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Prigogine's Dissipative Structures -- A Haimovician Analysis (Part II)

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Abstract

A system of decomposer organisms dissipating dead organic matter and parallel system of dead organic matter that contribute to the dissipation of the velocity of production of decomposer organisms is investigated. It is shown that the time independence of the contributions portrays another system by itself and constitutes the equilibrium solution of the original time independent system. A system of dead organic matter that reduces the dissipation coefficient of the decomposer organism is annexed to the oxygen consumption-terrestrial organism system towards the end of consummation of the system. With the methodology reinforced with the explanations, we write the governing equations with the nomenclature for the systems of plants-nutrients. Further papers extensively draw inferences upon such concatenation process, ipsofacto fait accompli.

Key words: Dissipative structures; Prigogine; Plants-nutrients; Dead organic matter; Decomposer organisms

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INTRODUCTION

In his celebrated paper Adolf Haimovici[1], studied the growth of a two species ecological system divided on age groups. In this paper, we establish that his processual regularities and procedural formalities can be applied for consummation of system of dead organic matter-decomposer organisms. Notations are changed towards the end of obtention of higher number of equations in the holistic study of the global climate models. Quintessentially, Haimovician diurnal dynamics was used in part 1 to draw interesting inferences, from the simple fact that terrestrial organisms consume oxygen due to cellular respiration.

Fritjof Capra[2] in his scintillating and brilliant synthesis of such scientific breakthroughs as the "Theory of Dissipative structures", 'Theory of complexity', 'Gaia theory', 'Chaos theory' in his much acclaimed 'The Web of life' elucidates dissipative structures as the new paradigm in ecology.

Heylighen F.[3] also concretises the necessity of self-organization and adaptability. Matsuit *et al.*[4] made a satellite based assessment of marine low cloud variability, atmospheric stability and diurnal cycle. Steven's B., Feingold G.[5] studied untangling aerosol effects on clouds and precipitation in a buffered system. Illan koren and Graham Feingold[6] studied the aerosol cloud precipitation system. One other study that eminently calls for such a study of application is by R. Wood[7] in which he studied the loss of cloud droplets by coalescence in warm clouds. On the same lines the investigation of Xue H., Fiengold G. where in indirect effects of aerosol on large eddy simulations of trade wind provides a rich repository and fertile ground for prosecution of investigation based on our theoretical analysis. Aerosol effects on clouds itself is a pointer to the food cycle - dissipative structure discussed by Prigogine.

At the centre of prigogines vision lies the coexistence of structure and change from being to becoming. It also is the principal frontier of stillness and motion.

To quote Prigogine in extensor "each great period of science has led to some model of nature. For classical science it was clock, for nineteenth century science the period of industrial revolutions, it was an engine running down. We believe that this will give our period its uniqueness"[11].

A living organism is characterized by continual flow and change in its metabolism, involving thousands of chemical reactions. Chemical and thermal equilibrium shall be extant and existential when all these process come to a halt. To put it differently, an organism in equilibrium is a dead organism. A state apart and far away from equilibrium is accomplished continually in a living organism, notwithstanding changes in components, the overall structure remain unaltered. Long range correlations appear at the precise point of transition from equilibrium to non equilibrium, and from that point systems behavioral pattern would be as if it is a holistic entity.

Far away from the point at which equilibrium is redeemed, the systems flow processes are entangled through multiple feedback loops. Thus towards the end of ameliorating and alleviating the non linearity in dissipative structures , a process of concatenation is resorted to that reduces the mathematics of complexity, and makes the equations amenable and tenable for deliberation and discussion.

All the studies centre on the possibility of application of Haimovician and Volterrian analysis to "dissipative structures". in this paper we study the following systems:

(a) Dead organic matter-Decomposer organisms (DOM-DO)

(b) Green plants-Nutrients

We elucidate the governing equations of (b). Methodology for obtention of Solution follows from the one herein given

In the next part we analyze the following systems:

(c) Solar radiation-Chemical process

(d) Systems structure-Change

Green plants play a vital role in the flow of energy through all ecological cycles. Their roots take in water and mineral salts from the earth, and the resultant juices rise up to the leaves, where they combine with CO_2 from air leading to the formulation of sugar and other organic compounds. Here solar energy is converted into chemical energy and encapsulated in organic substances, while oxygen is released in air to be taken up again by other plants and by animals in the process of cellular respiration. By the blend of water and minerals with sunlight and CO_2, green plants form link between earth and sky. Bulk of cellulose and the other organic compounds produced through photosynthesis consists of heavy carbon and oxygen atoms, which plants take directly from the air in the form of CO_2. Thus the weight of a wooden log comes almost entirely from air. A log burnt, combines oxygen and carbon combine once more in to CO_2, and in the light and heat of fire is recovered part of the solar energy that went into making the wood.

