The Dynamic and Collision Features of Microscopic Particles Described by the Nonlinear Schrödinger Equation in the Nonlinear Quantum Systems

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Abstract

The dynamic and collision features of microscopic particles described by nonlinear Schrödinger equation are investigated deeply using the analytic and the Runge-Kutta method of numerical simulation. The results show that the microscopic particles have a wave-corpuscle duality and are stable in propagation. When the two microscopic particles are collided, they can go through each other and retain their form after their collision of head-on from opposite directions, This feature is the same with that of the classical particles. However, a wave peak of large amplitude, which is a result of complicated superposition of two solitary waves, occurs in the colliding process. This displays the wave feature of microscopic particles. Therefore, the collision process shows clearly that the solutions of the nonlinear Schrödinger equation have a both corpuscle and wave feature, then the microscopic particles represented by the solutions have a wave-corpuscle duality. Obviously, this is due to the nonlinear interaction $b|\phi|^2\phi$. Thus we can determine the nonlinear Schrödinger equation can describe correctly the natures and properties of microscopic particles in quantum systems.

Key words: Microscopic particle Schrödinger equation; Wave-corpuscle duality; Nonlinear interaction; Collision; Propagation; Quantum mechanics

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INTRODUCTION

The Features of Microscopic Particle Described by the Linear Schrödinger Equation.

As it is known, the states of microscopic particles in quantum systems were as yet described by quantum mechanics, which was established by several great scientists, such as Bohr, Born, Schrödinger and Heisenberg, etc., in the early 1900s[1-10]. In quantum mechanics the dynamic equation of microscopic particles is the following Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{r},t)\psi \qquad (1)$$

Where $\hbar^2\nabla^2/2m$ is the kinetic energy operator, $V(\vec{r}, t)$ is the externally applied potential operator, m is the mass of the particles, $\psi(\vec{r},t)$ is a wave function describing the states of the particles, $\vec{r}$ is the coordinate or position of the particle. Equation (1) is a wave equation, if only the externally applied potential is known, we can find the solutions of the equation[7-9]. However, for all externally applied potentials, its solutions are always a linear or dispersive wave, for example, at $V(\vec{r}, t) = 0$, the solution is a plane wave as follows:

$$\psi(\vec{r},t) = A'\exp[i(\vec{k}\cdot\vec{r} - \omega t)] \qquad (2)$$

Where k is the wavevector of the wave, ω is its frequency, and A' is its amplitude. This solution denotes the state of a freely moving microscopic particle with an eigenenergy of

$$E = \frac{p^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2), (-\infty < p_x, p_y, p_y < \infty) \qquad (3)$$

This energy is continuous, this means that the probability of the particle to appear at any point in the space is a constant, thus the microscopic particle cannot be localized, can only propagate freely in a wave in total space. Then the particle has nothing about corpuscle feature.

If the potential field is further varied, i.e., $V(r,t) \neq 0$, the solutions of Equation (1) is still some waves with different features[5-12]. This shows clearly that the microscopic particles have only a wave feature and not corpuscle feature, which is inherent nature of particles in the quantum mechanics. These features of microscopic particles are not only incompatible with de Broglie relation of wave-corpuscle duality of $E = h\nu = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$ and Davisson and Germer's experimental result of electron diffraction on double seam in 1927[10-13] but also contradictory with regard to the traditional concept of particles[14-16]. These are just the limitations and difficulties of the quantum mechanics, which result in a duration controversy about a century in physics[8-12] and have not been solved up to now. This is very clear that the reasons having rise to these difficulties are the simplicity and approximation of quantum mechanics. As a matter of fact, the Hamiltonian operator of the system corresponding Equation (1) in quantum mechanics is represented by

$$\hat{H}(t) = \hbar^2 \nabla^2 / 2m + V(\vec{r}, t), \qquad (4)$$

Which is only composed of kinetic and potential operator of particles. The latter is not related to the wave function of state of the particle, thus it can only change the states of particles, such as amplitude, velocity and frequency, cannot influence their natures. The natures of particles can be only determined by the kinetic energy term in Equation (4), which but has a dispersive feature. Thus the microscopic particles have only a wave feature, not corpuscle feature. This is just the root and reason of the above limitations and difficulties quantum mechanics possesses.

On the other hand, the Hamiltonian of the systems and dynamic equation in Equations (1) and (4) contain only the kinetic energy and externally applied potential term. This means that we must incorporate all interactions, including nonlinear and complicated interactions, among particles or between particle and background field, such as the lattices in solids and nuclei in atoms and molecules, into the external potential by means of various approximate methods, such as the free electron and average field approximations, Born-Oppenheimer approximation, Hartree-Fock approximation, Thomas-Fermi approximation, and so on. This not only denotes the approximation of quantum mechanics but also is obviously incorrect[10-13]. The essence and substance of this method, in which these real interactions between them are replace by an average field, are that quantum mechanics freezes or blots out real motions of the microscopic particles and background fields and ignore completely real interactions between them, including nonlinear and other complicated interactions, which can influence the natures of the particles. This is just the essence and approximation of quantum mechanics. Therefore, quantum mechanics cannot be used to study the real properties of microscopic

particles in the system of many bodies and many particles, involving condensed matter, atoms and molecules[14-16].

These problems not only awake and evoke us that quantum mechanics must develop forward but also indicate the its direction of development.

In view of the above problems of quantum mechanics, we should take into account of the nonlinear interactions of the particles, which was ignored in quantum mechanics in Equations (1) and (4). As a matter of fact, the nonlinear interactions exist always in any realistic physics systems including the hydrogen atom, which is generated by the interaction between the particles and another particles or background field[17-28]. Therefore, once the real motions of the microscopic particles and background fields and their true interactions are considered, then the properties and states of microscopic particles cannot be described by Schrödinger Equation (1), but should be depicted by the following nonlinear Schrödinger equation in nonlinear quantum systems

$$i\hbar \frac{\partial \phi}{\partial t} == -\frac{\hbar^2}{2m}\nabla^2 \phi + V(\vec{r},t)\phi - b|\phi|^2 \phi, \qquad (5)$$

Where $\phi(\vec{r},t)$ is now a wave function representing the states of microscopic particles in the nonlinear systems, b is nonlinear interaction coefficient, the nonlinear interaction is now denoted by $b|\phi|^2\phi$, which is generated by interaction between the moved particle and background field. Therefore, Equation (5) represents correctly motion features of microscopic particle and its interactions with other particles or background field. In this case the nonlinear interaction and dispersive effect occur simultaneously in the dynamical equation, the deformational effect of nonlinear interaction on the wave can suppress its dispersive effect, thus dynamic Equation (5) have a soliton solution[29-30], which has both wave and corpuscle feature. Thus the microscopic particles have a wave-corpuscle duality in such a case. Therefore, Equation (5) not only describes really the motions of the particles and their interactions with other particles or background field but also represents correctly their wave-corpuscle duality. Then we have the sufficient reasons to believe the correctness of nonlinear Schrödinger equation for describing the properties of microscopic particles in quantum systems.

1. THE CHANGES OF PROPERTY OF MICROSCOPIC PARTICLES DESCRIBED BY THE NONLINEAR SCHRÖDINGER EQUATION

1.1 The Wave-Corpuscle Duality of Solutions of Nonlinear Schrödinger Equation

As it is known, the microscopic particles have only the wave feature, but not corpuscle property in the quantum

mechanics. Thus, it is very interesting what are the properties of the microscopic particles in the nonlinear quantum mechanics? We now study firstly the properties of the microscopic particles described by nonlinear Schrödinger equation in Equation (5). In the one-dimensional case, the Equation (5) at $V(x, t) = 0$ becomes as

$$i\phi_{t'} + \phi_{x'x'} + b|\phi|^2\phi = 0 \tag{6}$$

Where $x' = x/\sqrt{\hbar^2/2m}$, $t' = t/\hbar$. We now assume the solution of Equation (11) to be of the form

$$\phi(x',t') = \varphi(x',t')e^{i\theta(x',t')} \tag{7}$$

substituting Equation (7) into Equation (6) we get

$$\varphi_{x'x'} - \varphi\theta_{t'} - \varphi\theta_{x'}^2 - b\varphi^2\varphi = 0, (b > 0) \tag{8}$$

$$\varphi\theta_{x'x'} + 2\varphi_{x'}\theta_{x'} + \varphi_{t'} = 0 \tag{9}$$

If let

$$\theta = \theta(x' - v_c t'), \varphi = \varphi(x' - v_e t'), \quad \zeta = x' - v_c t',$$

$$\zeta' = x' - v_e t',$$

then Equations (8)-(9) become

$$\varphi_{x'x'} + v_c\varphi\theta_{x'} - \varphi\theta_{x'}^2 - b\varphi^3 = 0 \tag{10}$$

$$\varphi\theta_{x'x'} + 2\varphi_{x'}\theta_{x'} - v_e\varphi_{x'} = 0 \tag{11}$$

If fixing the time t' and further integrating Equation (11) with respect to x' we can get

$$\varphi^2(2\theta_{x'} - v_e) = A(t') \tag{12}$$

Now let integral constant $A(t') = 0$, then we can get $\theta_{x'} = v_e/2$. Again substituting it into Equation (10), and further integrating this equation we obtain

$$\int_{\varphi_0}^{\varphi} \frac{d\varphi}{\sqrt{Q(\varphi)}} = x' - v_e t' \tag{13}$$

Where $Q(\varphi) = -b\varphi^4/2 + (v_e^2 - 2v_e v_c)\varphi^2 + c'$. When $c' = 0$, $v_e^2 - 2v_c v_e > 0$, then $\varphi = \pm\varphi_0, \varphi_0 = [(v_e^2 - 2v_e v_c)/2b]$ is the roots of $Q(\varphi) = 0$ except for $\varphi = 0$. From Equation (13) Pang obtained the solution of Equations (8)-(9) to be

$$\varphi(x',t') = \varphi_0 \sec h[\sqrt{\frac{b}{2}}\varphi_0(x' - v_e t')] .$$

Pang[21-30] represented eventually the solution of nonlinear Schrödinger equation in Equation (6) in the coordinate of (x, t) by

$$\phi(x,t) = A_0 \sec h\left\{\frac{A_0\sqrt{bm}}{\hbar}[(x - x_0) - vt]\right\}e^{i[mv(x-x_0)-Et]/\hbar} \tag{14}$$

