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Toward a Complete Wave-Particle Duality: Do Matter Waves Have Inertia?

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Abstract

A quaternionic mass defining longitudinal mass (m_ℓ) and transverse mass ($\vec{m}_t$) of a particle is introduced. The longitudinal and transverse masses represent the mass of the particle and matter wave accompanying it, respectively. A massive particle is found to have a bigger longitudinal mass than transverse mass. The particle nature due to the transverse mass of a particle is described by a scalar wave satisfying the Klein-Gordon equation. The transverse mass vector is found to be along the direction of the particle's velocity ($\vec{v}$). A particle of an equal longitudinal and transverse mass travels at speed of light in vacuum. The transmission of matter wave energy is found to be opposite to the particle velocity. The momentum of the particle ($m_\ell\vec{v}$) and its matter wave ($\vec{m}_t c$) are equal. The direction of the transverse mass is found to be along the direction of the particle's velocity.

Key words: Matter wave; Particle-wave duality; De Broglie hypothesis; Complex mass

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INTRODUCTION

In Newtonian mechanics an object exhibits resistance to a change in its state of motion or rest. This resistance or tendency of an object to resist any change in its motion is termed inertia. Thus understood, inertia is a property of the object. However, Mach conjectured that inertia is not an object property but a measure of how the object interacts with the surrounding[1]. Consequently, a single object in the Universe should not exhibit inertia. In quantum mechanics a physical object has a dual wave-particle nature. The object, however, doesn't show these two properties simultaneously. When it exhibits one nature, it no longer shows the other. Hence, quantum mechanically, an object is neither a pure wave nor it is a pure particle. Interestingly no inertia is assumed to arise when this wave propagates[2]. The wave notion of a particle is a consequence of the Einstein's energy relation, viz., $E=mc^2$, that underlies that mass-energy equivalence. Hence, it is only logical to assume that such a "matter" wave have inertia too. The matter wave inertia has not been dealt with in the streamlines of physics. This issue has been addressed by Arbab in a recent publication devoted to the analogy between electromagnetism and matter wave[3]. In the framework of the wave theory, inertia can be modeled as a dissipative process, in a similar fashion to frictional force in particle mechanics[4]. As a consequence of this dissipation, the particle wave undergoes a wavepacket behavior. This can be compared to a pure wave due to a massless particle which is a plane wave, e.g., photon.

In this work, we aim at elucidating the distinction between the wave due to a field and a wave due to a particle. The particle wave is described by a scalar function (ψ_0) as well as a vector function ($\vec{\psi}$) so that an inertia could be associated with the particle and its accompanying wave. The wave inertia is represented by the vector transverse mass ($\vec{m}_t$), whereas that due to a particle is described by the longitudinal mass (m_ℓ). This follows from the fact that matter wave assumes certain direction for its propagation. However, the wave due to a charge is transverse. Hence one may expect it not to have a longitudinal mass. This proposition allows a non-zero mass for the photon as suggested by Schrödinger[5]. Though the photon is assumed to have a zero rest mass, yet Einstein's energy relation associates a mass with it ($E=mc^2$). Using such a reasoning, one may build a one-to-one analogy between a particle wave and a field wave.

With this motivation, we intend in this work to extend the concept of inertia to be associated with "matter" waves in addition to particles. Considering only the longitudinal mass of a particle, Arbab has derived Schrödinger, Dirac and Klein-Gordon equations from a single quaternionic Dirac's equation[6]. In that formalism, mass is considered as a scalar quantity. However, we trust that the appropriate wave equation for such a description is the quaternionic Dirac's equation provided that the mass is assumed to be a quaternion. The present work explores this possibility. Within this formulation, the effective mass of a particle turns out to be a combination of its longitudinal and transverse masses. Treating the mass as a quaternion is shown to lead to proper explanation of the Compton and photoelectric effects[7].

