CSCanada
**Advances in Natural Science**
Vol. 6, No. 2, 2013, pp. 13-17
**DOI:**10.3968/j.ans.1715787020120602.2121

ISSN 1715-7862 [PRINT]
ISSN 1715-7870 [ONLINE]
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# Thermodynamics Since Einstein

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Received 19 February 2013; accepted 02 June 2013

## Abstract

Relativistic thermodynamics is a relatively unknown theory. Thermodynamic laws apply only to quasi-static processes that quickly change between states that are in a long-term equilibrium. However, special relativity postulates that the propagation speed of physical signals is constrained, thus limiting the speed of change in thermal states. Einstein was especially interested in the concept of temperature and the transformation formula of thermodynamic quantities in a moving frame of reference, having inspired numerous investigations for two centuries. This article reviews the historical development of relativistic thermodynamics since Einstein, beginning from the initial idea of Planck-Einstein in which a moving body warms up, to the notion of Blanusa-Ott in which a moving body cools down, and to that of Landsberg in which the temperature remains unchanged—depending on how the observer's thermometer is defined. Current research focuses on identifying the correct form of relativistic Maxwell distribution to validate the related theory. Recent computational results using molecular dynamic simulations and their relevance to astrophysics are outlined as well.

**Key words:** Relativistic thermodynamics; Special relativity; Molecular dynamics

Chih-Yueh WANG (2013). Thermodynamics Since Einstein. *Advances in Natural Science, 6*(2), 13-17. Available from: http://www.cscanada.net/index.php/ans/article/view/ j.ans.1715787020130602.2121
DOI: http://dx.doi.org/10.3968/ j.ans.1715787020130602.2121

## 1. CONTRADICTION BETWEEN THERMODYNAMICS AND RELATIVITY

Einstein's famous paper on the electrodynamics of moving bodies, "Zur Elektrodynamik bewegter Körper" (1905) opened the theory of special relativity, which revealed that the Lorentz transformation inherent in the Maxwell equations can be derived from two simple postulates on motion: the relativity principle known since Galileo, and the invariance of the speed of light. However, the limitation that the speed of light is the ultimate velocity contradicts the notions in classical thermodynamics, since the laws of thermodynamics, described by exact differentials, apply to ideal states that are in a long-term equilibrium, or to quasi-static processes that can respond quickly to establish thermal equilibrium. One example is the Fourier Heat diffusion equation,

$\frac{\partial T}{\partial t}-\frac{k}{c_P\rho}\frac{\partial^2 T}{\partial x^2}=0$. According to the equation, the distance an energy packet propagates increases with $x\sim\sqrt{t}$, leading to a transport speed $v\sim 1/\sqrt{t}$, which can exceed the speed of light at a short time scale (Biro, 2011). One may propose that the diffusion is only valid for a long time scale, or that the applicable time scale depends on the properties of material, as in the case of diffusion constant $k$. However, such claims are not satisfying. Having a relativistic equation that approaches the classical diffusion limit is desirable.

The Lorentz transformation formulae of thermodynamic quantities, best known by temperature, were particularly interesting to Einstein and contemporary physicists. Various solutions have been suggested since Einstein's (1907) and Planck's (1907) initial proposal that the a moving body should appear cooler by a Lorentz factor, $\gamma\equiv 1/\sqrt{1-w^2/c^2}>1$, where $w$ is the speed of the observer relative to the object. In 1952, in private correspondence with Max von Laue (Liu, 1992), Einstein changed his view and argued that the body would become hotter by a Lorentz factor. In 1963, Heinrich Ott published the same claim (1963). In 1966, Peter Landsberg (1966) suggested that the temperature of a moving body should remain unchanged.

Following Landsberg's work, some pointed out that the apparent temperature must depend on how the temperature is measured or defined. Focuses of the question then become how thermal equilibrium is established between body parts; e.g., between parts of a gas cloud consisting of fast moving particles.