As terrestrial organisms dissipate oxygen in the atmosphere, due to cellular respiration the plants nutrients are passed through the food web, while energy is dissipated as heat through respiration and as waste through excretion. Dead animals and plants are disintegrated by decomposer organisms, which break them into basic nutrients to be taken up by plants. Nutrients and other basic elements continually cycle through the ecological system, while energy is dissipated at each stage in accord with Eugene Odum's dictum "matter circulates, energy dissipates". Waste generated by the ecological system as a whole is the heat energy of cellular respiration, which is radiated into the atmosphere and is reimbursed continually by photosynthesis.

Prigogine's theory interlinks and entangles the main characteristics of living forms in to a coherent, cogent conceptualization and mathematical framework. We give a model for his framework. Perhaps the most fundamental necessity of the systemic dynamics is the optimality considerations. Taking cognizance of the critical issues involved emphasizes need for setting out dynamic programming in order to capture systemic structural changes.

Axiomatic predications of systemic dynamics in question are essentially "laws of accentuation and dissipation". It includes once over change, continuing change, process of change, functional relationships, predictability, cyclical growth, cyclical fluctuations, speculation theory, cobweb analyses, stagnation thesis, perspective analysis etc.. Upshot of the above statement is data produce consequences and consequences produce data.

DEAD ORGANIC MATTER (DOM)

Assumptions

a) Dead organic matter are classified into three categories;
    1) Category 1 representative of the dead organic matter in the first interval vis-à-vis category 1 of terrestrial organisms (TO) and DO
    2) Category 2 (second interval) dead organic matter corresponding to category 2 of terrestrial organisms and DO
    3) Category 3 constituting dead organic matter which belong to higher age than that of category 1 and category 2.This is concomitant to category 3 of terrestrial organism and DO

In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the terrestrial organisms made out of the total quantum due to dead organic matter would be in the fitness of things, For category 3. "Over and above" nomenclature could be used to encompass a wider range of dead organic matter. Similarly, a "less than scale" for category 1 can be used.

b) The speed of growth of dead organic matter under category 1 is proportional to the total amount of dead organic matter under category 2. In essence the accentuation coefficient in the model is representative of the constant of proportionality between consumption due dead organic matter under category 1 and category 2 this assumptions is made to foreclose the necessity of addition of one more variable, that would render the systemic equations unsolvable

c) The dissipation in all the three categories is attributable to the following two phenomenon :
   1) **Aging phenomenon:** The aging process leads to transference of the balance of DOM vis-à-vis oxygen consumption due to cellular respiration to the next category, no sooner than the age of the terrestrial organism crosses the boundary of demarcation
   2) **Depletion phenomenon:** Disintegration of dead organic matter by decomposer organism dissipates the growth speed by an equivalent extent. The model is not concerned with the end uses of consumption due DOM-DO vis-à-vis to cellular respiration – dissipation other than for terrestrial organisms

Notation

G_{16} : Quantum of Dead Organic Matter (DOM) in category 1 of terrestrial organism vis-à-vis DO.
G_{17} : Quantum of dead organic matter in category 2 of terrestrial organism vis-à-vis DO.
G_{18} : Quantum of dead organic matter in category 3 of terrestrial organism vis-à-vis DO.
$(a_{16})^{(2)}$, $(a_{17})^{(2)}$, $(a_{18})^{(2)}$: Accentuation coefficients.
$(a'_{16})^{(2)}$, $(a'_{17})^{(2)}$, $(a'_{18})^{(2)}$: Dissipation coefficients.

Formulation of the System

In the light of the assumptions stated in the foregoing, we infer the following:

(a) The growth speed in category 1 is the sum of a accentuation term $(a_{16})^{(2)}G_{17}$ and a dissipation term $(a'_{16})^{(2)}G_{16} > 0$, the amount of dissipation taken to be proportional to the total quantum of dead organic matter in the concomitant category of terrestrial organisms and DO.

(b) The growth speed in category 2 is the sum of two parts $(a_{14})^{(2)}G_{16}$ and $-(a'_{17})^{(2)}G_{17}$ the inflow from the category 1 dependent on the total amount standing in that category.

(c) The growth speed in category 3 is equivalent to $(a_{18})^{(2)}G_{17}$ and $-(a'_{18})^{(2)}G_{18}$ dissipation ascribed only to depletion phenomenon.

Model makes allowance for the new quantum of dead organic matter due to new entrants in terrestrial organisms and deceleration in the oxygen consumption (OC) attributable and ascribable to death of terrestrial organisms leading to accentuation of dead organic matter

Governing Equations

The differential equations governing the above system can be written in the following form.