EQUATION OF MOTION

The quaternionic eigenvalue Dirac's equation with quaternionic mass can be written as

$$\tilde{P}\,\tilde{\psi} = \tilde{M}c\,\tilde{\psi}, \qquad \tilde{P} = (i\frac{E}{c}, \vec{p}), \qquad \tilde{\psi} = (\frac{i}{c}\psi_0\,,\vec{\psi})\,. \tag{1}$$

Using the quaternionic algebra, where for two quaternions $\tilde{A} = (a_0, \vec{a})$ and $\tilde{B} = (b_0, \vec{b})$, one has

$$\tilde{A}\tilde{B} = \left(a_0 b_0 - \vec{a}\cdot\vec{b}\,, a_0\vec{b} + \vec{a}b_0 + \vec{a}\times\vec{b}\right). \tag{2}$$

We propose here that the mass of an object (particle/field) can be expressed as

$$\tilde{M} = (im_\ell, \vec{m}_t), \tag{3}$$

where m_ℓ is the longitudinal and $\vec{m}_t$ is the transverse masses, respectively. Using eq.(1), eq.(2) and (3) and equating the real and imaginary parts of the two sides of the resulting equations yield

$$\vec{\nabla}\cdot\vec{\psi} - \frac{1}{c^2}\frac{\partial\psi_0}{\partial t} = 0, \tag{4}$$

$$\vec{\nabla}\psi_0 - \frac{\partial\vec{\psi}}{\partial t} - \frac{c^2}{\hbar}\vec{m}_t\times\vec{\psi} = 0, \tag{5}$$

$$\vec{\nabla}\times\vec{\psi} + \frac{m_\ell c}{\hbar}\vec{\psi} + \frac{1}{\hbar}\vec{m}_t\psi_0 = 0, \tag{6}$$

and $$c\vec{m}_t\cdot\vec{\psi} + m_\ell\psi_0 = 0, \tag{7}$$

where we have used the fact that $E = i\hbar\frac{\partial}{\partial t}$ and $\vec{p} = -i\hbar\vec{\nabla}$.

Using the commutator brackets recently given by Arbab and Yassein[8, 9], eqs.(4) - (7) yield

$$\frac{1}{c^2}\frac{\partial^2\psi_0}{\partial t^2} - \nabla^2\psi_0 + \left(\frac{m_t^2c^2}{\hbar^2} - \frac{m_\ell^2c^2}{\hbar^2}\right)\psi_0 = 0. \tag{8}$$

It is evident from eq.(8) that the scalar function of the particle, ψ_0, satisfies the Klien-Gordon wave equation with a rest mass, $m_0 = (m_t^2 - m_\ell^2)^{\frac{1}{2}}$. It is worthnoting that as $|\vec{m}_t| = m_\ell$ for the photon, it has the known rest mass $m_0 = 0$. In fact eq.(1) and (2) can, together, be thought of as an equation of a massless particle satisfying the transformation $E \to E - m_\ell c^2$ and $\vec{p} \to \vec{p} - \vec{m}_t c$. Such a transformation is adopted in electrodynamics when a charged particle (q) interacts with an electromagnetic wave (photon). The corresponding transformation is called minimal substitution[10]. It is given by $\vec{p} \to \vec{p} - q\vec{A}$ and $E \to E - q\varphi$, where $\vec{A}$ and φ are the vector field, and scalar potential of the photon, respectively. The above transformation can be thought of as a particle interacting with constant field proportional to its masses.