## 2. EINSTEIN'S 1907 ARGUMENT

In his most productive period, 1901-1909, Einstein published 28 papers, of which 16 were related to statistical thermodynamics. In his first derivation of the conversion of entropy and temperature in 1907, Einstein noted that the pressure and volume in the rest frame comoving with the object (denoted by subscript o) and those in the stationary frame satisfy

$$P = P_o,\ V = V_o / \gamma.$$

He quoted Planck's manuscript (1907) that was reviewed by him and marked that the entropy conversion satisfied

$$S = S_o,$$

and the transfer of heat satisfied

$$dQ = dQ_o/\gamma.$$

Because the scalar product of the four vector speed $\gamma(w,c)$ and energy momentum $(P,U/c)$ is unchanged after coordinate transformation, $\gamma(U-wP)=U_o$. Subsequently,

$$dQ = (dU_o - P_o dV_o)/\gamma = dU - w dP - PdV .$$

In the last step of deriving, Einstein used the relation between temperature and entropy for a reversible cyclic process as given in thermodynamics textbooks:

$$Tds = dQ = dQ_o/\gamma = (T_o/\gamma)dS_o .$$

Therefore, $T = T_o/\gamma$; i.e., the object is cooler when it is moving.

Einstein's theses also discussed the Lorentz transformation of black body radiation frequency. Von Mosengeil (1907) and Planck (1908) found that, given Kirchhoff's theorem and Lorentz invariance in Wien's law, the black radiation temperature of a moving body (in which case no heat transfer occurs) should also have the identical form of conversion. Later, Planck reached the same conclusion using Helmholtz free energy as a Lagrangian to deduce the temperature and entropy (1910). Planck-Einstein's formulation of relativistic thermodynamics considered the scenario that the black body is adiabatically accelerated to a speed (Treder, 1977). In the case of a non-adiabatic moving blackbody, others found that the same transformation law applied as well (Liu, 1992).

## 3. THE EINSTEIN-LAUE DEBATE

Half a century later, in 1952, during discussions with Max von Lau concerning Laue's revision of Laue's textbook on relativity, Einstein overruled his initial claim. Laue was awarded the Nobel Prize in physics in 1914 for the discovery of the X-ray diffraction of crystals. His communication with Einstein began in 1906, when he was Planck's assistant and Einstein was an employer of the patent office in Bern. As one of the first physicist to visit Einstein and respond to the theory of special relativity, their dialogues lasted until Einstein's decease. In the argument, Einstein employed a Carnot engine with two heat reservoirs, denoted by $U_o$ and $U$ and originally 'at rest' with the same (rest) temperature $T_o$, to illustrate how heat is transferred between them, if $U$ is accelerated to a velocity adiabatically along with an auxiliary machine. He concluded that because the auxiliary machine performed mechanical work and released heat to $U$, the temperature ($T$) of $U$ must be higher than its rest temperature. The discussion lasted about a year, unpublished and only disclosed in letters (Liu, 1992).

**Letter from Einstein to Laue** (translated from the original German script (Liu, 1992))

Dear Laue!

I cannot agree with your formula for the transformation of the absorbed heat $G$ (and temperature). Suppose there are two heat reservoirs $U_o$ and $U$, both of which are originally 'at rest' and have the same (rest) temperature $T_o$. $U$ is then brought to a velocity $v$ [we have used $w$ for the velocity] through an adiabatic process with its rest-frame temperature preserved. When observed from the rest system the temperature is $T$. The temperature must be well defined. I will proceed as you did: If an amount of heat input is transferred from $U_o$ to $U$ through a reversible cycle, via a machine that acquires only work without heat input from outside, then it should be $T/T_o = G/G_o$. [Work is done on the machine so that heat transfers from the cooler to the hotter reservoir. $G$ $(G_o)$ denotes the amount of heat transferred between reservoir $U$ $(U_o)$ and the machine.] This is equivalent to your treatment, that the reservoirs, when taken together, should experience no entropy increase. Let the 'Machine' be an auxiliary reservoir with an eigen temperature $T_o$ throughout. In the cyclic process: (a) the amount of heat $G_o$ is transferred from $U_o$ to the auxiliary reservoir; (b) the auxiliary reservoir is moving at the velocity $v$; (c) the heat $G$ is released to $U$ [from the auxiliary reservoir while mechanical work is done to the auxiliary reservoir], and the machine's original rest-heat content is restored; and (d) the auxiliary reservoir is back to rest. In the cycle the total energy of the two reservoirs has increased by the amount of $G - G_o$. The total amount of mechanical work done is

$$A = G_0\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

According to the first law it must be true that

$$G - G_0 = A = G_0\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right),$$

or

$$\frac{G}{G_0} = \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right). \qquad (2)$$

Take equation (1) [$T/T$o = $G/G$o] and (2) together,

$$\frac{G}{G_0} = \frac{T}{T_0} = \left(\frac{1}{\sqrt{1-v^2/c^2}}\right) \text{ but not } \sqrt{1-v^2/c^2}.$$

I have not studied your book closely enough to see where the difference comes from. This analysis is so simple that I can hardly imagine that it contains errors.