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - (a'_{16})^{(2)}G_{16} \quad (1)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - (a'_{17})^{(2)}G_{17} \quad (2)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - (a'_{18})^{(2)}G_{18} \quad (3)$$

$$(a_i)^{(2)} > 0 \quad , \quad i = 16,17,18 \quad (4)$$

$$(a'_i)^{(2)} > 0 \quad , \quad i = 16,17,18 \quad (5)$$

$$(a_{17})^{(2)} < (a'_{16})^{(2)} \quad (6)$$

$$(a_{18})^{(2)} < (a'_{17})^{(2)} \quad (7)$$

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{16}}{(a_{16})^{(2)}G_{17} - (a'_{16})^{(2)}G_{16}} = dt \quad (8)$$

$$\frac{dG_{17}}{(a_{17})^{(2)}G_{16} - (a'_{17})^{(2)}G_{17}} = dt \quad (9)$$

Or we write a single equation as

$$\frac{dG_{16}}{(a_{16})^{(2)}G_{17} - (a'_{16})^{(2)}G_{16}} = \frac{dG_{17}}{(a_{17})^{(2)}G_{13} - (a'_{17})^{(2)}G_{17}} = \frac{dG_{18}}{(a_{18})^{(2)}G_{17} - (a'_{18})^{(2)}G_{18}} = dt \quad (10)$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α, β, γ all positive we can write equation (10) as

$$\frac{\alpha dG_{16}}{\alpha\left((a_{16})^{(2)}G_{17} - (a'_{16})^{(2)}G_{16}\right)} = \frac{\beta dG_{17}}{\beta\left((a_{17})^{(2)}G_{16} - (a'_{17})^{(2)}G_{17}\right)} = \frac{\gamma dG_{18}}{\gamma\left((a_{18})^{(2)}G_{17} - (a'_{18})^{(2)}G_{18}\right)} = dt \quad (11)$$

The general solution of the DOM-DO consummated with consumption of oxygen due to cellular respiration – terrestrial organism system can be written in the form $\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t}$ Where i=16,17,18 and C_{16}, C_{17}, C_{18} are arbitrary constant coefficients.

Stability Analysis

Supposing $G_i(0) = G_i^0(0) > 0$, and denoting by $(\lambda_i)^{(2)}$

the characteristic roots of the system, it easily results that
1. If $(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} > 0$ all the components of the solution, i.e all the three parts of the DOM-DO vis-à-vis consumption of oxygen due to cellular respiration tend to zero, and the solution is stable with respect to the initial data.
2. If $(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ and $\left((\lambda_{17})^{(2)} + (a'_{16})^{(2)}\right)G^0_{16} - (a_{16})^{(2)}G^0_{17} \neq 0, \left((\lambda_{17})^{(2)} < 0\right)$, the first two components of the solution tend to infinity as $t \to \infty$, and $G_{18} \to 0$, i.e. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 dead organic matter tends to zero.
3. If $(a'_{13})^{(2)}(a'_{14})^{(2)} - (a_{13})^{(2)}(a_{14})^{(2)} < 0$ and $\left((\lambda_{17})^{(2)} + (a'_{16})^{(2)}\right)G^0_{16} - (a_{16})^{(2)}G^0_{17} = 0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i, the corresponding solution tends to infinity.

Actual food cycles can be understood on a much broader canvass, in which nutrient elements appear in a variety of chemical compounds.gia theory, has refined indicating interweaving of living and non living systems throughout the biosphere. Key to comprehension of such dissipative structures is that these systems maintain themselves in a "stable state" far from equilibrium. For instance chemical and thermal equilibrium exists when all these processes come to a halt. Organism in equilibrium is a dead organism. Living organisms, like terrestrial organisms, continually maintain themselves in a state far from equilibrium. Notwithstanding the fact, that such a maintained state is stable over a period of time, the same overall holistic structure is maintained, despite continual ongoing flow and change of components.

Prigogine realized that classical thermodynamics is not the appropriate tool to explain systems far from equilibrium, owing to the fact mathematical structure is linear. Close on the heels to equilibrium, there will be "fluxes", "vortices", however weak nevertheless. System shall evolve towards a stationary state in which generation of "entropy" (disorder) is as small as possible. By implication, there shall be a minimization problem mathematically, around the equilibrium state. In and around this range, linear equation would explain the characteristics of the system.

On the other hand, away from "equilibrium", the "fluxes" are more emphasized. Result is increase in "entropy". When this occurs, the system no longer tends towards equilibrium. On the contrary, it may encounter instabilities that culminate into newer orders that move away from equilibrium. Thus, dissipative structures revitalize and resurrect complex forms away from equilibrium state.

From the above analysis we infer the following:

1. The adjustment process is stable in the sense that the system of DOM vis-à-vis oxygen consumption converges to equilibrium.

