lambda_p^* y_{rp}^h = \sum_{p=1}^{n}\left(y_{rp}^h \left(\sum_{j=1}^{n} \lambda_{pj}^* \big/ n \right)\right) \ge \left(\sum_{j=1}^{n} y_{rj} \big/ n \right) \ge y_r / n$, which contradicts to the assumption that $(\sum_{j=1}^{n} d_{1j},...,\sum_{j=1}^{n} d_{sj}) \notin P$. From the above, both the sufficiency and necessity are proved in the setting of VRS.

Next, we take a close examination of CRS-setting. Once the market demand is forecasted, the exact value of d_{rj} is obtained. Hence, we can get $\max\left(\sum_{j=1}^{n} d_{rj} \Big/ \sum_{j=1}^{n} y_{rj}^h,\ r=1,2,...s \right) = L_r$ and $\max(d_{rj}/y_{rj}^h$, r=1,2,...s,j=1,2,...n)=L_{rj}, two largest ratios. Furthermore, $L_0=\max(L_r,L_{rj})$ will be achieved. Now, we consider the following linear equations for the j-th DMU:

$$\begin{pmatrix} y_{1,1}^h & y_{1,2}^h & \cdots & y_{1,n}^h \\ y_{2,1}^h & y_{2,2}^h & \cdots & y_{2,n}^h \\ \vdots & \vdots & \vdots & \vdots \\ y_{s,1}^h & y_{s,2}^h & \cdots & y_{s,n}^h \end{pmatrix} \begin{pmatrix} \lambda_{1,j} \\ \lambda_{2,j} \\ \vdots \\ \lambda_{n,j} \end{pmatrix} = L_0 \begin{pmatrix} y_{1,j}^h \\ y_{2,j}^h \\ \vdots \\ y_{s,j}^h \end{pmatrix} \tag{6}$$

It is obvious that the coefficient matrix and augmented matrix of (6) have the same rank, and then problem (6) is solvable. Suppose the solution to (6) is $(\lambda'_{1,j}\ \lambda'_{2,j} \cdots \lambda'_{n,j})^{\mathrm{T}}$ □ λ'_j. After we go through all the similar problem of (6) corresponding to every DMU, we will get the solution matrix of λ' □ $(\lambda'_1 \lambda'_2 \cdots \lambda'_n)$. However, we should note that 1) $L_0 y_{rj}^h \ge d_{rj}$ since $L_0 \ge L_{rj}$ and 2) $L_0 \sum_{j=1}^{n} y_{rj}^h \ge \sum_{j=1}^{n} d_{rj}$ since $L_0 \ge L_r$. 1) shows that the j-th outputs satisfy its market demand, and 2) shows that the i-th total output meets the corresponding demand of the whole market. Therefore, $x'_{ij} = \sum_{p=1}^{n} \lambda'_{pj} x_{ip}^h$ $y'_{rj} = \sum_{p=1}^{n} \lambda'_{pj} y_{rp}^h$, $\rho_{rj}^{(3)\prime} = 1$ and λ' is a feasible solution to (3) in the setting of CRS, while the rest variables all equal to 0. The boundness of is objective function is clearly as a result. Accordingly, optimal solution exists for (2) in the CRS situation no matter what the exact value of d_{rj} is.

Proof of Property 2

To begin with, new efficiency scores of all DMUs will be tackled first. Set x_{ij}^*, y_{rj}^*, λ_{pj}^*, $t_{jk}^{(r)*}$,$\rho_{rj}^{(l)*}$ (j, p, k=1,2,...n, i=1,2,...m, r=1,2,...s, l=1,2,3),as the optimal solution to model (3), then minimum objective value is $m^* = \sum_{i=1}^{m} c_i \sum_{j=1}^{n} x_{ij}^* + \sum_{j=1}^{n}\sum_{k=1}^{n} e_{jk} \sum_{r=1}^{s} t_{jk}^{(r)*}$. The efficiency of the q-th DMU after production planning, *i.e.,* new efficiency of the q-th DMU, under original PPS will be evaluated by the following model (7):

$$\min \theta_q$$
$$s.t.\begin{cases} \sum_{j=1}^{n} \lambda_j x_{ij}^h \le \theta_j x_{iq}^*, i=1,2,...m \\ \sum_{j=1}^{n} \lambda_j y_{rj}^h \ge y_{rq}^*, r=1,2,...s \\ \mu_1\left(\sum_{j=1}^{n} \lambda_j \right) = \mu_1 \\ \lambda_j \ge 0, j=1,2,...n \end{cases} \tag{7}$$

Assume that there exists at least one DMU, namely DMU_{j0}, whose new efficiency score of certain kind is strictly less than one. Then suppose the optimal value of model (7) for DMU_{j0} as θ^*_{j0},λ^*_j. Hence, we know that θ^*_{j0}<1, However, from model (7), we have $\sum_{j=1}^{n} \lambda_j^* x_{ij}^h \le x_{ij0}^* + (\theta_{j0}^* - 1)x_{ij0}^*$ and $\sum_{j=1}^{n} \lambda_j^* y_{rj}^h \ge y_{rj0}^*$. Therefore, it is easy to find out that $x_{ij0}^* + (\theta_{j0}^* - 1)x_{ij0}^*, \lambda_j^*$, with other decision variables unchanged, is a feasible solution to model (3). Then the objective value of this feasible solution is $m^{*'} = \sum_{i=1}^{m} c_i \sum_{j=1}^{n} x_{ij}^* + \sum_{j=1}^{n}\sum_{k=1}^{n} e_{jk} \sum_{r=1}^{s} t_{jk}^{)(r\,*} + (\theta_{j0}^* - 1)\sum_{i=1}^{m} c_i x_{ij0}^*$. Since θ^*_{j0}<1, it is obvious that $m^{*'}\ m^*$, which contradicts to the pre-supposed condition that m^* is the minimum objective value to model (3). As a consequence, the assumption that there exists at least one DMU with new efficiency score of certain return-to-scale technology strictly less than one is wrong. Then, the property is proved. □