Where $A_0 = \sqrt{(mv^2/2 - E)/2b}$, v is the velocity of motion of the particle, $E = \hbar\omega$. This solution is completely different from Equation (3), and consists of an envelop and carrier waves, the former is $\varphi(x,t) = A_0 \sec h\left\{A_0\sqrt{bm}[(x - x_0) - vt]/\hbar\right\}$ and a bell-type non-topological soliton with an amplitude A_0, the latter is $\exp\{i[mv(x - x_0) - Et]/\hbar\}$. This solution is shown in Figure 1a. Therefore, the particles described by nonlinear

Schrödinger Equation (6) are solitons. The envelop $\varphi(x, t)$ is a slow varying function and is a mass centre of the particles; the position of the mass centre is just at x_0, A_0 is its amplitude, and its width is given by $W' = 2\pi\hbar/A_0\sqrt{2m}$. Thus, the size of the particle is $A_0 W' = 2\pi\hbar/\sqrt{2m}$ and a constant. This shows that the particle has exactly a determinant size and is localized at x_0. Its form resemble a wave packet, but differ in essence from both the wave solution in Equation (1) and the wave packet mentioned above in linear quantum mechanics due to invariance of form and size in its propagation process. According to the soliton theory[29-30], the bell-type soliton in Equation (14) can move freely over macroscopic distances in a uniform velocity v in space-time retaining its form, energy, momentum and other quasi-particle properties. However, the wave packet in linear quantum mechanics is not so and will be decaying and dispersing with increasing time. Just so, the vector $\vec{r}$ or x in the representation in Equation (14) has definitively a physical significance, and denotes exactly the positions of the particles at time t. Thus, the wave-function $\phi(\vec{r}, t)$ or $\varphi(x, t)$ can represent exactly the states of the particle at the position $\vec{r}$ or x at time t. These features are consistent with the concept of particles. Thus the microscopic particles depicted by Equation (6) display outright a corpuscle feature[31-38].

Using the inverse scattering method Zakharov and Shabat[39-40] obtained also the solution of Equation (6), which was represented as

$$\varphi(x',t') = 2\left(\frac{2}{b}\right)^{1/2}\eta \sec h\left[2\eta(x' - x_0') + 8\eta\xi t'\right]\times$$

$$\exp\left[-4i(\xi^2 - \eta^2)t' - i2\xi x' - i\theta\right] \tag{15}$$

in the coordinate of (x', t'), where η is related to the amplitude of the microscopic particle, ξ relates to the velocity of the particle, $\theta = \arg\gamma$, $\lambda = \xi + i\eta$, $x_0' = (2\eta)^{-1}\log(|\gamma|/2\eta)$, γ is a constant. We now re-write it as following form[23-27]:

$$\phi(x',t') = 2\sqrt{\frac{2}{b}}k \sec h\left\{2k\left[(x' - x_0') - v_e t'\right]\right\}e^{iv_e[(x'-x_0')-v_c t']/2} \tag{16}$$

Where v_e is the group velocity of the particle, v_c is the phase speed of the carrier wave in the coordinate of (x', t'). For a certain system, v_e and v_c are determinant and do not change with time. We can obtain $2^{3/2}k/b^{1/2} = A_0$,

$$A_0 = \sqrt{\frac{v_e^2 - 2v_c v_e}{2b}} .$$

According to the soliton theory[29-30], the soliton shown in Equation (16) has determinant mass, momentum and energy, which can be represented by[21-28]

$$N_s = \int_{-\infty}^{\infty}|\phi|^2 dx' = 2\sqrt{2}A_0,$$

$$p = -i \int_{-\infty}^{\infty} \left(\phi^* \phi_{x'} - \phi \phi_{x'}^* \right) dx' = 2\sqrt{2} A_0 v_e = N_s v_e = cons ,$$

$$E = \int_{-\infty}^{\infty} \left[|\phi_{x'}|^2 - \frac{1}{2} |\phi|^4 \right] dx' = E_0 + \frac{1}{2} M_{sol} v_e^2 \qquad (17)$$

Where $M_{sol} = N_s = 2\sqrt{2} A_0$ is just effective mass of the particles, which is a constant. Thus we can confirm that the energy, mass and momentum of the particle cannot be dispersed in its motion, which embodies concretely the corpuscle features of the microscopic particles. This is completely consistent with the concept of classical particles. This means that the nonlinear interaction, $b|\phi|^2 \phi$, related to the wave function of the particles, balances and suppresses really the dispersion effect of the kinetic term in Equation (6) to make the particles become eventually localized. Thus the position of the particles, $\vec{r}$ or x, has a determinately physical significance. However, the envelope of the solution in Equations (14)-(16) is a solitary wave. It has a certain wavevector and frequency as shown in Figure 1(b), and can propagate in space-time, which is accompanied with the carrier wave. Its feature of propagation depends on the concrete nature of the particles. Figure 1(b) shows the width of the frequency spectrum of the envelope φ(x, t) which has a localized distribution around the carrier frequency ω_0. This shows that the particle has also a wave feature[21-28]. Thus we believe that the microscopic particles described by nonlinear quantum mechanics have simultaneously a wave-corpuscle duality. Equations (14)-(16) and Figure 1(a) are just the most beautiful and perfect representation of this property, which consists also of de Broglie relation, $E = h\upsilon = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$, wave-corpuscle duality and Davisson and Germer's experimental result of electron diffraction on double seam in 1927 as well as the traditional concept of particles in physics[10-13]. Thus we have reasons to believe the correctness of nonlinear quantum mechanics proposed by Pang.[21-28]

1.2 The Wave-Corpuscle Duality of Solution of the Nonlinear Schrödinger Equation with Different Potentials

We can verify that the nature of wave-corpuscle duality of microscopic particles is not changed when varying the externally applied potentials. As a matter of fact, if $V(x') = \alpha x' + c$ in Equation (5), where α and c are some constants, in this case Pang[31-38, 41-43] replaced Equation (8) by

$$\varphi_{x'x'} - \varphi\theta_{t'} - \varphi\theta_{x'}^2 - b\varphi^2\varphi = \alpha x' + c. \qquad (18)$$

Now let

$$\varphi(x',t') = \varphi(\xi), \xi = x' - u(t'), u(t') = -\alpha(t')^2 + vt' + d \qquad (19)$$

Where $u(t')$ describes the accelerated motion of $\varphi(x', t')$. The boundary condition at $\xi \to \infty$ requires $\varphi(\xi)$ to approach zero rapidly, then Equation (9) can be written as

$$-\dot{u} \frac{\partial\varphi}{\partial\xi} + 2 \frac{\partial\varphi}{\partial\xi} \frac{\partial\theta}{\partial\xi} + \varphi \frac{\partial^2\theta}{\partial\xi^2} = 0 \qquad (20)$$

Where $\dot{u} = \frac{du}{dt}$. If $2\partial\theta/\partial\xi - \dot{u} \neq 0$, Equation (20) may be denoted as

$$\varphi^2 = \frac{g(t')}{(\partial\theta/\partial\xi - \dot{u}/2)} \text{ or } \frac{\partial\theta}{\partial x'} = \frac{g(t')}{\varphi^2} + \frac{\dot{u}}{2} \qquad (21)$$

Integration of Equation (26) yields

$$\theta(x',t') = g(t') \int_0^{x'} \frac{dx''}{\varphi^2} + \frac{\dot{u}}{2} x' + h(t') \qquad (22)$$

Where $h(t')$ is an undetermined constant of integration. From Equation (22) Pang[41-43] can get

$$\frac{\partial\theta}{\partial t'} = \dot{g}(t') \int_0^{x'} \frac{dx''}{\varphi^2} - \frac{g\dot{u}}{\varphi^2} + \frac{g\dot{u}}{\varphi^2}\Big|_{x'=0} + \frac{\ddot{u}}{2} x' + \dot{h}(t') \qquad (23)$$

Substituting Equations (22)-(23) into Equation (17), we have:

$$\frac{\partial^2\varphi}{\partial(x')^2} = \left[(ax'+c) + \frac{\ddot{u}}{2} x' + \dot{h}(t') + \frac{\dot{u}^2}{4} + \dot{g} \int_0^{x'} \frac{dx''}{\varphi^2} + \right.$$

$$\left. \frac{g\dot{u}}{\varphi^2}\Big|_{x'=0} \right] \varphi - b\varphi^3 + \frac{g^2}{\varphi^3} \qquad (24)$$

Since $\frac{\partial^2\varphi}{\partial(x')^2} = \frac{d^2\varphi}{d\xi^2}$, which is a function of ξ only. In order for the right-hand side of Equation (29) is also a function of ξ only, it is necessary that $g(t') = g_0 = $ constant, and

$$(ax'+c) + \frac{\ddot{u}}{2} x' + \dot{h}(t') + \frac{\dot{u}^2}{4} + \frac{g\dot{u}}{\varphi^2}\Big|_{x'=0} = \overline{V}(\xi) \qquad (25)$$

Next, we assume that $V_0(\xi) = \overline{V}(\xi) - \beta$, where β is real and arbitrary, then

$$ax'+c = V_0(\xi) - \frac{\ddot{u}}{2} x' + \left[\beta - \frac{g\dot{u}}{\varphi^2}\Big|_{x'=0} - \dot{h}(t') - \frac{\dot{u}^2}{4} \right] \quad (26)$$

Clearly, in the discussed case $V_0(\xi) = 0$, and the function in the brackets in Equation (26) is a function of t¢. Substituting Equation (26) into Equation (4), we can get:

$$\frac{d^2\overline{\varphi}}{d\xi^2} = \beta\overline{\varphi} - b\overline{\varphi}^{-3} + \frac{g_0}{\overline{\varphi}^{-3}} \qquad (27)$$

Where β is a real parameter and defined by

$$V_0(\xi) = \overline{V}(\xi) - \beta \qquad (28)$$

with

$$(\alpha x'+c) + \frac{\ddot{u}}{2} x' + \dot{h}(t') + \frac{\dot{u}^2}{4} + \frac{g\dot{u}}{\varphi^2}\Big|_{x'=0} = \overline{V}(\xi) \qquad (29)$$