2.The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point

3. Conditions 1 and 2 are independent of the size and direction of initial disturbance

4. The actual shape of the time path of dead organic matter by the DO vis-à-vis terrestrial organism is determined by efficiency parameter, the strength of the response of the portfolio in question, and the initial disturbance

5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold

6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question, in the present case DOM-DO vis-à-vis terrestrial organisms – oxygen consumption-dead organic matter available for decomposer organisms

7. Some authors Nober F J, Agee, Winfree were interested in such questions, whether growing system could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine strato cumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

DECOMPOSER ORGANISM

Assumptions

Decomposer organisms are classified into three categories analogous to the stratification that was resorted to in dead organic matter sector. When decomposer organisms in a particular category is transferred to the next sector, (such transference is attributed to the aging process of decomposer organisms), dead organic matter from that category apparently would have become qualified for classification in the corresponding category, because we are in fact classifying DO vis-à-vis terrestrial organisms based on stratification of dead organic matter vis-à-vis terrestrial organism.

(1) Category 1 is representative of decomposer organisms corresponding to dead organic matter under category 1.
(2) Category 2 constitutes those decomposer organisms whose age is higher than that specified under the head category 1 and is in correspondence with the similar classification of DOM vis-à-vis oxygen consumption due to cellular respiration, of DO vis-à-vis terrestrial organism in corresponding category.
(3) Category 3 of decomposer organisms encompasses

those decomposer organisms with respect to category 3 of DOM vis-à-vis oxygen Consumption due to cellular respiration of terrestrial organisms and DO vis-à-vis terrestrial organism in corresponding category.

It is assumed for the sake of simplicity that amount of oxygen taken in water is slowly divided into that of utilization due to terrestrial organisms, Cellular respiration, clouds, etc..

a) The speed of growth of DO vis-à-vis terrestrial organism sector in category 1 is a linear function of the amount of DO vis-à-vis terrestrial organism sector in category 2 at the time of reckoning. As before the accentuation coefficient that characterizes the speed of growth in category 1 is the proportionality factor between balance in category 1 and category 2.

   The dissipation coefficient in the growth model is attributable to two factors;

   1. With the progress of time DO vis-à-vis terrestrial organism sector gets aged and become eligible for transfer to the next Category. Notwithstanding Category 3 does not have such a provision for further transference
   2. DO vis-à-vis Terrestrial organism sector when become irretrievable(dead from which no cells can be obtained) are the other outlet that decelerates the speed of growth of DO vis-à-vis terrestrial organism sector

b) Inflow into category 2 is only from category 1 in the form of transfer of balance of DOM vis-à-vis OC due to terrestrial organism sector from the category 1.This is evident from the age wise classification scheme. As a result, the speed of growth of category 2 is dependent upon the amount of inflow, which is a function of the quantum of balance of terrestrial organism sector under the category 1.

c) The balance of DO vis-à-vis terrestrial organism sector in category 3 is because of transfer of balance from category 2. It is dependent on the amount of terrestrial organism sector under category 2, as also DOM thereof.

Notation

T_{16} : Balance standing in the category 1 of DO vis-à-vis terrestrial organism and DOM vis-à-vis OC

T_{17} : Balance standing in the category 2 of DO vis-à-vis terrestrial organism and DOM vis-à-vis OC

T_{18} : Balance standing in the category 3 of DO vis-à-vis terrestrial organism and DOM vis-à-vis OC

$(b_{16})^{(2)}$, $(b_{17})^{(2)}$, $(b_{18})^{(2)}$: Accentuation coefficients

$(b'_{16})^{(2)}$, $(b'_{17})^{(2)}$, $(b'_{18})^{(2)}$: Dissipation coefficients

Formulation of the System

Under the above assumptions, we derive the following :

a) The growth speed in category 1 is the sum of two parts:

1. A term $(b_{16})^{(2)}T_{17}$ proportional to the amount of balance of DO vis-à-vis terrestrial organisms in the category 2 of DO vis-à-vis OC

2.A term $-(b'_{16})^{(2)}T_{16}$ representing the quantum of balance dissipated from category 1 .This comprises of terrestrial organisms which have grown old qualifiable to be classified under category 2 and loss of terrestrial organisms due to death of terrestrial organism (dead organic matter- for concatenated equations see end of the paper) and concomitant DO.

b) The growth speed in category 2 is the sum of two parts:

1.A term $(b_{17})^{(2)}\ T_{16}$ constitutive of the amount of inflow from the category 1.

2.A term $-(b'_{17})^{(2)}T_{17}$ the dissipation factor arising due to aging of terrestrial organism Corresponding DOM and the oxygen saved on account of death of terrestrial organisms.

c) The growth speed under category 3 is attributable to inflow from category 2 and DO vis-à-vis oxygen consumption stalled irrevocably and irretrievable due to death of the terrestrial organisms, and hence cannot deplete oxygen quantum in the atmosphere due to cellular respiration any further.

