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Fold Catastrophe Model of Fracture Propagation of Hydraulic Fracturing

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Abstract
According to energy conservation from the destruction of rock catastrophe, a new calculation method of the length of fracture propagation in hydraulic fracturing is proposed, and assuming the crack extends to approximate ellipse, the width calculation model of fracture extension is established. By using this new method in Changsheng well of Jilin oilfield, And the maximum error between the productivity that calculated by using the new method and the actual oil production is anastomosing. the method is an innovation in parameter calculation of crack propagation with the destruction of rock catastrophe, and it supposes some new theoretical foundation for the design of fracture parameter optimization.

Key words: Hydraulic fracture; Fold catastrophe; Fracture parameters

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INTRODUCTION

After several years of development, many scholars put forward many kinds of theoretical calculation and on-well site empirical formula of fracture parameters, such as Howard-Fast parallel-plate model[1], PKN and KGD model[2]. Otherwise, the width calculation model of fracture fractal extension is established considered fractal characteristics of fracture by some scholars. But the results of calculation are not satisfied in practice. The calculation of fracture parameters based on continuous and progressive destruction rather than catastrophe mechanics of start propagation of fracture is the main reason that causes the deviation. There will inevitably be big deviation between the calculated fracture parameters and the actual fracture parameters. When hydraulic fracturing is carrying out, and fracturing fluids are acting on the underground rock, fractures are induced through continually exerting pump pressure, so the state of rock is instable and develops to another equilibrium state. Following this sudden change, the rock macroscopically shows energy release, which causes instability and damage of rock, and finally the new fracture is formed. It is appropriate and accurate to research jump instability of rock fracturing process by using catastrophe theory.

Cusp catastrophe model and fold catastrophe model are proposed by some scholars in the development of catastrophe theory. Cusp catastrophe model is used to research the change of equilibrium state which has reversible system change, fold catastrophe model describes a system which has a irreversible destruction process. The author proposes fold catastrophe model of the fracture propagation in hydraulic fracturing based on energy conservation of fracturing, and establishes the width and length calculation method of fracture propagation in the fold catastrophe model. This method is an innovation in parameter calculation of fracture propagation with the destruction of rock catastrophe. The well in a block of Jilin Oilfield are selected to calculate with the new model, the new model results are in agreement with the practical results.

1. ROCK MASS DEFORMATION PROCESS OF HYDRAULIC FRACTURING

In Figure 1, the stress-strain of hydraulic fracturing in rock mass can be generalized about: OA-the gradual compaction in the fracture of rock; AB-the state of elastic deformation of rock; BC-the plastic deformation of rock mass, this is the first state of destruction; CD-the state of rock fracturation and crack propagation.

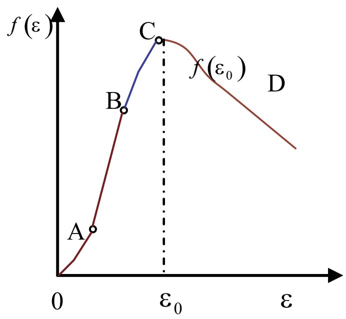

Figure 1
The Stress-Strain Curve of Rock Mass

2. THE FOLD CATASTROPHE MODEL OF HYDRAULIC FRACTURING

According to the law of energy conservation, the fracture energy balance equation of per unit volume of rock in fracturing process is:

$$\int_0^{\varepsilon} f_1^{-1}(\varepsilon)d\varepsilon-\int_{\varepsilon_e}^{\varepsilon} f_2(\varepsilon)d\varepsilon=0 \tag{1}$$

In the equation: $f_1^{-1}(\varepsilon)$-the external force about per unit volume of the rock mass under the fracturing fluids pressure and the stress of rock mass; $f_2(\varepsilon)$-the stress of rock mass occurred plastic deformation; ε_e-the elastic strain of rock mass.

According to the theory of fracture mechanics, $f_1^{-1}(\varepsilon)$ is expressible in the form:

$$f_1^{-1}(\varepsilon)=\delta\cdot p_f(\varepsilon)-\sigma_h \tag{2}$$

In the equation: δ-Fracturing fluid compressibility factor, δ = 0.8; $p_f(\varepsilon)$-Bottom hole pressure of fracture fluids; σ_k-Minimum principal stress of the wellhole rock.