Clearly, in the discussed case, $V_0(\xi) = 0$. Obviously, $\overline{\varphi} = \varphi(\xi)$ is the solution of Equation (27) when β and g are

constant. For large $|\xi|$, we may assume that $\overline{\varphi} \leq \beta / |\xi|^{1+\Delta}$, when Δ is a small constant. To ensure that $d^2\overline{\varphi} / d\xi^2$, and $\overline{\varphi}$ approach zero when $|\xi| \to \infty$, only the solution corresponding to $g_0 = 0$ in Equation (27) is kept stable. Therefore we choose $g_0 = 0$ and obtain the following from Equation (26)

$$\frac{\partial \theta}{\partial x'} = \frac{\dot{u}}{2} \qquad (30)$$

thus, we obtain from Equation (34)

$$\alpha x' + c = -\frac{\ddot{u}}{2}x' + \beta - \dot{h}(t') - \frac{\dot{u}^2}{4},$$
$$h(t') = (\beta - v^2/4 - c)t' - \alpha^2 t'^3/3 + v\alpha t'^2/2]\} \qquad (31)$$

Substituting Equation (31) into Equations (23) and (29), we obtain

$$\theta = [(-\alpha t' + v/2)x' + (\beta - v^2/4 - c)t' - \alpha^2 t'^3/3 + v\alpha t'^2/2 \qquad (32)$$

Finally, substituting the above into Equation (33), we can get

$$\frac{d^2\overline{\varphi}}{d\xi^2} - \beta\overline{\varphi} + b\varphi^3 = 0 \qquad (33)$$

When $\beta > 0$, Pang gives the solution of Equation (38), which is of the form

$$\overline{\varphi} = \sqrt{\frac{2\beta}{b}} \sec h(\sqrt{\beta}\,\zeta) \qquad (34)$$

Pang[41-43] finally obtained the complete solution in this condition, which is represented as

$$\phi(x',t') = \sqrt{\frac{2\beta}{b}} \sec h\{\sqrt{\beta}[(x' - x_0') + (\alpha t'^2 - vt' - d)]\} \times$$
$$\exp\{i[-\alpha t' + v/2(x' - x_0') + (\beta - v^2/4 - c)t' - \alpha^2 t'^3/3 + v\alpha t'^2/2]\} \qquad (35)$$

This is a soliton solution. If $V(x') = c$, the solution can be represented as

$$\phi(x',t') = \sqrt{\frac{2\beta}{b}}k \sec h\{\sqrt{\beta}[(x' - x_0') - v(t' - t_0')]\} \times$$
$$\exp\{i[v(x' - x_0')/2 - (\beta - v^2/4 - c)t']\} \qquad (36)$$

At $V(x') = 2\alpha x'$ and $b = 2$, we can also get a corresponding soliton solution from the above process. However, Chen and Liu[44-45] adopted the following transformation

$$\phi(x',t') = \phi'(\tilde{x}',\tilde{t}')\exp[-2i\alpha\tilde{x}'\tilde{t}' + 8i\alpha^2\tilde{t}'^3/3], x' =$$
$$\tilde{x}' - 2\alpha\tilde{t}'^2, t' = \tilde{t}' \qquad (37)$$

to make Equation (5) become

$$i\phi'_{\tilde{t}'} + \phi'_{\tilde{x}'\tilde{x}'} + 2|\phi'|^2 \phi' = 0 \qquad (38)$$

Thus Chen and Liu[44-45] represented the solution of Equation (5) at $V(x') = ax', b = 2$ by

$$\phi(x',t') = 2\eta \sec h\{2\eta[(x' - x_0') + (2\alpha t'^2 - 4\xi't')]\} \times$$

$$\exp\{-i[2(\xi' - \alpha t')(x' - x_0') + 4(\xi'^2 - \eta^2)t' + 4\alpha^2 t'^3/3 - 4\xi'\alpha t'^2] + \theta_0\} \qquad (39)$$

At the same time, utilizing the above method, Pang[25-27,41] found also the soliton solution of Equation (5) at $V(x) = kx^2 + A(t)x + B(t)$, which could be represented as

$$\phi = \varphi(x - u(t))e^{i\theta(x,t)} \qquad (40)$$

Where

$$\varphi(x',t') = \sqrt{\frac{2\beta}{b}} \sec h\{\sqrt{\beta}[(x' - x_0') - u(t')]\},$$
$$u(t') = 2\cos(2\sqrt{k}t' + \gamma) + u_0(t') \qquad (41)$$
$$\theta(x',t') = [-2\alpha \sin(2\sqrt{k}t' + \gamma) + u_0(t')/2](x' - x_0') +$$
$$\int_0^{t'} \{[-\alpha^2(2\cos(2\sqrt{k}t'' + \gamma) + u_0(t'')/2)]^2 - B(t'') +$$
$$[-2\alpha \sin(2\sqrt{k}t'' + \gamma) + u_0(t'')/2]\}dt'' + \overline{L}t' + \theta_0$$

Where $\overline{L}$ is a constant related to $A(t')$. When $A(t) = B(t) = 0$, the solution is still Equation (40), but

$$u(t') = 2\cos(2\sqrt{k}t') + u_0(t'),$$
$$\theta(x',t') = [-2\sqrt{k}\sin(2\sqrt{k}t') + u_0(t')/2](x' - x_0') +$$
$$\int_0^{t'} \{[-k(2\cos(2\sqrt{k}t'') + u_0(t'')/2)]^2 +$$
$$[-2\sqrt{k}\sin(2\sqrt{k}t'') + u_0(t'')/2]\}dt'' + \theta_0$$

For the case of $V_0(x') = \alpha^2 x'^2$ and $b = 2$, where α is constant, Chen and Liu[44-45] assume $u(t') = (2\xi/\alpha)\sin(2\alpha t')$, thus they represent the soliton solution in this condition by

$$\phi(x',t') = 2\eta \sec h\{2\eta[(x' - x_0') - (2\xi'/\alpha)\sin(2\alpha t')]\}$$
$$\times \exp\{i[2\xi'(x' - x_0')\cos 2\alpha(t' - t_0') - +4\eta^2(t' - t_0') -$$
$$(\xi'^2/\alpha)\sin[4\alpha(t' - t_0') + \theta_0]\} \qquad (42)$$

Where $2\eta = \sqrt{\beta}$ is the amplitude of microscopic particles, $4\xi'$ is related to its group velocity in Equations (39) and (42), and ξ' is the same as ξ in Equation (15). From the above results we see clearly that these solutions of nonlinear Schrödinger Equation (5) under influences of different potentials, $V(x) = c$, $V(x') = \alpha x'$, $V(x') = \alpha x' + c$, $V(x) = kx^2 + A(t)x + B(t)$ and $V(x') = \alpha^2 x'^2$, still consist of envelop and carrier waves, which are analogous to Equation (14), some bell-type solitons with a certain amplitude A_0, group velocity v_e and phase speed v_c, and have a mass center and determinant amplitude, width, size, mass, momentum and energy. If inserting these solutions, Equations (35), (38), (39), (41) and (42) into Equation (16) we can find out the effective masses, moments and energies of these microscopic particles, respectively, which all have determinant values. Therefore we can determine that the microscopic particles described by these dynamic equations still possess a wave-corpuscle

duality as shown in Figure 1, although they are acted by different external potentials. These potentials change only amplitude, size, frequency, phase and group and phase velocities of the particles, in which velocity and frequencies of some particles are further related to time and oscillatory[47-50]. These results indicate that in Equation (5) the kinetic energy term decides the wave feature of the particles, the nonlinear interaction determines its corpuscle feature, their combination results in its wave-corpuscle duality, but the external potentials influence only wave form, phase and velocity of particles, but cannot affect the wave-corpuscle duality. These results verify directly and clearly the necessity and correctness for describing the properties of microscopic particles using the nonlinear Schrödinger equation, or the nonlinear quantum mechanics proposed by Pang[17-24].
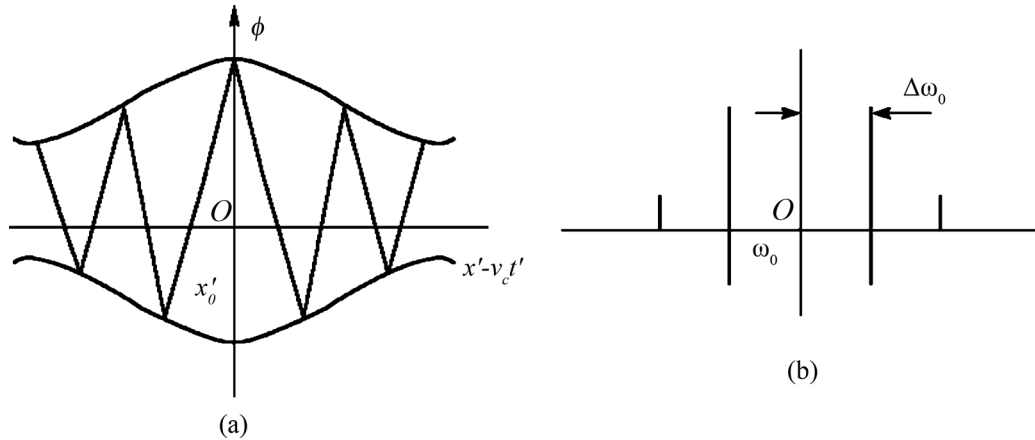

Figure 1
The Solution of Equation (6) and Its Features

2. THE CLASSICAL FEATURES OF MOTION OF MICROSCOPIC PARTICLES

2.1 The Feature of Newton's Motion of Microscopic Particles

Since the microscopic particle described by the nonlinear Schrödinger Equation (5) has a corpuscle feature and is also quite stable as mentioned above. Thus its motion in action of a potential field in space-time should have itself rules of motion. Pang[51-55] studied deeply this rule of motion of microscopic particles in such a case.

We know that the solution Equation (14), shown in Figure 1, of the nonlinear Schrödinger Equation (5) with different potentials, has the behavior of $\frac{\partial \phi}{\partial x'} = 0$ at $x' = x'_0$.