In the main body of this paper, we carefully deliberate how to obtain the parameters in model (3). Followings are the data. Since the history production data, i.e., a two-input two-output production process, is presented by Lozano and Villa (2004), here we omit them for simplicity. The randomly generated input prices are 0.6038 and 0.2722. The unit transportation price matrix for scene 5 is given in Table 5. Here we consider it as a symmetric matrix. Predicted market demand for next time period is listed in Table 6. Table 7 represents outputs targets for CCR and BCC to complete data in Table 2 and Table 3.

Table 5
Unit transportation Price Matrix for Scene 5

m*	DC1	DC2	DC3	DC4	DC5	DC6	DC7	DC8	DC9	DC10
DC1	0	0.9501	0.7621	0.6154	0.4057	0.0579	0.2028	0.0153	0.4187	0.8381
DC2	0.9501	0	0.2311	0.4565	0.7919	0.9355	0.3529	0.1987	0.7468	0.8462
DC3	0.7621	0.2311	0	0.0196	0.6068	0.0185	0.9218	0.9169	0.8132	0.6038
DC4	0.6154	0.4565	0.0196	0	0.4451	0.5252	0.6813	0.486	0.8214	0.7382
DC5	0.4057	0.7919	0.6068	0.4451	0	0.4103	0.0099	0.2722	0.9318	0.2027
DC6	0.0579	0.9355	0.0185	0.5252	0.4103	0	0.3795	0.8913	0.4447	0.1763
DC7	0.2028	0.3529	0.9218	0.6813	0.0099	0.3795	0	0.8937	0.1389	0.1988
DC8	0.0153	0.1987	0.9169	0.486	0.2722	0.8913	0.8937	0	0.466	0.6721
DC9	0.4187	0.7468	0.8132	0.8214	0.9318	0.4447	0.1389	0.466	0	0.8318
DC10	0.8381	0.8462	0.6038	0.7382	0.2027	0.1763	0.1988	0.6721	0.8318	0

Table 6
Predicted Market Demand for Next Time Period

	DC1	DC2	DC3	DC4	DC5	DC6	DC7	DC8	DC9	DC10
Output1	2	3	2	5	4	3	6	8	1	3
Output2	1	1	2	3	4	3	6	2	6	5

Table 7
Outputs Targets for CCR and BCC in Next Period

CCR	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
SCENE1	8	3	3	5	2	1	2	6	4	3
	4	5	6	1	6	2	2	1	3	3
SCENE2	8	3	3	5	2	1	2	6	4	3
	4	5	6	1	6	2	2	1	3	3
SCENE3	8	3	3	5	2	1	2	6	4	3
	4	5	6	1	6	2	2	1	3	3
SCENE4	9	3	3	5	2.8	1	1.2	5	4	3
	5.4	5	4	3	6	0.6	2	1	3	3
SCENE5	12.222	3	4	5	3.6	0	0.4	1.778	4	3
	7.333	4.933	4	3	6	0	0.667	1.067	3	3
SCENE6	5	3	7	1	2	1	6	6	3	3
	3.4	3	7	1	2	1	6	3.6	3	3
SCENE7	5	3	2	5	2	2	6	6	3	3
	3.4	3	2	5	2	2	6	3.6	3	3
SCENE8	5	3	7	1	2	1	6	6	3	3
	3.4	3	7	1	2	1	6	3.6	3	3
SCENE9	8	5	2	5	6	1	2	1	4	3
	5	5	2	5	6	1	2	1	3	3
BCC	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
SCENE1	7	3	3	5	2	1	2	7	4	3

To be continued

Continued

CCR	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
	4	5	6	1	6	2	2	1	3	3
SCENE2	7	3	3	5	2	1	2	7	4	3
	4	5	6	1	6	2	2	1	3	3
SCENE3	7	3	3	5	2	1	2	7	4	3
	4	5	5	1	5	3	3	1	3	3
SCENE4	5	3	3	5	2	4	2	6	5	3
	3	5	3	3	5	3	3	2	3	3
SCENE5	5	4	3	5	2	4	2	5	4	3
	3	4	3	3	5	3	3	3	3	3
SCENE6	5	4	3	5	2	4	2	5	4	3
	3	4	3	3	5	3	3	3	3	3
SCENE7	5	4	3	5	2	4	2	5	4	3
	3	4	3	3	5	3	3	3	3	3
SCENE8	5	4	3	5	2	4	2	5	4	3
	3	4	3	3	5	3	3	3	3	3
SCENE9	5	4	3	5	2	4	2	5	4	3
	3	4	3	3	5	3	3	3	3	3