According to the yield criteria, the plastic yield function represents:

$$F(f_2(\varepsilon^p),\varepsilon^p)=\varepsilon^p-3\sqrt{3}\alpha f_2(\varepsilon^p)-\sqrt{3}k=0 \tag{3}$$

If F<0, the rock mass is in elastic state, when the yield surface is achieved, the rock represent plastic.

Assuming the corresponding strain of the inflection point in the stress-strain curve is ε_0, Taylor expansion is made at ε_0, according to the certainty principle[12], the system stability are discussed with rounding higher order terms than ε_2, and the mutation model of equilibrium equations under sudden rupture of rock fracturing are get as following equations:

$$\left[\varepsilon-\varepsilon_0-\frac{f_1^{-1}(\varepsilon_0)-f_2(\varepsilon_0)}{\left[f_1^{-1}(\varepsilon_0)\right]'-f_2'(\varepsilon_0)}\right]^2+\int_0^{\varepsilon_0} f_1^{-1}(\varepsilon)d\varepsilon-\int_{\varepsilon_e}^{\varepsilon_0} f_2(\varepsilon)d\varepsilon-\left[\frac{f^{-1}(\varepsilon_0)-f_2(\varepsilon_0)}{\left[f_1^{-1}(\varepsilon_0)\right]'-f_2'(\varepsilon_0)}\right]^2=0 \tag{4}$$

Equation (3) turns into standard form of fold catastrophe:

$$\Pi'(x)=x^2+a=0 \tag{5}$$

In the equation:

$$x=\left[\varepsilon-\varepsilon_0-\frac{f_1^{-1}(\varepsilon_0)-f_2(\varepsilon_0)}{\left[f_1^{-1}(\varepsilon_0)\right]'-f_2'(\varepsilon_0)}\right];$$

$$a=\int_0^{\varepsilon_0} f_1^{-1}(\varepsilon)d\varepsilon-\int_0^{\varepsilon_0} f_2(\varepsilon)d\varepsilon-\left[\frac{f^{-1}(\varepsilon_0)-f_2(\varepsilon_0)}{\left[f_1^{-1}(\varepsilon_0)\right]'-f_2'(\varepsilon_0)}\right]^2$$

Discussion:

If a > 0, $x=\pm\sqrt{a}i$, state variable is virtual function that can characterize the virtual state of fracturing engineering.

If $a \le 0$, $x=\pm\sqrt{-a}$, the differential equation of potential energy is a parabola.

According to Equation (5) we can know, the hydraulic fracturing system exists two states which are the rock state before instability and destruction and the state after instability and destruction. x_1, x_2 can be got from solving Equation (5).

The total potential energy function with the fold catastrophe model of the rock destruction in fracturing can be got from integrated with Equation (4):

$$\Pi(x)=\frac{1}{3}x^3+ax=0 \tag{6}$$

In the equation, x is the state variable which describes the fracture of rock; a is the control variable which describes the fracture of rock.

The following equation can be got from the second derivative on Equation (5):

$$\frac{\partial^2\Pi}{\partial x^2}=2x \tag{7}$$

According to the rule of Dirichlet, if x < 0, $\frac{\partial^2\Pi}{\partial x^2}<0$, the system is in an instability state, if x > 0, $\frac{\partial^2\Pi}{\partial x^2}>0$, the system will get a new stability state.

The total potential energy of the two points is got from the potential energy of fracture before initiate fracturing:

$$\Pi_1=\frac{1}{3}x_1^3+ax_1=0 \tag{8}$$

$$\Pi_2 = \frac{1}{3}x_2^3 + ax_2 = 0 \tag{9}$$

According to the former equations, the energy release of the equilibrium position skipped from point 1 to point 2 can be got:

$$\Delta\Pi = \Pi_1 - \Pi_2 \tag{10}$$

3. THE RESEARCH OF FRACTURE PROPAGATION BASED ON THE CATASTROPHE MODEL

3.1 The Length Calculation Model of Fracture Propagation

Assume that the fracture is tensile fracture, the increasing strain energy of the fracture is to be given:

$$\Delta\Pi = 2\int_0^{L_f} G dA = 2\frac{\left(1-\nu^2\right)}{E}\int_0^{L_f} K_1^2 da \tag{11}$$