Therefore, $\phi^*\phi dx' = \rho(x')dx'$ can be regarded as the mass in the interval of x' to $x' + dx'$. Thus the position of mass centre of a microscopic particle at x'_0 in nonlinear quantum mechanics can be represented by

$$\langle x' \rangle = x'_g = \ x'_0 = \int_{-\infty}^{\infty}\phi^{'*}x'\phi dx'/\int_{-\infty}^{\infty}\phi^*\phi dx' \qquad (43)$$

Then the velocity ofb the microscopic particle is represented by

$$v = v_g = \frac{d}{dt'}\{\int_{-\infty}^{\infty}\phi^*x'\phi dx'/\int_{-\infty}^{\infty}\phi^*\phi dx'\} = $$
$$\int_{-\infty}^{\infty}(\phi_t^*x'\phi + \phi^*x'\phi_{t'})dx'/\int_{-\infty}^{\infty}\phi^*\phi dx' \qquad (44)$$

then from Equation (5) and its conjugate equation

$$-i\hbar\frac{\partial \phi^*}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\phi^* \pm b|\phi|^2\phi^* + V(\vec{r},t)\phi^* \qquad (45)$$

We can get that the velocity of mass centre of microscopic particle can be denoted by

$$v_g = d\langle x'\rangle/dt' = -2i\int_{-\infty}^{\infty}\phi^*\phi_{x'}dx'/\int_{-\infty}^{\infty}\phi^*\phi dx' \qquad (46)$$

We now determine the acceleration of the mass center of the microscopic particles and its rules of motion in an externally applied potential.

We can obtain from the above equation

$$\frac{d}{d'}\int_{-\infty}^{\infty}\phi^*\phi_x dx' = \int_{-\infty}^{\infty}\phi_t^*\phi_x dx' + \int_{-\infty}^{\infty}\phi^*(\phi_t)_x dx' = $$
$$i\int_{-\infty}^{\infty}\{\phi^*\frac{\partial}{\partial x'}[\phi_{xx'} + b\phi^*\phi^2 - V\phi] - V\phi^*]\phi_x\}dx' = $$
$$i\int_{-\infty}^{\infty}\phi^*\frac{\partial V}{\partial x'}\phi dx' \qquad (47)$$

Where $x' = x/\sqrt{\hbar^2/2m}$, $t' = t/\hbar$. We here utilize the following relations and the boundary conditions:

$$\int_{-\infty}^{\infty}(\phi^*\phi_{x'x'x'} - \phi_{x'x'}^*\phi_{x'})dx' = 0, \int_{-\infty}^{\infty}b(\phi^{*2}\phi\phi_{x'} + \phi^*\phi^2\phi_{x'}^*)dx' = 0,$$

$$\int\phi^*\phi dx' = \text{constant (or a function of t') and}$$

$$\lim_{|x'|\to\infty}\phi^*x'\frac{\partial \phi}{\partial x'} = \lim_{|x'|\to\infty}\frac{\partial \phi^*}{\partial x'}x'\phi = 0,$$

$$\underset{|x'|\to\infty}{Lim}\phi(x',t') = \lim_{|x'|\to\infty}\phi_{x'}(x',t') = 0$$

Where $\phi_{x'} = \dfrac{\partial \phi}{\partial x'}, \phi_{x'x'x'} = \dfrac{\partial^3 \phi}{\partial x'^3}$. Thus, we can get

$$\frac{d}{dt'}\int_{-\infty}^{\infty}\phi^* x'\phi\,dx' = \int_{-\infty}^{\infty}\left(\frac{\partial\phi^*}{\partial t'}x'\phi + \phi^* x'\left(\frac{\partial\phi}{\partial t'}\right)\right)dx' = -2i\int_{-\infty}^{\infty}\phi^*\phi_{x'}\,dx'$$

(48)

In the systems, the position of mass centre of microscopic particle can be represented by Equation (43), thus the velocity of mass centre of microscopic particle is represented by Equation (44). Then, the acceleration of mass centre of microscopic particle can also be denoted by

$$\frac{d^2}{dt'^2}\langle x'\rangle = -2i\frac{d}{dt'}\left\{\int_{-\infty}^{\infty}\phi^*\phi_{x'}\,dx' \middle/ \int_{-\infty}^{\infty}\phi^*\phi\,dx' \right. =$$

$$-2\int_{-\infty}^{\infty}\phi^* V_x\phi^*\,dx' = -2\left\langle\frac{\partial V}{\partial x'}\right\rangle$$

(49)

If ϕ is normalized, i.e., $\int_{-\infty}^{\infty}\phi^*\phi\,dx' = 1$, then the above conclusions also are not changed.

Where V = V(x') in Equation (49) is the external potential field experienced by the microscopic particles. We expand $\dfrac{\partial V}{\partial x'}$ at the mass centre $x' = \langle x'\rangle = x'_0$ as

$$\frac{\partial V(x')}{\partial x'} = \frac{\partial(\langle x'\rangle)}{\partial\langle x'\rangle} + (x' - \langle x'\rangle)\frac{\partial^2 V(\langle x'\rangle)}{\partial\langle x'\rangle^2} +$$

$$\frac{1}{2!}(x' - \langle x'\rangle)^2\frac{\partial^3 V(\langle x'\rangle)}{\partial\langle x'\rangle^3} + \cdots$$

Taking the expectation value on the above equation, we can get

$$\left\langle\frac{\partial V(x')}{\partial x'}\right\rangle = \frac{\partial V(\langle x'\rangle)}{\partial\langle x'\rangle} + \frac{1}{2!}\langle(x' - \langle x'\rangle)^2\rangle\frac{\partial^3 V(\langle x'\rangle)}{\partial\langle x'\rangle^3}$$

where

$$\Delta_{ij} = \langle((x' - \langle x'\rangle)^2)\rangle = \langle(x'_i - x'_0)(x'_j - x'_0)\rangle =$$

$$\langle(x'_i - \langle x'_i\rangle)(x'_j - \langle x'_j\rangle)\rangle$$

For the microscopic particle described by Equation (5) or Equation (6), the position of the mass center of the particle is known and determinant, which is just $\langle x'\rangle = x'_0 = $ constant, or 0. Since we here study only the rule of motion of the mass centre x_0, which means that the terms containing x'_0 in $\langle x'^2\rangle$ are considered and included, then $\langle(x' - \langle x'\rangle)^2\rangle = 0$ can be obtained. Thus

$$\left\langle\frac{\partial V(x')}{\partial x'}\right\rangle = \frac{\partial V(\langle x'\rangle)}{\partial\langle x'\rangle}$$

Pang[17-28] finally obtained the acceleration of mass center of microscopic particle in the nonlinear quantum mechanics, Equation (49), which is denoted as

$$\frac{d^2}{dt'^2}\langle x'\rangle = -2\frac{\partial V(\langle x'\rangle)}{\partial\langle x'\rangle}$$

(50)

Returning to the original variables, the Equation (50) becomes

$$m\frac{d^2 x_0}{dt^2} = -\frac{\partial V}{\partial x_0}$$

(51)

Where $x'_0 = \langle x'\rangle$ is the position of the mass centre of microscopic particle. Equation (50) is the equation of motion of mass center of the microscopic particles in the nonlinear quantum mechanics. It resembles quite the Newton-type equation of motion of classical particles, which is a fundamental dynamics equation in classical physics. Thus it is not difficult to conclude that the microscopic particles depicted by the nonlinear quantum mechanics have a property of the classical particle.

The above equation of motion of particles can also derive from Equation (5) by another method. As it is known, the momentum of the particle depicted by Equation (5) is obtained from Equation (17) and denoted by $P = \dfrac{\partial L}{\partial\phi} = -i\int_{-\infty}^{\infty}\left(\phi^*\phi_{x'} - \phi_{x'}^*\phi\right)dx'$. Utilizing Equation (5) and Equations (43)-(45) Pang obtained[18-27,51]

$$\frac{dP}{dt'} = \int_{-\infty}^{\infty}2V(x')\frac{\partial}{\partial x'}|\phi|^2\,dx' = -2\int_{-\infty}^{\infty}\frac{\partial V}{\partial x'}|\phi|^2\,dx' =$$

$$-2\left\langle\frac{\partial V(x')}{\partial x'}\right\rangle$$

(52)

Where the boundary condition of $\phi(x') \to 0$ as $|x'| \to \infty$ is used. Utilizing again the above result of $\left\langle\dfrac{\partial V(x')}{\partial x'}\right\rangle = \dfrac{\partial V(\langle x'\rangle)}{\partial\langle x'\rangle}$ we can get also that the acceleration of the mass center of the particle is the form of

$$\frac{dP}{dt'} = -2\frac{\partial V(x'_0)}{\partial x'_0} \quad \text{or} \quad m\frac{d^2 x_0}{dt^2} = -\frac{\partial V}{\partial x_0}$$

(53)

Where x'_0 is the position of the center of the mass of the macroscopic particle. This is the same as Equation (51). Therefore, we can confirm that the microscopic particles in the nonlinear quantum mechanics satisfy the Newton-type equation of motion for a classical particle.