Governing Equations

Following are the differential equations that govern the growth in the DO vis-à-vis terrestrial organisms portfolio.

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - (b'_{16})^{(2)}T_{16} \tag{12}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - (b'_{17})^{(2)}T_{17} \tag{13}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - (b'_{18})^{(2)}T_{18} \tag{14}$$

$$(b_i)^{(1)} > 0 \quad , \quad i = 13,14,15 \tag{15}$$

$$\left(b'_i\right)^{(2)} > 0 \quad , \quad i = 16,17,18 \tag{16}$$

$$(b_{17})^{(2)} < (b'_{16})^{(2)} \tag{17}$$

$$(b_{18})^{(2)} < (b'_{17})^{(2)} \tag{18}$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is $\alpha'_i T_i + \beta'_i T_i + \gamma'_i T_i = C'_i e_i{}^{\lambda'_i t}$, $i = 16, 17, 18$ where $C'_{16}, C'_{17}, C'_{18}$ are arbitrary constant coefficients and $\alpha'_{16}, \alpha'_{17}, \alpha'_{18}, \gamma'_{16}, \gamma'_{17}, \gamma'_{18}$ corresponding multipliers to the characteristic roots of the DOM vis-à-vis terrestrial organism system.

Dom – Do Vis-À-Vis Oxygen Consumption (Oc) Due To Cellular Respiration – Terrestrial Organism(To) – Dual System Analysis

In the previous section, we studied the growth of DOM-DO concatenated with oxygen consumption due to cellular respiration and terrestrial organisms separately. In this section, we study the two-portfolio model comprising six-storey DOM-DO vis-à-vis oxygen consumption due to cellular respiration and terrestrial organisms. Scheme of age wise classification however remains the same. We make an explicit assumption that only category 2 of DO vis-à-vis terrestrial organisms is responsible for the increase in the dissipation coefficient of the DOM vis-à-vis oxygen consumption due to cellular respiration. DO vis-à-vis Terrestrial organisms of three categories dissipating three portfolios of DOM vis-à-vis oxygen consumption due to cellular respiration levels follows by mere substitution of corresponding variables. Dissipation coefficients of the DO vis-à-vis terrestrial organisms portfolio are diminished by the contribution of all three categories of DOM vis-à-vis oxygen consumption due to cellular respiration portfolio of terrestrial organisms vis-à-vis DO. This is to facilitate circumvention of the nonlinearity of the equations and consequent unsolvability We will denote

1) By $T_i\ (t)$,i=16,17,18 , the three parts of the DO vis-à-vis terrestrial organisms system analogously to the G_i of the DOM vis-à-vis consumption of oxygen due to cellular respiration
2) By $\left(a_i''\right)^{(2)}(T_{17},t)\ \ (T_{17}\geq 0, t\geq 0)$,the contribution of the DO vis-à-vis terrestrial organisms to the dissipation coefficient of the DOM vis-à-vis oxygen consumption due to cellular respiration of terrestrial organisms vis-à-vis DO
3) B y $\left(-b_i''\right)^{(2)}(G_{16}\ ,G_{17}\ ,G_{18},t)=-\left(b_i''\right)^{(2)}\left((G_{19})\ ,t\ \right)$, the contribution of the DOM vis-à-vis consumption of oxygen due to cellular respiration to the dissipation coefficient of the DO vis-à-vis terrestrial organisms

Dead Organic Matter (DOM) - Decomposer Organism (DO) System Governing Equations

The differential system of this model is now

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[(a_{16}')^{(2)}+(a_{16}'')^{(2)}(T_{17},t)\right]G_{16} \tag{19}$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[(a_{17}')^{(2)}+(a_{17}'')^{(2)}(T_{17},t)\right]G_{17} \tag{20}$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[(a_{18}')^{(2)}+(a_{18}'')^{(2)}(T_{17},t)\right]G_{18} \tag{21}$$

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[(b_{16}')^{(2)}-(b_{16}'')^{(2)}\left((G_{19}),t\right)\right]T_{16} \tag{22}$$

$$\frac{dT_{17}}{dt}=(b_{17})^{(2)}T_{16}-\left[(b_{17}')^{(2)}-(b_{17}'')^{(2)}\left((G_{19}),t\right)\right]T_{17} \tag{23}$$

$$\frac{dT_{18}}{dt}=(b_{18})^{(2)}T_{17}-\left[(b_{18}')^{(2)}-(b_{18}'')^{(2)}\left((G_{19}),t\right)\right]T_{18} \tag{24}$$

$+(a_{16}'')^{(2)}(T_{17},t)$ = First augmentation factor attributable to DOM vis-à-vis cellular respiration of DO vis-à-vis terrestrial organism, to the dissipation of oxygen consumption due to disintegration of DOM by DO.