The connection between static stress intensity factor and net pressure of the fracture around rock mass is:

$$K_1 = f_2\left(\varepsilon\right)\sqrt{\pi L_f} \tag{12}$$

The propagation length equation of fracture with catastrophe under hydraulic fracturing can be given from integrated with Equation (13):

$$L_f = \sqrt{\frac{\Delta\Pi \cdot E}{1-\nu^2}\pi f_2\left(\varepsilon\right)} \tag{13}$$

The Width Calculation Model of Fracture

Assume that any position along the length of fracture with propagation have the same permanent width. The shape of the vertical section of fracture is ellipse, the width model of fracture at any time of propagation can be given:

$$w(x,t,z) = w_{\max}(t)\sqrt{(1-\frac{x^2}{L_f^2})(1-\frac{4z^2}{h_f{}^2})} \tag{14}$$

According to the mass conservation law, the relationship with the maximum fracture width and the initial maximum fracture width can be given:

$$\frac{\pi^2}{4}\rho h_f L_f\left[w_{\max}(0) - w_{\max}(t)\right] = \rho\int_0^t\int_{-L_f}^{L_f}\frac{4ch_f}{\sqrt{t}}\mathrm{d}x\mathrm{d}t + \int_0^t 1.414r^2\pi\sqrt{\rho}\frac{\sqrt{[p(t)-p_o]}}{\sqrt{(1+\xi-\frac{r^4}{R^4})}}\mathrm{d}t \tag{15}$$

4. CASE STUDY

4.1 The Engineering Situation

The experimental condition and results are showed in Table 1.

Table 1
The Foundation Parameter of Core Triaxial Test

Well number	Depth of coring removal m	Test conditions: Peripheral pressure MPa	Test conditions: Hole pressure MPa	Test results: Young's modulus MPa	Test results: Poisson's ratio	Test results: Coefficient of volume compressibility $\times 10^{-4}$/MPa	Test results: Grain comprssibility $\times 10^{-5}$/MPa	Test results: Porous elastic coefficient
Changshen 4	3,504.45 -3,504.56	62.4	41.7	29,600	0.32	1.19	2.65	0.78

4.2 The Calculation Results of Fracture Parameter

The fracture parameters of fracturing construction program are simulated by using the triaxial fracturing design software-FracproPT. To compare the simulation results with the calculation results of fracture parameter based on the catastrophe model supposed from this paper, the comparative results are showed in Table 2.

Table 2
The Comparative Results of Fracture Parameter Calculation

	Fracture: Semi-major axis (m)	Fracture: Average width of fracture(mm)
Actual simulation	148.85	7.34
Theoretical calculation	144.37	6.71
Relative error(%)	3.01	8.58

4.3 The comparative results of productivity

The following conclusions can be made from the comparative results of productivity: The calculation results of the new model are close to the PKN model at initial period of production after fracturing, as production time went on, the error between the result calculated from regular model and the actual test result is big, but the simulation results are closely with them, that could be the advantage of the new model.

CONCLUSIONS

(a) Energy dissipation of catastrophe destruction of rock mass cause the formation of hydraulic fracturing fracture, the destruction process of rock mass is an irreversible process of energy dissipation. The fracture propagation of rock mass is the characteristic of energy liberation of destruction with catastrophe of rock mass. This irreversible process on failure catastrophe can be accurately explained by using the fold catastrophe theory.

(b) Fracture propagation is induced by failure catastrophe on rock mass of hydraulic fracturing, the rock mass is skipping from one critical stable state to another stable state, the additional strain energy can be made from the difference between the total potential energy.

(c) The fold catastrophe model on failure of rock mass in hydraulic fracturing is established by using the catastrophe theory, the failure from of rock mass is characterized by the characteristic change of state variable on this model.

(d) Based on actual engineering of fracture propagation on hydraulic fracturing, a new calculation method about length of fracture propagation and width of fracture is proposed. The results error between the new calculation method and the software simulation are 3.01% and 8.58%.

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