2.2 Lagrangian and Hamilton Equations of Microscopic Particle

Using the above variables ϕ in Equation (5) and ϕ^* in Equation (45) one can determine the Poisson bracket and write further the equations of motion of microscopic particles in the form of Hamilton's equations. For Equation (5), the variables ϕ and ϕ^* satisfy the Poisson bracket:

$$\left\{\phi^{(a)}(x), \phi^{(b)}(y)\right\} = i\delta^{ab}\delta(x - y)$$

(54)

where $\{A, B\} = i\int_{-\infty}^{\infty}\left(\dfrac{\delta A}{\delta\phi}\dfrac{\delta B}{\delta\phi^*} - \dfrac{\delta B}{\delta\phi}\dfrac{\delta A}{\delta\phi^*}\right)$

The Lagrange density function, L', corresponding to Equation (5) is given as follows:

$$L' = \frac{i\hbar}{2}\left(\phi^*\partial_t\phi - \phi\partial_t\phi^*\right) -$$

$$\frac{\hbar^2}{2m}\left(\nabla\phi\cdot\nabla\phi^*\right) - V(x)\phi\phi^* + (b/2)\left(\phi\phi^*\right)^2 \qquad (55)$$

where $L'=\mathcal{L}$. The momentum density of the particle system is defined as $P = \partial\mathcal{L}/\partial\phi$. Thus, the Hamiltonian density, $\mathcal{H} = H'$, of the systems is as follows

$$\mathcal{H} = \frac{i\hbar}{2}\left(\phi^*\partial_t\phi - \phi\partial_t\phi^*\right) - \mathcal{L} = \frac{\hbar^2}{2m}\left(\nabla\phi\cdot\nabla\phi^*\right) +$$

$$V(x)\phi\phi^* - (b/2)\left(\phi\phi^*\right)^2 \qquad (56)$$

From the Lagrangian density $L' = L'$ in Equation (55) Pang[18-27, 51-55] find out

$$\frac{\partial L'}{\partial\phi^*} = \frac{i\hbar}{2}\phi_t - V(x)\phi + b(\phi^*\phi)\phi$$

and $\partial_t(\frac{\partial L'}{\partial\phi_t^*}) + \nabla\cdot(\frac{\partial L'}{\partial\nabla\phi^*}) = -\frac{i\hbar}{2}\phi_t - \frac{\hbar^2}{2m}\nabla\cdot\nabla\phi$

Thus we can obtain

$$\frac{\partial L'}{\partial\varphi^*} - \partial_t(\frac{\partial L'}{\partial\varphi_t^*}) - \nabla\cdot(\frac{\partial L'}{\partial\nabla\varphi^*}) = i\hbar\varphi_t + \frac{\hbar^2}{2m}\nabla^2\varphi - V(x)\varphi +$$

$$b(\varphi^*\varphi)\varphi \qquad (57)$$

Through comparison with Equation (5) Pang gets

$$\frac{\partial L'}{\partial\phi^*} = \partial_t(\frac{\partial L'}{\partial\phi_t^*}) + \nabla\cdot(\frac{\partial L'}{\partial\nabla\phi^*})$$

or

$$\frac{\partial L'}{\partial\phi} = \partial_t(\frac{\partial L'}{\partial\phi_t}) + \nabla\cdot(\frac{\partial L'}{\partial\nabla\phi}) \qquad (58)$$

Equation (58) is just the well-known Euler-Lagrange equation for this system. This show that the nonlinear Schrödinger equation amount to Euler-Lagrange equation in nonlinear quantum mechanics, in other word, the dynamic equation, or the nonlinear Schrödinger equation can be obtained from the Euler-Lagrange equation in nonlinear quantum mechanics, if the Lagrangian function of the system is known. This is different from quantum mechanics, in which the dynamic equation is the linear Schrödinger equation, instead of the Euler-Lagrange equation.

On the other hand, Pang[18-27, 51-55] got also the Hamilton equation of the microscopic particle from the Hamiltonian density of this system in Equation (56). In fact, we can obtain from Equation (56)

$$\frac{\delta H'}{\delta\phi^*} = -\frac{\hbar^2}{2m}\nabla^2\phi + V(x)\phi - b(\phi^*\phi)\phi \qquad (59)$$

Where $H' = H'$. Then from Equation (56) we can give

$$i\hbar\frac{\partial\phi}{\partial t} = \frac{\delta H'}{\delta\phi^*} = -\frac{\hbar^2}{2m}\nabla^2\phi + V(x)\phi - b(\phi^*\phi)\phi$$

Thus

$$i\hbar\frac{\partial\phi}{\partial t} = \frac{\delta H'}{\delta\phi^*}, \text{ or } i\hbar\frac{\partial\phi^*}{\partial t} = -\frac{\delta H'}{\delta\phi} \qquad (60)$$

Equation (60) is just the complex form of Hamilton equation in nonlinear quantum mechanics. In fact, the Hamilton equation can be also represent by the canonical coordinate and momentum of the particle. In this case the canonical coordinate and momentum are defined by

$$q_1 = \frac{1}{2}\left(\phi + \phi^*\right), \quad p_1 = \frac{\partial L'}{\partial\left(\partial_t q_1\right)}, q_2 = \frac{1}{2i}\left(\phi - \phi^*\right),$$

$$p_2 = \frac{\partial L'}{\partial\left(\partial_t q_2\right)}$$

Thus, the Hamiltonian density of the system in Equation (56) takes the form

$$H' = \sum_i p_i\partial_t q_i - L'$$

and the corresponding variation of the Lagrangian density $L' = \mathcal{L}$ can be written as

$$\delta L' = \sum_i \frac{\delta L'}{\delta q_i}\delta q_i + \frac{\delta L'}{\delta\left(\nabla q_i\right)}\delta\left(\nabla q_i\right) + \frac{\delta L'}{\delta\left(\partial_t q_i\right)}\delta\left(\partial_t q_i\right)$$

From Equation (17), the definition of p_i, and the Euler-Lagrange equation,

$$\frac{\partial L'}{\partial q_i} = \nabla\cdot\frac{\partial L'}{\partial\nabla q_i} + \frac{\partial p_i}{\partial t}$$

one obtains the variation of the Hamiltonian in the form of $\delta H' = \sum_i \int \left(\partial_t q_i\delta p_i - \partial_t p_i\delta q_i\right)dx'$.

Thus, one pair of dynamic equations can be obtained and expressed by

$$\frac{\partial q_i}{\partial t'} = \delta H'/\delta p_i, \quad \frac{\partial p_i}{\partial t'} = -\delta H'/\delta q_i \qquad (61)$$

This is analogous to the Hamilton equation in classical mechanics and has same physical significance with Equation (60), but the latter is used often in nonlinear quantum mechanics. This result shows that the nonlinear Schrödinger equation describing the dynamics of microscopic particle can be obtained from the classical Hamilton equation in the case, if the Hamiltonian of the system is known. Obviously, such methods of finding dynamic equations are impossible in the quantum mechanics. As it is known, the Euler-Lagrange equation and Hamilton equation are fundamental equations in classical theoretical (analytic) mechanics, and were used to describe laws of motions of classical particles. This means that the microscopic particles possess evidently classical features in nonlinear quantum mechanics. From this study we seek also a new way of finding the equation of motion of the microscopic particles in nonlinear systems, i.e., only if the Lagrangian or Hamiltonian of the system is known, we can obtain the equation of motion of microscopic particles from the Euler-Lagrange or Hamilton equations.

On the other hand, from de Broglie relation $E = h\upsilon = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$, which represent the wave-

corpuscle duality of the microscopic particles in quantum theory, we see that the frequency ω and wavevector $\vec{k}$ can play the roles as the Hamiltonian of the system and momentum of the particle, respectively, even in the nonlinear systems and has thus the relation:

$$\frac{d\omega}{dt'} = \frac{\partial\omega}{\partial k}\Big|_{x'}\frac{dk}{dt'} + \frac{\partial\omega}{\partial x'}\Big|_k\frac{\partial x'}{\partial t'} = 0$$

as in the usual stationary media. From the above result we also know that the usual Hamilton equation in Equation (57) for the nonlinear systems remain valid for the microscopic particles. Thus, the Hamilton equation in Equation (57) can be now represented by another form[25-27, 56-57]:

$$\frac{dk}{dt'} = -\frac{\partial\omega}{\partial x'}\Big|_k \text{ and } \frac{dx'}{dt'} = \frac{\partial\omega}{\partial k}\Big|_{x'} \quad (62)$$

in the energy picture, where $k = \partial\theta/\partial x'$ is the time-dependent wavenumber of the microscopic particle, $\omega = -\partial\theta/\partial t'$ is its frequency, θ is the phase of the wave function of the microscopic particles.

2.3 Confirmation of Correctness of the Above Conclusions

We now use some concrete examples to confirm the correctness of the laws of motion of the microscopic particles mentioned above in the nonlinear quantum mechanics[18-27, 51-55].

(1) For the microscopic particles described by Equation (5), $V = 0$ and constant, of which the solutions are Equation (15) or (16) and (36), respectively, we obtain that the acceleration of the mass centre of microscopic particle is zero because of $m\frac{d^2}{dt^2}\langle x\rangle = -\frac{\partial V(<x>)}{\partial <x>} = 0$ in this case. This means that the velocity of the particle is a constant. In fact, if inserting Equation (15) into Equation (53) we can obtain the group velocity of the particle $v_g = d<x'>/dt' = -2i\int_{-\infty}^{\infty}\phi^*\phi_x dx' = v_e = -4\xi = $ constant. This manifests that the microscopic particle moves in uniform velocity in space-time, its velocity is just the group velocity of the soliton, thus the energy and momentum of the microscopic particles can retain in the motion process. These properties are the same with classical particle.

On the other hand, if the dynamic Equation (62) is used we can obtain from Equation (15) that the acceleration and velocity of the microscopic particle are

$$\frac{dk}{dt'} = 0 \text{ and } v_g = \frac{dx'}{dt'} = \frac{\partial\omega}{\partial k} = v_e = -4\xi,$$

respectively, where
$$\omega = -\partial\theta/\partial t' = 4(\xi^2 - \eta^2) = k^2 - 4\eta^2, k = \partial\theta/\partial x' = -2\xi,$$
$$\theta = -4(\xi^2 - \eta^2)t' - 2\xi x' + \theta_0$$

For the solution in Equation (36) at V = constant, $\theta = v_e(x'-x_0')/2 - (\beta - v_e^2/4 - C)t'$, then $\omega = (\beta - v_e^2/4 - C)$,

$k = v_e/2$, thus $\frac{dk}{dt'} = 0$ and $v_g = \frac{dx'}{dt'} = \frac{\partial\omega}{\partial k} = -2k = -v_e$.

These results of the acceleration and velocity of microscopic particle are same with the above data obtained from Equations (15) and (51). This indicates that these moved laws shown in Equations (50), (51), (53), (60), (61), and (62) are self-consistent, correct and true in nonlinear quantum mechanics.