Now taking the dot product of $\vec{m}_t$ with left the hand side of eq.(5) and differentiating eq.(7) partially with respect to time, one gets

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0, \qquad \vec{J} = \rho\vec{v}, \tag{9}$$

where

$$\rho = \psi_0, \qquad \vec{v} = \frac{\vec{m}_t}{m_\ell}c. \tag{10}$$

Equation (9) is the ordinary continuity equation relating the current (probability) density, $\vec{J}$, to the charge (probability) density, ρ. It is worthwhile to recall that in the ordinary quantum mechanics the probability density of the particle is linearly proportional to the square of the wavefunction. However, eq.(10) states that the probability density is linearly proportional to the particle's wavefunction. Interestingly, the transverse mass is parallel to the particle's velocity. A particle with $m_t = m_\ell$ travels at speed of light, c. The second part of eq.(10) can be seen as a momentum balance between the particle momentum and the wave momentum, *viz.*, $m_\ell\vec{v} = \vec{m}_t c$. Therefore, the particle and wave nature are concomitant in the sense that the momentum of the wave is equal to the momentum of the particle representing it. The longitudinal mass (m_ℓ) is the mass of the particle traveling at the speed $\vec{v}$, whereas $\vec{m}_t$ is the mass of the wave that is assumed to travel with speed of light. In special relativity, the relativistic mass of an object increases with velocity, but not linearly as in eq.(10). The relation in eq.(10) has been shown very recently to result from the interaction of a Klien-Gordon's particle interacting with a constant vector field that is proportional to the particle's velocity[11].

It is evident from eq.(8) that such a particle is described by the wave equation

$$\frac{1}{c^2}\frac{\partial^2\psi_0}{\partial t^2} - \nabla^2\psi_0 = 0. \tag{11}$$

Now apply eq.(10) in (7) to obtain

$$\psi_0 = -\vec{v}\cdot\vec{\psi}. \qquad (12)$$

This implies that the scalar field, ψ_0, propagates along the projection of the vector field, $\vec{\psi}$, and the particle's velocity. Now applying eq.(10) in eq.(4) yields

$$\vec{v} = -c^2\frac{\vec{\psi}}{\psi_0}. \qquad (13)$$

This implies that the particle field propagates with the same velocity as the particle but in opposite direction. Equation (13) is tantamount to a relativistic particle with a velocity given by

$$\vec{v} = c^2\frac{\vec{p}}{E}. \qquad (14)$$

Now multiply eq.(4) by ψ_0, take the dot product of $\vec{\psi}$ with eq.(5), and add the two resulting equations to obtain energy conservation equation

$$\vec{\nabla}\cdot\vec{S} + \frac{\partial u}{\partial t} = 0, \qquad (15)$$

where

$$\vec{S} = -\psi_0\vec{\psi}, \qquad u = \frac{1}{2}\left(\frac{\psi_0^2}{c^2} + \psi^2\right). \qquad (16)$$

Here $\vec{S}$ is the Poynting vector pointing along the direction of energy flow, and u is the energy density of the particle's wave. Apparently, the direction of the energy transmission is antiparallel to the particle's velocity as momentum conservation requires. Thus, the particle energy is transmitted along the vector matter field direction, $-\vec{\psi}$. Similar equations, as in eq.(15) and (16), have been obtained recently as a result of invariance of Lorenz gauge under space-time derivatives[4, 3,12]. Equation (15) is analogous to the energy conservation of an electromagnetic wave in free space (vacuum). For an electromagnetic wave in vacuum, one has the energy equation, *viz*.,

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}, \qquad u = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} + B^2\right). \qquad (17)$$

While the matter's energy is transmitted antiparallel to the field direction, the electromagnetic energy is transmitted in a direction normal to the fields. Thus, matter waves are longitudinal, whereas electromagnetic waves are transverse[3, 12].

CONCLUDING REMARKS

We presented here a full description of the particle-wave duality that was hypothesized by de Brogile in 1924. In de Broglie theory, the details of the matter wave nature were not specified. We made here a one-to-one correspondence between the particle dynamics and its de Broglie wave. We associated a vector transverse mass with the matter wave and a scalar longitudinal mass with the particle. We have shown that a particle with equal transverse and longitudinal mass travels with speed of light in vacuum. However, in standard wave theory only massless particle travels at speed of light.

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