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Relation Equations in the Set of Finite Natural Numbers and Its Strunk of Solutions

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Abstract

Definition of relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ in the set of finite natural numbers is given. Based on the discovery of the maximum solution of relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ in the set of finite natural numbers, the strunk of solutions of relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ in the set of finite natural numbers is provided.

Key words: Set of finite natural numbers; Relation equations; Maximum solution; Strunk of solutions

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INTRODUCTION

$N=\{0,1,2,\ldots\}$ is a set of natural numbers (Hua, 1963; Chen, 1978). $N_{n^*}=\{0,1,2,\ldots,n^*\}$ is a set of finite natural numbers, among them, n^* is an arbitrary natural number larger than zero. Starting from Jieshengjishi, human ancestors began to use a set of finite natural numbers. Since the discovery of the set of natural numbers, the set of rational numbers, the set of real numbers, the set of finite natural numbers seem to fade out of sight. The emergence of computer, and the line of sight of people back to the set of finite natural numbers. In the computer, the set $\{0,1\}$ is the most simple set of finite natural numbers. In the image processing, the set $\{0, 1, 2,\ldots, 255\}$ is the set of finite natural numbers. The relation equation on the set of natural numbers has been discussed very mature (Hua, 1957), but the relation equation on the set of finite natural numbers is only mentioned in paper (Zhang, Dong, & Luo, 2014). One of the reasons is that conventional operators on the set of natural numbers does not satisfy closed on the set of finite natural numbers. But that information processing dependent on the set of finite natural numbers show the importance of relation equations on the set of finite natural numbers (Liu, 2003).

In the paper (Zhang, Dong, & Luo, 2014), Addition and multiplication on elements in the set $N_{n^*}=\{0, 1, 2,\ldots,n^*\}$ of finite natural numbers is defined as follows:

Definition 1 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, multiplication is defined as follows

$$m\times n=min\{m, n\}= m\wedge n ,. \tag{1}$$

Definition 2 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, addition is defined as follows

$$m+n=max\{m, n\}= m\vee n. \tag{2}$$

Definition 3 With $F(N_{n^*})$ representative power set in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers, for given set $U=\{u_1, u_2,\ldots, u_m\}\in F(N_{n^*})$ and $V=\{ v_1, v_2,\ldots, v_n\}\in F(N_{n^*})$, the product set of U and V is defined as follows

$$P=U\times V=\{p(u,v)|u\in U,v\in V\}$$

among them, $0\leq u_1\leq u_2\leq\ldots\leq u_m\leq n^*$ and $0\leq v_1\leq v_2\leq\ldots\leq v_n\leq n^*$.

Definition 4 With $\boldsymbol{F}(U\times V)$ representative all product set on the U and V,for given product set $Q=\{q(u,v)|u\in U,v\in V\}\in\boldsymbol{F}(U\times V)$,$X=\{x(u,v)|u\in U,v\in V\}\in\boldsymbol{F}(V\times W)$ and $S=\{s(u,v)|u\in U,v\in V\}\in\boldsymbol{F}(U\times W)$, relationship equations for opertor $(\vee,\wedge)$in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers is defined as follows

$$Q_{uv}\circ X_{vw}=S_{uw} \tag{3}$$

among them $S_{uw}=[s_{uw}]=[\vee_v(q_{u}\wedge_{u,w} r_{w}]$

Definition 5 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, operator α is defined as follows

$$m\alpha n=\begin{cases} n & m>n \\ n^* & m\le n \end{cases} \quad (4)$$

Definition 6 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, operator β is defined as follows

$$m\beta n=\begin{cases} n & m\ge n \\ 0 & m<n \end{cases} \quad (5)$$

Definition 7 For given product set $Q=\{q(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(U\times V)$,$R=\{r(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(V\times W)$, opertoration for opertor $(a, \wedge)$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers is defined as follows

$$S=Q@R\in \boldsymbol{F}(U\times W) \quad (6)$$

among them $S_{uw}=[s_{uw}]=[\underset{v}{\wedge}(q_{uv}\underset{u,w}{\alpha} r_{vw})]$

Definition 8 For given product set $Q=\{q(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(U\times V)$,$R=\{r(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(V\times W)$, opertoration for opertor $(\beta,\vee)$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers is defined as follows

$$S=Q\&R\in \boldsymbol{F}(U\times W) \quad (7)$$

among them $S_{uw}=[s_{uw}]=[\underset{v}{\vee}(q_{uv}\underset{u,w}{\beta} r_{vw})]$.