(2) For the case of $V(x') = \alpha x'$, the solution of Equation (5) is Equation (39) by Chen and Liu[44-45], which is also composed of a envelope and carrier wave. The mass centre of the particle is at x_0', which is its localized position. From Equation (51) we can determined the accelerations of the mass center of the microscopic particle in this case, which is given by

$$\frac{d^2 x_0'}{dt'^2} = -2\frac{\partial V(\langle x'\rangle)}{\partial\langle x'\rangle} = -2\alpha = \text{constant} \quad (63)$$

On the other hand, from Equation (39) we know that

$$\theta = 2(\xi - \alpha t')x' + \frac{4\alpha^2 t'^3}{3} - 4\alpha\xi t'^2 + 4(\xi^2 - \eta^2)t' + \theta_0, \quad (64)$$

where ξ is same with ξ' in Equation (39). Utilizing again Equation (62) we can find

$$k = 2(\xi - \alpha t'), \omega = 2\alpha x' - 4(\xi - \alpha t')^2 + (2\eta)^2 = 2\alpha x' - k^2 + (2\eta)^2$$

Thus, the group velocity of the microscopic particle is found out from

$$v_g = \frac{d\overline{x'}}{dt'} = \frac{\partial\omega}{\partial k} = 4(\xi - \alpha t') \quad (65)$$

Then its acceleration is given by

$$\frac{d^2\tilde{x}}{dt'^2} = \frac{dk}{dt'} = -2a = \text{constant}, \text{ here}(x_0' = \tilde{x}) \quad (66)$$

Comparing Equation (63) with Equation (64) we find that they are also same. This indicates that Equations (51), (53), (60), (61) and (62) are correct. In such a case the microscopic particle moves in a uniform acceleration. This is similar with that of classical particle in an electric field.

(3) For the case of $V(x') = \alpha^2 x'^2$, which is a harmonic potential, the solution of Equation (5) in this case is Equation (42) obtained by Chen and Liu[44-45]. This solution contain also an envelop and carrier wave, and has also a mass centre, its position is at x'_0, which is the position of the microscopic particle. When Equation (51) is used to determine the properties of motion of the particle in this case Pang[18-28] found out that the accelerations of the center of mass of the particle is

$$\frac{d^2 x_0'}{dt'^2} = -4\alpha^2 x_0' \quad (67)$$

At the same time, from Equation (42) we gain that

$$\theta = 2\xi(x' - x_0')\cos[2\alpha(t' - t_0')] + 4\eta^2(t' - t_0') - (\xi^2/\alpha)\sin[4\alpha(t' - t_0')] + \theta_0 \quad (68)$$

where ξ is same with ξ' in Equation (42). From Equations (67) and (68) we can find

$$k = 2\xi \cos 2\alpha\left(t' - t_0'\right),$$

$$\omega = 4a\xi x' \sin 2\alpha\left(t' - t_0'\right) - 4\xi^2 \cos 4\alpha\left(t' - t_0'\right) - 4\eta^2$$

$$= 2\alpha x'\left(4\xi^2 - k^2\right)^{1/2} - 2k^2 + 4\left(\xi^2 - \eta^2\right),$$

Thus, the group velocity of the microscopic particle is

$$v_g = \frac{\partial \omega}{\partial k}\Big|_x = \frac{\alpha x'}{\xi}\frac{k}{\sqrt{1 - k^2\big/4\xi^2}} - 2k = 2\alpha x' ctg[2\alpha x(t' - t_0')]$$

$$- 4\xi \cos[2\alpha(t' - t_0')]$$

While its acceleration is

$$\frac{dk}{dt'} = -\frac{\partial \omega}{\partial x'}\Big|_k = -2\alpha\sqrt{4\xi^2 - k^2} = -4\xi\alpha \sin[2\alpha(t' - t_0')]$$

(69)

Since $\dfrac{d^2\tilde{x}'}{dt'^2} = \dfrac{dk}{dt'}$, here $\left(\tilde{x}' = x_0'\right)$, we have

$$\frac{dk}{dt'} = \frac{d^2\overline{x'}}{dt'^2} = -4\xi\alpha \sin[2\alpha(t' - t_0')]$$

and

$$\tilde{x}' = \frac{\xi}{\alpha} \sin[2\alpha(t - t_0')]$$

(70)

Finally, the acceleration of the microscopic particle is

$$\frac{d^2\overline{x'}}{dt'^2} = \frac{dk}{dt'} = -4\alpha^2\overline{x'}$$

(71)

Equation (98) is also the same with Equation (67). Thus we confirm also the validity of Equations (48), (51), (53), (58), (60) and (61)-(62). In such a case the microscopic particle moves in harmonic form. This resembles also with the result of motion of classical particle.

From the above studies we draw the following conclusions[18-27, 51-55]. (1) The motions of microscopic particles in the nonlinear quantum mechanics can be described by not only the nonlinear Schrödinger equation but also Hamiltonian principle, Lagrangian and Hamilton equations, its changes of position with changing time satisfy the law of motion of classical particle in both uniform and inhomogeneous. This not only manifests that the natures of microscopic particles described by nonlinear quantum mechanics differ completely from those in the linear quantum mechanics but displays sufficiently the corpuscle nature of the microscopic particles.

(2) The external potentials can change the states of motion of the microscopic particles, although it cannot vary its wave-corpuscle duality, for example, the particle moves with a uniform velocity at $V(x') = 0$ or constant, or in an uniform acceleration at $V(x') = ax'$, which corresponds to the motion of a charge particle in a uniform electric field, but when $V(x') = a^2x'^2$ the microscopic particle performs localized vibration with a frequency of

2 α and an amplitude of ξ/α, the corresponding classical vibrational equation is $x' = x_0' \sin\omega t'$ with $\omega = 2a$ and $x_0' = \xi/\alpha$. The laws of motion of the center of mass of microscopic particles expressed by Equation (53) and Equations (58), (60) and (62) in the nonlinear quantum mechanics are consistent with the equations of motion of the macroscopic particles.

The correspondence between a microscopic particle and a macroscopic object shows that microscopic particles described by the nonlinear quantum mechanics have exactly the same moved laws and properties as classical particles. These results not only verify the necessity of development and correctness of the nonlinear quantum mechanics, but also exhibit clearly the limits and approximation of the linear quantum mechanics and can solve these difficulties of the linear quantum mechanics and problems of contention in it as described in Introduction. Therefore, the results mentioned above have important significances in physics and nonlinear science.

3. THE COLLISION PROPERTY OF MICROSCOPIC PARTICLES DESCRIBED BY NONLINEAR SCHRÖDINGER EQUATION

As a matter of fact, the properties of collision of soliton solutions of nonlinear Schrödinger Equations (5) at $b = 1 > 0$ and b < 0 firstly were analytically studied by Zakharov and Shabat[39-40] using inverse scattering method and Zakharov and Shabat equation. They found from calculation that the mass centre and phase of soliton occur only change after this collision at $V(x) = 0$ and $b = 1$. The translations of the mass centre x_{0m} and phase θ_m of mth soliton, which moves to a positive (or final) direction after this collision, can be represented, respectively, by

$$x_{0m}^+ - x_{0m} = \frac{1}{\eta_m} \prod_{p=m+1}^{N} \left|\frac{\lambda_m - \lambda_p}{\lambda_m - \lambda_p^*}\right| < 0, \text{ and}$$

$$\theta_m^+ - \theta_m = -2 \prod_{p=m+1}^{N} \arg\left(\frac{\lambda_m - \lambda_p}{\lambda_m - \lambda_p^*}\right)$$

(72)

Where η_m and λ_m are some constants related to the amplitude and eigenvalue of mth soliton, respectively. The Equation (72) shows that shift of position of mass centre of solitons and their variation of phase are constants after the collision of two solitons moving with different velocities and amplitudes. The collision process of the two solitons can be described from Equation (72) as follows. Before the collision and in the case of $t \to -\infty$ the slowest soliton is in the front while the fastest at the rear, they collide with each other at t' = 0, after the collision and $t \to \infty$, they are separated and their positions are just reversed. Zakharov and Shabat[39-40] obtained that as the time t varies from $-\infty$ to ∞, the relative change of mass

centre of two solitons, Δx_{0m}, and their relative change of phases, $\Delta \theta_m$, can, respectively, denoted as

$$\Delta x'_{0m} = x'^{+}_{0m} - x'^{-}_{0m} = \frac{1}{\eta_m} \left(\sum_{k=m+1}^{N} \ln \left| \frac{\lambda_m - \lambda_p}{\lambda_m - \lambda_p^*} \right| - \sum_{k=1}^{m-1} \ln \left| \frac{\lambda_m - \lambda_p}{\lambda_m - \lambda_p^*} \right| \right)$$

(73)

and

$$\Delta \theta_m = \theta_m^{+} - \theta_m^{-} = 2 \prod_{k=1}^{m-1} \arg \left(\frac{\lambda_m - \lambda_p}{\lambda_m - \lambda_p^*} \right) - 2 \prod_{k=m+1}^{N} \arg \left(\frac{\lambda_m - \lambda_p}{\lambda_m - \lambda_p^*} \right)$$

(74)

Where x_{0m}^{-} and phase θ_m^{-} are the mass centre and phase of mth particles at initial position, respectively. Equation (73) can be interpreted by assuming that the solitons collide pairwise and every soliton collides with others. In each paired collision, the faster soliton moves forward by an amount of $\eta_m^{-1} \ln \left| (\lambda_m - \lambda_k^*) / \lambda_m - \lambda_k \right|$, $\lambda_m > \lambda_k$, and the slower one shifts backwards by an amount of $\eta_k^{-1} \ln \left| (\lambda_m - \lambda_k^*) / \lambda_m - \lambda_k \right|$. The total shift is equal to the algebraic sum of their shifts during the paired collisions. So that there is not effect of multi-soliton collisions at all. In other word, in the collision process in each time the faster soliton moves forward by an amount of phase shift, and the slower one shifts backwards by an amount of phase. The total shift of the solitons is equal to the algebraic sum of those of the pair during the paired collisions. The situation is the same with the phases. This rule of collision of the solitons described by the nonlinear Schrödinger Equation (6) is the same as that of classical particles, or speaking, meet also the collision law of classical particles, i.e., during the collision these solitons interact and exchange their positions in the space-time trajectory as if they had passed through each other. After the collision, the two solitons may appear to be instantly translated in space and/or time but otherwise unaffected by their interaction. The translation is called a phase shift as mentioned above. In one dimension, this process results from two solitons colliding head-on from opposite directions, or in one direction between two particles with different amplitudes or velocities. This is possible because the velocity of a soliton depends on the amplitude. The two solitons surviving a collision completely unscathed demonstrate clearly the corpuscle feature of the solitons. This property separates the solitons described by the nonlinear Schrödinger Equation (6) from the microscopic particles in quantum mechanical regime. Thus this demonstrates the classical feature of the solitons.