Best Wishes. Yours, A. E.

**End of Letter**

In the Carnot cycle, the work done by the engine (machine) is related to the change in volume. Einstein adopted an auxiliary thermal reservoir rather than an ordinary Carnot engine such that the machine absorbs heat from $U$o without volume change. Because the heat $G$ - $G$o transferred in the cycle corresponds to a mass of

$$\frac{G_0}{c^2}\left(\sqrt{1-v^2/c^2} - 1\right),$$

it must be equal to $A$ (the total amount of mechanical work). This equivalently assumes that $G$ equals

$$G_0 / \sqrt{1-v^2/c^2}.$$

Thus, G/Go=$T/T$o=$\gamma$; a moving object warms up. Einstein did not specifically derive the amount of work done to/by the auxiliary machine. He might have had a detailed proof unknown to the world, or he might have simply been wrong in assuming the amount of work, which Laue tried to convince Einstein was a miscalculation.

## 4. BLANUSA-OTT AND LANDSBERG'S THOUGHTS

After half a century's silence, in 1963, Henrich Ott (Sommerfeld's student) proposed that a moving object would become warmer (1963) and this result reopened the discussions of relativistic thermodynamics. Ott thought that, since entropy corresponds to the number of thermodynamic states, it must be Lorentz invariant. Therefore,

$T = \gamma T$o , $S = S$o , $P = P$o , $U = \gamma U$o , $Q = \gamma Q$o .

Ott's results were found to reflect the argument that Einstein made in 1953. Furthermore, as early as 1947, Croatian mathematician Danilo Blanusa (1947) had presented the same idea in a local journal. From 1966 to 1968, Peter Landsberg published a series of papers in Nature (1966, 1967, 1968) and raised a new related issue: he suggested that when the heat source moves transversely relative to the observer, the temperature change would not involve Doppler's effect, which can be determined only by considering how high-speed mobile objects reach thermal equilibrium. Should the relative velocities of the particles in different coordinate systems be offset relative to each other, the temperature would not change. Thus, applying the correct logical deduction, temperature must be Lorentz invariant.

Note that modern experiments could not test the three schools of thoughts of Planck-Einstein, Blanusa-Ott, and Landsberg to decide which is true for a fast-moving body. The situation appears similar to the twin paradox in relativity. A paper of Balescu (1968) summarized the highlights of the controversy, and listed and compared all pertinent transformation formulae.

Notably, in his final correspondence with Laue in 1952-1953, Einstein once again changed his perspective because he saw no reason to regard one formulation rather than any other to be true; "I am tempted to understand the notion of temperature with reference to a co-moving thermometer"; therefore, "the temperature should be treated in every case as an invariant" (Liu, 1992).

In 1992, Van Kampen used four dimensional covariant theories to describe the laws of thermodynamics, in which all thermodynamic variables were Lorentz invariant (Van Kampen, 1992). If temperature is not invariant, then the heat flow direction (from high temperature to low temperature) may change depending on the observer's frame of reference. To the contrary, in 1964, J. L. Anderson proposed that thermodynamics should not focus on the Lorentz transformations, because a Lorentz transformation is not needed, as in a stationary frame of coordinates, to meet the principal of special relativity (Anderson, 1964). Anderson believed that the covariant theory limited the possible forms of physics laws, and not all theories of relativistic physics required Lorentz transformation. In three papers in 1981, Landsberg summarized and reviewed the history of the discussion of relativistic transformation and its impact on the development of statistical thermodynamics (Landsberg, 1981).

## 5. JÜTTNER AND MODIFIED JÜTTNER FUNCTION

Given that pressure, temperature, and all the other thermodynamic state variables are related to the speed of the gas molecules, recent studies in thermodynamics accent the search of the correct formulation of relativistic Maxwell distribution. At low temperature (low speed), a dilute gas in equilibrium has a velocity distribution that follows the Maxwellian probability density function (PDF):