$-(b_{16}'')^{(2)}\left((G_{19}),t\right)$= First detrition factor contributed by DOM vis-à-vis oxygen consumption to the dissipation of DO vis-à-vis terrestrial organisms.

Where we suppose

(A) $(a_i)^{(2)}$, $(a'_i)^{(2)}$, $(a''_i)^{(2)}$, $(b_i)^{(2)}$, $(b'_i)^{(2)}$, $(b''_i)^{(2)}>0$

i, j = 16, 17, 18

(B) The functions $(a''_i)^{(2)}$,$(b''_i)^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a''_i)^{(2)}(T_{17},t)\leq (p_i)^{(2)}\leq (\hat{A}_{16})^{(2)} \tag{25}$$

$$(b''_i)^{(2)}(G,t)\leq (r_i)^{(2)}\leq (b'_i)^{(2)}\leq (\hat{B}_{16})^{(2)} \tag{26}$$

$$\text{(C)}\ \lim_{T_2\to\infty}\left(a_i''\right)^{(2)}(T_{17},t)=(p_i)^{(2)} \tag{27}$$

$$\lim_{G\to\infty}\left(b_i''\right)^{(2)}\left((G_{19}),t\right)=(r_i)^{(2)} \tag{28}$$

Definition of $(\hat{A}_{16})^{(2)}$,$(\hat{B}_{16})^{(2)}$:

Where $(\hat{A}_{16})^{(2)}$,$(\hat{B}_{16})^{(2)}$, $(p_i)^{(2)}$, $(r_i)^{(2)}$ are positive constants and (i =16,17,18). They satisfy Lipschitz condition:

$$|(a''_i)^{(2)}(T_{17},t)-(a''_i)^{(2)}(T_{17},t)|\leq (\hat{k}_{16})^{(2)}|T_{17}-T'_{17}|e^{-(\hat{M}_{16})^{(2)}t} \tag{29}$$

$$|(b''_i)^{(2)}((G_{19})',t)-(b''_i)^{(2)}((G_{19}),T_{19})|<(\hat{k}_{16})^{(2)}\|(G_{19})-(G_{19})'\|e^{-(\hat{M}_{16})^{(2)}t} \tag{30}$$

With the Lipschitz condition, we place a restriction on the behavior of functions$(a''_i)^{(2)}(T'_{17},t)$ and $(a''_i)^{(2)}(T_{17},t)$. (T'_{17},t) and (T_{17},t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a''_i)^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)}=1$ then the function $(a''_i)^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to DO vis-à-vis terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}$, $(\hat{k}_{16})^{(2)}$:

(D) $(\hat{M}_{16})^{(2)}$, $(\hat{k}_{16})^{(2)}$, are positive constants (31)

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\ ,\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}<1$$

Definition of $(\hat{P}_{13})^{(2)}$, $(\hat{Q}_{13})^{(2)}$:

(E) There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ and which together with $(\widehat{M}_{16})^{(2)}$, $(\widehat{k}_{16})^{(2)}$, $(\widehat{A}_{16})^{(2)}$, and $(\widehat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}$, $(a'_i)^{(2)}$, $(b_i)^{(2)}$, $(b'_i)^{(2)}$, $(p_i)^{(2)}$, $(r_i)^{(2)}$, i = 16, 17, 18, satisfy the inequalities

$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}[(\mathrm{a_i})^{(2)} + (\mathrm{a}_i')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{\mathrm{k}}_{16})^{(2)}] < 1 \qquad (32)$$

$$\frac{1}{(\widehat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\widehat{B}_{16})^{(2)} + (\widehat{Q}_{16})^{(2)}(\widehat{k}_{16})^{(2)}] < 1 \qquad (33)$$

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \le (\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \le (\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $T_i(0) = T_i^0 > 0$

Proof

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\widehat{P}_{16})^{(2)}$, $T_i^0 \le (\widehat{Q}_{16})^{(2)}$ (34)

$0 \le G_i(t) - G_i^0 \le (\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ (35)

$0 \le T_i(t) - T_i^0 \le (\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ (36)

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a_{16}')^{(2)} + a_{16}'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)} \qquad (37)$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)} \qquad (38)$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)} \qquad (39)$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b_{16}')^{(2)} - (b_{16}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)} \qquad (40)$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b_{17}')^{(2)} - (b_{17}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)} \qquad (41)$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b_{18}')^{(2)} - (b_{18}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)} \qquad (42)$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying 34, 35, 36 into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\widehat{P}_{16})^{(6)} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\widehat{P}_{16})^{(2)}}{(\widehat{M}_{16})^{(2)}} \left(e^{(\widehat{M}_{16})^{(2)} t} - 1\right) \qquad (43)$$