At the same time, Desem et al.[56] and Tan et al.[57] pay attention to the features of the above solitons in collision process by Zakharov and Shabat's approach.

Zakharov and Shabat[39-40] also discussed analytically the properties of collision of two particles depicted by the nonlinear Schrödinger Equation (6) at b < 0 using Zakharov and Shabat equation. The result shows that the feature of collision of the two solitons in this case is basically similar with the above properties. In the meanwhile, Aossey et al.[25] investigated numerically the detailed structure, mechanism and rules of collision of the microscopic particles described by the nonlinear Schrödinger Equation (6) at b < 0 and obtained the rules of collision of the two solitons by a macroscopic model.

In fact, the properties of collisions of microscopic particles can be also obtained by numerically solving Equation (6). Numerical simulation can reveal more detailed feature of collision between two microscopic particles. We here studied numerically the features of collision of microscopic particles described by the nonlinear Schrödinger Equation (5) at b > 0 by fourth-order Runge-Kutta method[58-59]. For this purpose we began in one dimensional case by dividing Equation (5) into the following two-equations

$$i\hbar \frac{\partial \phi}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \phi}{\partial x^2} = \chi \phi \frac{\partial u}{\partial x},$$

(75)

$$M \left(\frac{\partial^2 u}{\partial t^2} - v_0 \frac{\partial^2 u}{\partial x^2} \right) = \chi \frac{\partial}{\partial x} \left(|\phi|^2 \right).$$

(76)

Equations (75)-(76) may be thought to describe the features of motion of studied particle and another particle (such as, phonon) or background field (such as, lattice) with mass M and velocity v_0, u is the characteristic quantity of another particle (such as, phonon) or of vibration (such as, displacement) of the background field. The coupling between the two modes of motion is caused by the deformation of the background field through the studied particle – background field coupling, such as, dipole-dipole interaction, χ is the coupling coefficient between them and represents the change of interaction energy between the studied particle and background field due to a unit variation of the background field. The relation between the two modes of motion due to their interaction can be represented by

$$\frac{\partial u}{\partial x} = \frac{\chi}{M(v^2 - v_0^2)} |\phi|^2 + A$$

(77)

If inserting Equation (77) into Equation (75) yields just the nonlinear Schrödinger Equation (5) at V(x) = constant, where $b = \dfrac{\chi^2}{M(v^2 - v_0^2)}$ is a nonlinear coupling coefficient, $V(x) = A\chi$, A is an integral constant. This investigation shows clearly that the nonlinear interaction $b|\phi|^2 \phi$ comes from the coupling interaction between two particles or the studied particle and background field. Very clearly, in this model the real motion of background field and its interaction with the particle are completely considered, instead of are replacement by an average field, which is fully different from that in quantum mechanics. Just

so, we can say that the nonlinear interaction $b|\phi|^2\phi$ or the nonlinear Schrödinger Equation (5) represent the real motions of particles and background field and their interactions, which are completely different from quantum mechanics. Therefore, we can believe that the nonlinear Schrödinger Equation (5) describe correctly the natures and properties of microscopic particles in the quantum mechanical systems.

In order to use fourth-order Runge-Kutta method[58-59] to solve numerically Equation (5) we must further discretize Equations (75) and (76), which can be denoted as

$$i\hbar\dot{\phi}_n(t) = \varepsilon\phi_n(t) - J[\phi_{n+1}(t)+\phi_{n-1}(t)] + (\chi/2r_0)[u_{n+1}(t)-u_{n-1}(t)]\phi_n(t) \tag{78}$$

$$M\ddot{u}_n(t) = W[u_{n+1}(t)-2u_n(t)+u_{n-1}(t)] + (\chi/2r_o)[|\phi_{n+1}|^2-|\phi_{n-1}|^2] \tag{79}$$

where the following transformation relation between continuous and discrete functions are used

$$\phi(x,t)\rightarrow\phi_n(t) \text{ and } u(x,t)\rightarrow u_n(t)$$

$$\phi_{n\pm1}(t) = \phi_n(t) \pm r_0\frac{\partial\phi_n(t)}{\partial x} + \frac{1}{2!}r_0^2\frac{\partial^2\phi_n(t)}{\partial x^2} \pm ....$$

$$u_{n\pm1}(t) = u_n(t) \pm r_0\frac{\partial u_n(t)}{\partial x} + \frac{1}{2!}r_0^2\frac{\partial^2 u_n(t)}{\partial x^2} \pm .... \tag{80}$$

Here $\varepsilon = \hbar^2/mr_0 + A\chi$, $J = \hbar^2/2mr_0^2$, $W = Mv_0^2/r_0^2$, r_0 is distance between neighbouring two lattice points. If using transformation: $\phi_n \rightarrow \phi_n\exp(i\varepsilon t/\hbar)$ we can eliminate the term $\varepsilon\phi_n(t)$ in Equation (78). Again making a transformation: $\phi_n(t)\rightarrow a_n(t) = a(t)r_n + ia(t)i_n$, then Equations (78)-(79) become

$$\hbar\dot{ar}_n = -J(ai_{n+1}+ai_{n-1}) + (\chi/2r_0)(u_{n+1}-u_{n-1})ai_n \tag{81}$$

$$-\hbar\dot{ai}_n = -J(ar_{n+1}+ar_{n-1}) + (\chi/2r_0)(u_{n+1}-u_{n-1})ar_n \tag{82}$$

$$\dot{u}_n = y^n/M \tag{83}$$

$$\dot{y}_n = W(u_{n+1}-2u_n+u_{n-1}) + (\chi/2r_o)(ar_{n+1}^2+ai_{n+1}^2-ar_{n-1}^2-ai_{n-1}^2) \tag{84}$$

$$|a_n|^2 = |ar_n|^2 + |ai_n|^2 = |\phi_n|^2 \tag{85}$$

Where ar_n and ai_n are real and imaginary parts of a_n. Equations (81)-(85) can determine states and behaviors of the microscopic particle. Their solutions can be found out, which are denoted in Appendix. There are four equations for one structure unit. Therefore, for the quantum systems constructed by N structure units there are 4N associated equations. When the fourth-order Runge-Kutta method[58-59] is used to numerically calculate the solutions of the above equations we should further discretize them. Thus the n is replaced by j and let the time be denoted by n, the step length of the space variable is denoted by h in the above equations. An initial excitation is required in this calculation, which is chosen as, $a_n(o) = A\text{sech}[(n-n_0)(\chi/2r_0)^2/4JW]$ (where A is the normalization constant) at the size n, for the applied lattice, $u_n(0) = y_n(0) = 0$. In the numerical simulation it is required that the total energy and the norm (or particle number) of the system must be conserved. The system of units, ev for energy, $\overset{o}{A}$ for length and ps for time are proven to be suitable for the numerical solutions of Equations (81)-(85). The one dimensional system is fixed, N is chosen to be N = 200, and a time step size of 0.0195 is used in the simulations. Total numerical simulation is performed through data parallel algorithms and MALAB language.

If the values of the parameters, M, ε, J, W, χ and r_0, in Equations (78)-(79) are appropriately chosen we can calculate the numerical solution of the associated Equations (81)-(85) by using the fourth-order Runge-Kutta method[26-27] in the systems, thus the changes of $|\phi_n(t)|^2 = |a_n(t)|^2$, which is probability or number density of the microscopic particles occurring at the nth structure unit, with increasing time and position in time-place can be also obtained. This result is shown in Figure 2. This figure shows that the amplitude of the solution can retain constancy, i.e., the solution of Equations (78)-(79) or Equation (5) at V(x) = constant is very stable while in motion. In the meanwhile, we give the propagation feature of the solutions of Equations (81)-(85) in the cases of a long time period of 250ps and long spacings of 400 in Figure 3, which indicates that the states of solution are also stable in the long propagation. According to the soliton theory[29-30] we can obtain that Equations (78)-(79) have exactly a soliton solution, thus the microscopic particles described by nonlinear Schrödinger Equations (5) are a soliton and have a wave-corpuscle feature.
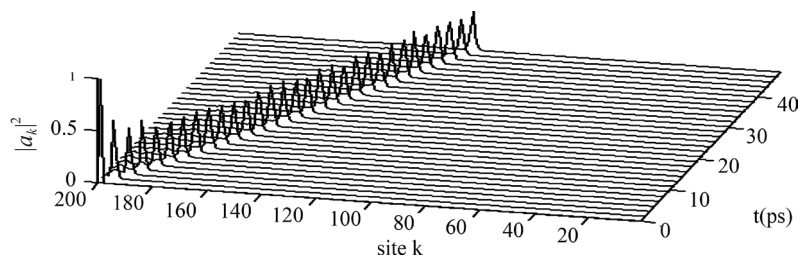

Figure 2
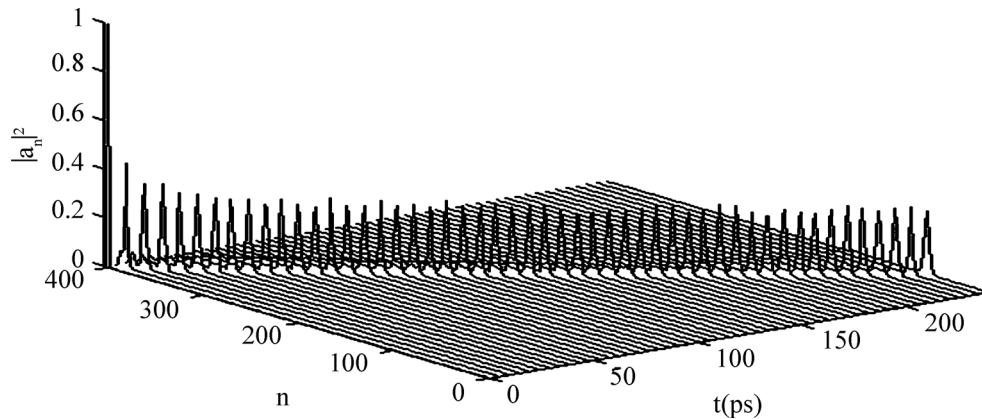**Motion of Soliton Solution of Equations (78)-(79)**