$$f_M(v;m,\beta)=[\beta m/(2\pi)]^{d/2}\exp[-\beta_J m\gamma(v)]/Z_J,$$

where $m$ is the rest mass of the particle, $T=1/k\beta$, $k$ is the Boltzmann constant, $d$ is the dimension of space, and Z is the appropriate normalization constant. After the relativity was published, Planck and others immediately noted the conflict between the postulate on the speed of light and the Maxwellian PDF, which has nonzero populations at extreme velocities. A first attempt to solve this problem was made by Jüttner in 1911, who applied the maximum entropy principle to obtain the following relativistic generalization of Maxwell's PDF (Jüttner, 1911):

$$f_J(v;m,\beta)=m^d\gamma(v)^{2+d}\exp[-\beta_J m\gamma(v)]/Z_J \quad .$$

The Jüttner's function has an M-shape, bipolar distribution, such that the number of particles drops to zero at extreme velocities. Despite having no rigorous microscopic derivation, due to the difficulty of formulating a relativistic consistent Hamilton mechanics of interacting particles, Jüttner's function was widely accepted until the last quarter of the 20th century.

Doubts about Jüttner's function starts to rise in the 1980s, when Horwitz et al. (1981; 1989) introduced a ''manifestly covariant'' relativistic Boltzmann equation that shows a different mean energy-temperature relation in the ultra-relativistic regime of infinite temperature. Since then, studies as to which distribution is the correct generalization of the Maxwellian PDF have been contradictory and further deepen the confusion. Amendments on Jüttner's functions, or modified Jüttner function, based on the principle of maximum relative entropy and Lorentz symmetry at high speeds, was proposed (Dunkel, Talkner, & Hänggi, 2007):

$$f_{MJ}(v;m,\beta_{MJ})=\frac{m^d}{Z_{MJ}}\frac{\gamma(v)^{2+d}}{m\gamma(v)}\exp[-\beta_{MJ}m\gamma(v)].$$

Compared with Jüttner's function at the same parameters, the modified PDF exhibits a significantly lower particle population in the high energy tail because of the additional factor of $1/E$.

## 6. MOLECULAR DYNAMIC SIMULATIONS

With the advances in computer technology, studies of relativistic thermodynamics are starting to engage in molecular dynamics simulations using a microscopic approach. The simulations assume that the collisions between each pair of particles are elastic, following the gas kinetic theory. Assuming that the particles have a random initial distribution in velocity and space, the computation determines when next collisions occur, obtains the momentum and energy after the collisions, and then progresses to the next time step repeatedly. Two-species one-dimensional models and one-species two-dimensional models have been presented (Cubero, Casado-Pascual, Dunkel, Talkner, & Hänggi, 2007; Ghodrat & Montakhab, 2011; Ghodrat & Montakhab, 2010). In two-dimensional simulations, the collision probability between point-like particles is almost zero, unable to reach equilibrium, so the particles are assumed to be disk-like. When two hard disks collide, the forces delineated by the force potentials act along the line connecting their center of mass, leaving the vertical component of momentum unchanged, and so the change of momentum can be decomposed as in a one-dimensional collision.

Identifying the relativistic velocity distribution in equilibrium is essential to understanding the phenomena that involves hyper-energetic particles. Interpretations of the heavy ion collision described by the relativistic Langevin equation, the cosmic microwave background radiation caused by hot electron thermalization of inverse Compton scattering (Sunyaev-Zeldovich effect) (Itoh, Kohyama, & Nozawa, 1998), and many other astrophysical phenomena all rely on the relativistic Maxwell distribution function. However, distinguishing the correct form of the function may be challenging through astronomical observations (Prokhorov et al., 2011).

The present concept of temperature follows Landsberg's view that the notion of temperature should depend on how the observer defines the thermometer. In the case that the temperature is taken as $T = 1/k\beta J$, the moving observer can use an inner thermometer to simultaneously measure the velocities of various particles and decide the temperature inherent in the system. Since such a thermometer presumes Lorentz invariant equipartition theorem, a moving object appears neither cold nor hot. One problem of current molecular dynamics simulations is that the computed results are consistent neither with Jüttner function nor modified Jüttner function. These computations impose certain physical and numerical assumptions, such as specific forms of force potential and simplified numerical setup, to achieve quickly converged solutions. In addition, the total number of particles under consideration is merely ~100. How the initial random condition relaxes to a state of equilibrium is not answered. Also, the major relativistic effect such as the retardation effect is not implemented due to numerical complexity. Discussions on relativistic thermodynamics still require further detailed examinations.

## ACKNOWLEDGEMENT

I thank Yu-Hsien Chang for searching historical documents on the Einstein-Laue debate.

 

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