From which it follows that (44)

$$(G_{16}(t) - G_{16}^0) e^{-(\widehat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[\left((\widehat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\widehat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\widehat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for G_{17}, G_{18}, T_{16}, T_{17}, T_{18}

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose $(\widehat{P}_{16})^{(2)}$ and $(\widehat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[(\widehat{P}_{16})^{(2)} + \left((\widehat{P}_{16})^{(2)} + G_j^0\right) e^{-\left(\frac{(\widehat{P}_{16})^{(2)} + G_j^0}{G_j^0}\right)}\right] \le (\widehat{P}_{16})^{(2)} \qquad (45)$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[\left((\widehat{Q}_{16})^{(2)} + T_j^0\right) e^{-\left(\frac{(\widehat{Q}_{16})^{(2)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{16})^{(2)}\right] \le (\widehat{Q}_{16})^{(2)} \qquad (46)$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34, 35, 36 into itself. The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right), \left((G_{19})^{(2)}, (T_{19})^{(2)}\right)\right) = \sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\widehat{M}_{16})^{(2)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\widehat{M}_{16})^{(2)} t}\} \qquad (47)$$

Indeed if we denote

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: (48)

$$\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$$

It results

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a_{16}')^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} + \tag{49}$$

$$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval [0, t]. From the hypotheses on 25, 26, 27, 28 and 29 it follows

$$|(G_{19})^{(1)} - (G_{19})^{(2)}|e^{-(\widehat{M}_{16})^{(2)} t} \leq \frac{1}{(\widehat{M}_{16})^{(2)}} ((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}) d(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})) \tag{50}$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis (34, 35, 36) the result follows

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$, i =16,17,18 depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ (52)

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of

$\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$:

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18}. indeed if (53)

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows

$\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16}, G_{18} and G_{16}, G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18}. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. (54)

Remark 5: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. (55)

Definition of $(m)^{(2)}$ and ε_2:

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, \; T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} t$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if

$$\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42.

Behavior of the solutions of equation 37 to 42 (56)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$:

(a) $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a_{16}')^{(2)} + (a_{17}')^{(2)} - (a_{16}'')^{(2)}(T_{17}, t) + (a_{17}'')^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)} \tag{57}$$

$$-(\tau_2)^{(2)} \leq -(b_{16}')^{(2)} + (b_{17}')^{(2)} - (b_{16}'')^{(2)}\left((G_{19}), t\right) - (b_{17}'')^{(2)}\left((G_{19}), t\right) \leq -(\tau_1)^{(2)} \tag{58}$$

Definition of $(\nu_1)^{(2)} (\nu_2)^{(2)} (u_1)^{(2)} (u_2)^{(2)}$:

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$, the roots (b) of the equations

$$(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0 \tag{60}$$

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (61)

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$:

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ theroots of the equations

$$(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0 \tag{62}$$

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ (63)

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$: (64)

(c) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ (65)

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$,

and $(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ (66)

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***If*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ (67)

and analogously $(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$,

if $(u_0)^{(2)} < (u_1)^{(2)}$ (68)

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$, and

$(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}$ (69)

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ (70)

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{16}^0 e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t} \tag{71}$$

$(p_i)^{(2)}$ is defined by equation 25

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \tag{72}$$

$$\left(\frac{(a_{18})^{(2)}G_{16}^0}{(m_1)^{(2)}\left((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)}\right)}\left[e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq \frac{(a_{18})^{(2)}G_{16}^0}{(m_2)^{(2)}\left((S_1)^{(2)} - (a_{18}')^{(2)}\right)}\left[e^{(S_1)^{(2)}t} - e^{-(a_{18}')^{(2)}t}\right] + G_{18}^0 e^{-(a_{18}')^{(2)}t}\right) \tag{73}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t}} \tag{74}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t} \tag{75}$$

$$\frac{(b_{18})^{(2)}T_{16}^0}{(\mu_1)^{(2)}\left((R_1)^{(2)} - (b_{18}')^{(2)}\right)}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b_{18}')^{(2)}t} \leq$$

$$T_{18}(t) \leq \frac{(a_{18})^{(2)}T_{16}^0}{(\mu_2)^{(2)}\left((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)}\right)}\left[e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$ (78)

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)}$ (79)

$(R_2)^{(2)} = (b_{18}')^{(2)} - (r_{18})^{(2)}$

Proof : From 19,20,21,22,23,24 we obtain

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\nu^{(2)} = \frac{G_{16}}{G_{17}}$

It follows

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \tag{80}$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:

(a) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}},$$

$$(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}} \tag{81}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}$$

$$(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}} \tag{82}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$

(b) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, (83)

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)} \tag{84}$$

(c) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain

$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)} \quad (85)$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}} \quad (86)$$

In a completely analogous way, we obtain

Definition of $u^{(2)}(t)$:

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}} \quad (87)$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

Particular case:

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$, and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if, in addition, $(\nu_0)^{(2)} = (\nu_1)^{(2)}$, then $(\nu_0)^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$. Analogously if, and then $(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if, in addition, $(u_0)^{(2)} = (u_1)^{(2)}$then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$. This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$.