Figure 3
State of Motion of Microscopic Particle Described by Equations (78)-(79) in the Cases of a Long Time Period and Long Spacings

In order to verify the wave-corpuscle feature of microscopic particles described by nonlinear Schrödinger Equations (5) we should also study their collision property in accordance with the soliton theory[29-30]. Thus we further simulated numerically the collision behaviors of two particles described by nonlinear Schrödinger Equations (5) at $V(x) = \varepsilon = \hbar^2 / mr_0 + A\chi$ = constant using the fourth-order Runge-Kutta method[58-59]. This process resulting from two microscopic particles colliding head-on from opposite directions, which are set up from opposite ends of the channel, is shown in Figure 4, where the above initial conditions simultaneously motivate the opposite ends of the channels. From this Figure we see clearly that the initial two particles with clock shapes separating 50 unit spacings in the channel collide with each other at about 8ps and 25 units. After this collision, the two particles in the channel go through each other without scattering and retain their clock shapes to propagate toward and separately along itself channels. The collision properties of microscopic particles described by the nonlinear Schrödinger Equation (5) are same with those obtained by Zakharov and Shabat[39-40] and Asossey *et al.*[60] as mentioned above. Meanwhile, this collision is same with the rules of collision of macroscopic particles. Thus, we can conclude that microscopic particles described by nonlinear Schrödinger Equations (5) have a corpuscle feature.
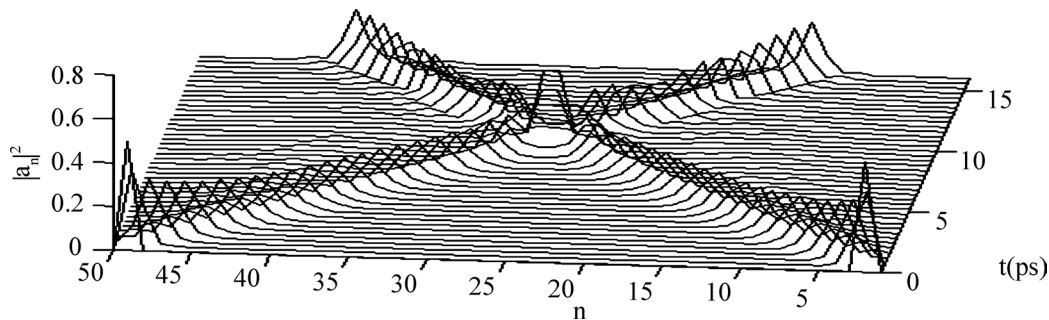

Figure 4
The Features of Collision of Microscopic Particles Described by Nonlinear Schrödinger Equation (5)

However, we see also from Figure 4 to occur a wave peak of large amplitude in the colliding process, which appears also in Desem and Chu's numerical result[56]. Obviously, this is a result of complicated superposition of two solitary waves, which thus displays the wave feature of the microscopic particles. Therefore, the collision process shown in Figure 4 represent clearly that the solutions of the nonlinear Schrödinger equation have a both corpuscle and wave feature, then the microscopic particles denoted by the solutions have also a wave-corpuscle duality, which is due to the nonlinear interaction $b|\phi|^2\phi$. Thus we can again determine the nonlinear Schrödinger equation can describe correctly the natures and properties of microscopic particles in quantum systems.

CONCLUSION

In this paper we used a nonlinear Schrödinger equation, instead of Schrödinger equation in quantum mechanics, to describe the properties of microscopic particles. In the dynamic equation the nonlinear interaction is denoted by $b|\phi|^2\phi$, which is caused by the interaction between

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the particle and other particles or background field in the case of their real motions to be considered. The nonlinear interaction can suppress the dispersive effect of kinetic energy in the dynamic equation and change also the natures of microscopic particles. Thus we investigated deeply the dynamic and collision features of microscopic particles described by nonlinear Schrödinger equation using the analytic and the Runge-Kutta method of numerical simulation through finding the soliton solutions of the equation. The results obtained show that the microscopic particles have a wave-corpuscle duality and are stable in propagation. When the two microscopic particles are collided, they can go through each other and retain their form after their collision of head-on from opposite directions, which is the same with that of the classical particles. However, a wave peak of large amplitude, which is a result of complicated superposition of two solitary waves, occurs in the colliding process. This displays the wave feature of microscopic particles. Therefore, the collision process shows clearly that the solutions of the nonlinear Schrödinger equation have a both corpuscle and wave feature, then the microscopic particles represented by the solutions have a wave-corpuscle duality. Clearly, this nature is due to the nonlinear interaction $b|\phi|^2\phi$. Thus we can determine the nonlinear Schrödinger equation can describe correctly the natures and properties of microscopic particles in quantum systems. This result has an important significance in physics. Then a new and nonlinear quantum mechanical theory can be established based on these results.

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APPENDIX

The Solutions of Equations **(81)-(85)**

From Equations (81)-(84) we can easily find out its solutions which are as follows

$$ar_{j,n+1} = ar_{jn} + \frac{h}{6}(K1_j + 2K2_j + 3K3_j + K4_j)$$

$$ai_{j,n+1} = ai_{jn} + \frac{h}{6}(L1_j + 2L2_j + 2L3_j + L4_j)$$

$$u_{j,n+1} = u_{jn} + \frac{h}{6}(M1 + 2M2_j + 2M3_j + M4_j)$$

$$y_{j,n+1} = y_{jn} + \frac{h}{6}(N1_j + 2N2_j + 2N3_j + N4_j)$$

where

$$K1_j = -\frac{J}{\hbar}(ai_{j+1} + ai_{j-1n}) + \frac{\chi}{2r_0\hbar}(u_{j+1n} - u_{j-1n})ai$$

$$K2_j = K1_j - \frac{Jh}{2\hbar}(L1_{j+1} + L1_{j-1}) + \frac{\chi h}{4r_0\hbar}(u_{j+1n} - u_{j-1n})L1_j + \frac{\chi h}{4r_0\hbar}(M1_{j+1} - M1_{j-1})(ai_{jn} + \frac{h}{2}L1_j)$$

$$K3_j = K1_j - \frac{Jh}{2\hbar}(L2_{j+1} + L2_{j-1}) + \frac{\chi h}{4r_0\hbar}(u_{j+1,n} - u_{j-1,n})L2_j + \frac{\chi h}{4r_0\hbar}(M2_{j+1} - M2_{j-1})(ai_{jn} + \frac{h}{2}L2_j)$$

$$K4_j = K1_j - \frac{Jh}{\hbar}(L3_{j+1} + L3_{j-1}) + \frac{\chi h}{2r_0\hbar}(u_{j+1,n} - u_{j-1,n})L3_j + \frac{\chi h}{2r_0\hbar}(M3_{j+1} - M3_{j-1})(ai_{jn} + hL3_j)$$

$$L1_j = \frac{J}{\hbar}(ar_{j+1,n} + ar_{j-1n}) - \frac{\chi}{2r_0\hbar}(u_{j+1n} - u_{j-1n})ar_{jn}$$

$$L2_j = K1_j + \frac{Jh}{2\hbar}(K1_{j+1} + K1_{j-1}) - \frac{\chi h}{4r_0\hbar}(u_{j+1n} - u_{j-1n})K1_j - \frac{\chi h}{4r_0\hbar}(M1_{j+1} - M1_{j-1})(ar_{jn} + \frac{h}{2}K1_j)$$

$$L3_j = L1_j + \frac{Jh}{2\hbar}(K2_{j+1} + K2_{j-1}) - \frac{\chi h}{4r_0\hbar}(u_{j+1,n} - u_{j-1,n})K2_j - \frac{\chi h}{4r_0\hbar}(M2_{j+1} - M2_{j-1})(ar_{jn} + \frac{h}{2}K2_j)$$

$$L4_j = K1_j + \frac{Jh}{\hbar}(K3_{j+1} + K3_{j-1}) - \frac{\chi_1 h}{2r_0\hbar}(u_{j+1,n} - u_{j-1,n})K3_j - \frac{\chi h}{2r_0\hbar}(M3_{j+1} - M3_{j-1})(ar_{jn} + hK3_j)$$

$$M1_j = y_{jn}/M ;$$

$$M2_j = M1_j + \frac{h}{2M}N1_j ;$$

$$M3_j = M1_j + \frac{h}{2M}N2_j ;$$

$$M4_j = M1_j + \frac{h}{M}N3_j ;$$

$$N1_j = W(u_{j+1,n} - 2u_{jn} + u_{j-1,n}) + (\chi/r_0)(ar_{j+1,n}^2 + ai_{j+1,n}^2 - ar_{j-1,n}^2 - ai_{j-1,n}^2)$$

$$N2_j = W[(u_{j+1,n} - 2u_{jn} + u_{j-1,n}) + \frac{h}{2}(M1_{j+1} - 2M1_j + M1_{j-1})]$$

$$+ (\chi/r_0)[(ar_{j+1,n} + \frac{h}{2}K1_{j+1})^2 + (ai_{j+1,n} + \frac{h}{2}L1_{j+1})^2 - (ar_{j-1,n} + \frac{h}{2}K1_{j-1})^2 - (ai_{j-1,n} + \frac{h}{2}L1_{j-1})^2]$$

$$N3_j = W[(u_{j+1,n} - 2u_{jn} + u_{j-1,n}) + \frac{h}{2}(M2_{j+1} - 2M2_j + M2_{j-1})]$$

$$+ (\chi/r_0)[(ar_{j+1,n} + \frac{h}{2}K2_{j+1})^2 + (ai_{j+1,n} + \frac{h}{2}L2_{j+1})^2 - (ar_{j-1,n} + \frac{h}{2}K2_{j-1})^2 - (ai_{j-1,n} + \frac{h}{2}L2_{j-1})^2]$$

$$N4_j = W[(u_{j+1,n} - 2u_{jn} + u_{j-1,n}) + h(M3_{j+1} - 2M3_j + M3_{j-1})]$$

$$+ (\chi/r_0)[(ar_{j+1,n} + hK3_{j+1})^2 + (ai_{j+1,n} + hL3_{j+1})^2 - (ar_{j-1,n} + hK3_{j-1})^2 - (ai_{j-1,n} - hL3_{j-1})^2]$$

These are just base equations we make numerical simulation by computer and Runge-Kutta method.