In the paper (Zhang, Dong, & Luo, 2014), the maximum solution of the relation equation in the set of finite natural numbers be given as follows.

Theorem 1. If the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers has solutions R_{vw},then

$$[Q^T@S_{uw}]=[\underset{u}{\wedge}(q_{vu}\underset{v,w}{\alpha} s_{uw})]\cap[\underset{u}{\vee}(q_{vu}\underset{v,w}{\beta} s_{uw})] \quad (8)$$

is the maximum solution of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers.

In this paper, the strunk of solutions of the relation equation in the set of finite natural numbers will be discussed.

1. THE STUNK OF SOLUTIONS OF THE RELATION EQUATIONON IN THE SET OF FINITE NATURAL NUMBERS

Theorem 2. If the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers has solutions R_{vw},then

$$[Q^T@S_{uw}]\cap[Q^T\&S_{uw}]=[\underset{u}{\wedge}(q_{vu}\underset{v,w}{\alpha} s_{uw})]\cap[\underset{u}{\vee}(q_{vu}\underset{v,w}{\beta} s_{uw})] \quad (9)$$

is another solution of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers.

Proof. If the relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers has solutions R_{vw}, let $q^T_{uv}=q_{vu}$, on the basis of the theorem 1 we know that relation

$$[Q^T@S_{uw}]=[\underset{u}{\wedge}(q_{vu}\underset{v,w}{\alpha} s_{uw})]$$

is a resolution of relation equations. For any $(u,v)\in U\times V$, then

$$Q_{uv}\circ[Q^T@S_{uw}]=S_{uw}.$$

For given $(u^*,w^*)\in U\times W$, then

$$Q_{u*v}\circ[Q^T@S_{uw*}]=S_{u*w*}$$

This means

$$[q_{u^*v}]\circ[\underset{u}{\wedge}(q_{vu}\underset{v,w^*}{\alpha} s_{uw^*})]=[s_{u^*w^*}].$$

That is

$$[\underset{v}{\vee}(q_{u^*v}\wedge(\underset{u}{\wedge}(q_{vu}\underset{v,w^*}{\alpha} s_{uw^*})))]=[s_{u^*w^*}] \quad (10).$$

We must prove

$$Q_{u*v}\circ[Q^T@S_{u*w*}]\cap[Q^T\&S_{u*w*}]=[q_{u^*v}]\circ[[\underset{u}{\wedge}(q_{vu}\underset{v,w^*}{\alpha} s_{uw^*})]\cap[\underset{u}{\vee}(q_{vu}\underset{v,w^*}{\beta} s_{uw^*})]].$$
$$=[\underset{v}{\vee}(q_{u^*v}\wedge((\underset{u}{\wedge}(q_{vu}\underset{v,w^*}{\alpha} s_{uw^*}))\wedge(\underset{u}{\vee}(q_{vu}\underset{v,w^*}{\beta} s_{uw^*}))))]$$

Or

$$Q_{u*v}\circ[Q^T@S_{u*w*}]\cap[Q^T\&S_{u*w*}]=[\underset{v}{\vee}(q_{u^*v}\wedge((\underset{u}{\wedge}(q_{vu}\underset{v,w^*}{\alpha} s_{uw^*}))\wedge(\underset{u}{\vee}(q_{vu}\underset{v,w^*}{\beta} s_{uw^*}))))]. \quad (11)$$

We subdivide proof into two steps:

(a) For any $u\in U$, if $q_{uv}\le s_{uw*}$ then according to definition 5 we have

$$(\underset{u}{\wedge}(q_{vu}\underset{v,w^*}{\alpha} s_{uw^*}))=n^* \quad (12).$$

In this case, Formula (10) can be simplified as follows:

$$[\underset{v}{\vee}(q_{u^*v})]=[s_{u^*w^*}] \quad (13).$$

According to Formula (13), we know $\$v_0\in V$ satisfies Formula (14) and Formula (15) as follows.

$$q_{u^*v_0}=s_{u^*w^*} \quad (14)$$

$$\max_v\{q_{u^*v}\}=s_{u^*w^*} \quad (15)$$

According to Formula (12), Formula(11) can be simplified as follows.

$$Q_{u*v}\circ[Q^T@S_{u*w*}]\cap[Q^T\&S_{u*w*}]=[\underset{v}{\vee}(q_{u^*v}\wedge(\underset{u}{\vee}(q_{vu}\underset{v,w^*}{\beta} s_{uw^*})))] \quad (16)$$

On the one hand, according to Definition 4 and Formula (14) we have

$$[\underset{v}{\vee}(q_{u^*v}\wedge(\underset{u}{\vee}(q_{vu}\underset{v,w^*}{\beta} s_{uw^*})))]\le[\underset{v}{\vee}(s_{u^*w^*}\wedge(\underset{u}{\vee}(q_{vu}\underset{v,w^*}{\beta} s_{u^*w^*})))]\le[\underset{v}{\vee}(s_{u^*w^*})]=[s_{u^*w^*}]$$

On the another hand, according to Definition 4 and Formula (15) we have

$$[\underset{v}{\vee}(q_{u^*v}\wedge(\underset{u}{\vee}(q_{vu}\ \underset{v,w^*}{\beta}\ s_{uw^*})))] \ge [q_{u^*v_0}\wedge(\underset{u}{\vee}(q_{v_0u}\ \underset{v_0,w^*}{\beta}\ s_{uw^*}))]$$

$$\ge [q_{u^*v_0}\wedge(q_{v_0u^*}\ \underset{v_0,w^*}{\beta}\ s_{u^*w^*})] = [q_{u^*v_0}\wedge s_{u^*w^*}] = [s_{u^*w^*}]$$

and thus

$$[\underset{v}{\vee}(q_{u^*v}\wedge(\underset{u}{\vee}(q_{vu}\ \underset{v,w^*}{\beta}\ s_{uw^*})))] = [s_{u^*w^*}].$$

Because $[s_{u*w*}] = S_{u*w*}$, for every given $(u^*, w^*) \in U\times W$ we have

$Q_{u*v}\circ[Q^T @ S_{u*w*}]\cap[Q^T \& S_{u*w*}] = S_{u*w*}$.

(b) For $1\le i\le n$, if has $u_i \in$ U, satisfies

$$q_{vu_i} > s_{u_iw^*} \qquad (17)$$

By definition 3 we have

$$(\underset{u}{\wedge}(q_{vu}\ \underset{v,w^*}{\alpha}\ s_{uw^*})) = \min_{1\le i\le n}\{s_{u_iw^*}\}.$$

In this case, Formula (10) can be simplified as follows:

$$[\underset{v}{\vee}(q_{u^*v}\wedge \min_{1\le i\le n}\{s_{u_iw^*}\}] = [s_{u^*w^*}] \qquad (18)$$

By definitions of operator Ù and operator Ú, and by formula (18) we have

$$\min_{1\le i\le n}\{s_{u_iw^*}\} = s_{u^*w^*} \qquad (19)$$

In the similar way we know that for $1 \le j \le m$, if has $v_j \in$ V, satisfies

$$q_{u^*v_j} \ge s_{u^*w^*} \qquad (20)$$

By definition 3, Formula (17), Formula (19) and Formula (20), we have

$$(\underset{u}{\wedge}(q_{vu}\ \underset{v,w^*}{\alpha}\ s_{uw^*})) = s_{u^*w^*} \qquad (21)$$

By definition 4, Formula (17), Formula (19) and Formula (20), we have

$$(\underset{u}{\vee}(q_{vu}\ \underset{v,w^*}{\beta}\ s_{uw^*})) = \max_{1\le i\le n}\{q_{vu_i}\ \underset{v,w^*}{\beta}\ s_{u_iw^*}\} \ge \min_{1\le i\le n}\{s_{u_iw^*}\} = s_{u*w*} \qquad (22)$$

By Formula (19), (20) , (21) and Formula (22), we can simplify Formula (11) as follows:

$$Q_{u*v}\circ[Q^T @ S_{u*w*}]\cap[Q^T \& S_{u*w*}] = [q_{u^*v}]\circ[[\underset{u}{\wedge}(q_{vu}\ \underset{v,w^*}{\alpha}\ s_{uw^*})]\cap[\underset{u}{\vee}(q_{vu}\ \underset{v,w^*}{\beta}\ s_{uw^*})]]$$

$$= [\underset{v}{\vee}(q_{u^*v}\wedge((\underset{u}{\wedge}(q_{vu}\ \underset{v,w^*}{\alpha}\ s_{uw^*}))\wedge(\underset{u}{\vee}(q_{vu}\ \underset{v,w^*}{\beta}\ s_{uw^*}))))]$$

$$= [\underset{v}{\vee}(q_{u^*v}\wedge s_{u^*w^*})] = [s_{u^*w^*}]$$

Hence for every given $(u^*, w^*) \in U\times W$, we have

$Q_{u*v}\circ[Q^T @ S_{u*w*}]\cap[Q^T \& S_{u*w*}] = [s_{u^*w^*}]$.

This conclusion means that $[Q^T @ S_{uw}]\cap[Q^T \& S_{uw}]$ is a resolution of the relation equations $Q_{uv}\circ X_{vw}=S_{uw}$.

Inference. If the relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ have resolutions and for any relation R_{vw} satisfies

$[Q^T @ S_{uw}]\cap[Q^T \& S_{uw}]\subseteq R_{vw}\subseteq[Q^T @ S_{uw}]$,

then the relation R_{vw} is a resolution of the relation equations $Q_{uv}\circ X_{vw}=S_{uw}$.

Proof. Because the relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ have resolutions, on the basis of the theorem 1 and the theorem 2 we know that the relation $Q^T @ S_{uw}$ and the relation $[Q^T @ S_{uw}]\cap[Q^T \& S_{uw}]$ are resolutions of the relation Equations $Q_{uv}\circ X_{vw}=S_{uw}$. At the same time we know that the relation $Q^T @ S_{uw}$ is the maximum resolution of the relation equations $Q_{uv}\circ X_{vw}=S_{uw}$.

Let the relation R_{vw} satisfies

$[Q^T @ S_{uw}]\cap[Q^T \& S_{uw}]\subseteq R_{vw}\subseteq[Q^T @ S_{uw}]$.

According to the definition of the composition operator we have

$Q_{uv}\circ [Q^T @ S_{uw}]\cap[Q^T \& S_{uw}]\subseteq Q_{uv}\circ R_{vw}\subseteq Q_{uv}\circ [Q^T @ S_{uw}]$.

So we see

$S_{uw}\subseteq Q_{uv}\circ R_{vw}\subseteq S_{uw}$.

That is

$Q_{uv}\circ R_{vw} = S_{uw}$.

Therefore the set of relations R_{vw} is strunk resolutions of the relation equations $Q_{uv}\circ X_{vw}=S_{uw}$.

CONCLUSION

Because that information processing dependent on the set of finite natural numbers shows importance of relation equations in the set of finite natural numbers. This paper provides a method of solving the strunk of solutions of relation equations in the set of finite natural numbers. In the future, we will discuss other properties of the relation equations in the set of finite natural numbers.

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