Stationary Solutions And Stability

Stationary solutions and stability curve representative of the variation of DOM vis-à-vis oxygen consumption due to cellular respiration of terrestrial organism's vis-à-vis that of DO vis-à-vis terrestrial organism variation curve lies below the tangent at $(G_{19})=(G_{19})_0$ for $(G_{19})<(G_{19})_0$ and above the tangent for $(G_{19})>(G_{19})_0$ .Wherever such a situation occurs the point $(G_{19})_0$ is called the "point of inflexion". In this case, the tangent has a positive slope that simply means the rate of change of DOM vis-à-vis oxygen consumption due to cellular respiration is greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

Stationary value :

In all the cases $(G_{19})=(G_{19})_0$, $(G_{19})<(G_{19})_0$, $(G_{19})>(G_{19})_0$ the condition that the rate of change of DOM vis-à-vis oxygen consumption is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

1. A necessary and sufficient condition for there to be stationary value of (G_{19}) is that the rate of change of DOM vis-à-vis oxygen consumption function at $(G_{19})_0$ is zero.
2. A sufficient condition for the stationary value at $(G_{19})_0$, to be maximum is that the acceleration of the DOM vis-à-vis oxygen consumption of TO vis-à-vis DOM system is less than zero.
3. A sufficient condition for the stationary value at $(G_{19})_0$, be minimum is that acceleration of DOM vis-à-vis oxygen consumption of TO vis-à-vis DOM system is greater than zero.
4. With the rate of change of (G_{19}) namely DOM vis-à-vis oxygen consumption defined as the accentuation term and the dissipation term, we are sure that the rate of change of DOM vis-à-vis oxygen consumption is always positive.
5. Concept of stationary state is mere methodology although there might be closed system exhibiting symptoms of stationariness.

We can prove the following

Theorem 3: If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ and are independent on t , and the conditions (with the notations 25,26,27,28)

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad (88)$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0 \ ,$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

$with$ $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation 25 are satisfied , then the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad (89)$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad (90)$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad (91)$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad (92)$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad (93)$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad (94)$$

has a unique positive solution, which is an equilibrium solution for (19 to 24).

Proof:

(1) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0 \quad (95)$$

Definition and uniqueness of T^*_{17}:

After hypothesis $f(0)<0$, $f(\infty)>0$ and the functions $(a''_i)^{(2)}(T_{19})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(\mathrm{T}^*_{17}) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}\mathrm{G}_{17}}{\left[(a'_{16})^{(2)}+(a''_{16})^{(2)}(\mathrm{T}^*_{17})\right]} ,$$

$$G_{18} = \frac{(a_{18})^{(2)}\mathrm{G}_{17}}{\left[(a'_{18})^{(2)}+(a''_{18})^{(2)}(\mathrm{T}^*_{17})\right]} \tag{96}$$

(b) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - \left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0 \tag{97}$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis φ(0)>0 ,φ(∞) <0 it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$

Finally we obtain the unique solution of 89 to 94 G^*_{17} given by $\varphi((G_{19})^*) = 0$, T^*_{17} given by $f(\mathrm{T}^*_{17}) = 0$ and

$$\mathrm{G}^*_{16} = \frac{(\mathrm{a}_{16})^{(2)}\mathrm{G}^*_{17}}{\left[(\mathrm{a}'_{16})^{(2)}+(\mathrm{a}''_{16})^{(2)}(\mathrm{T}^*_{17})\right]} ,$$

$$\mathrm{G}^*_{18} = \frac{(\mathrm{a}_{18})^{(2)}\mathrm{G}^*_{17}}{\left[(\mathrm{a}'_{18})^{(2)}+(\mathrm{a}''_{18})^{(2)}(\mathrm{T}^*_{17})\right]} \tag{98}$$

$$\mathrm{T}^*_{16} = \frac{(\mathrm{b}_{16})^{(2)}\mathrm{T}^*_{17}}{\left[(\mathrm{b}'_{16})^{(2)}-(\mathrm{b}''_{16})^{(2)}((G_{19})^*)\right]} ,$$

$$\mathrm{T}^*_{18} = \frac{(\mathrm{b}_{18})^{(2)}\mathrm{T}^*_{17}}{\left[(\mathrm{b}'_{18})^{(2)}-(\mathrm{b}''_{18})^{(2)}((G_{19})^*)\right]} \tag{99}$$

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24.

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(\mathrm{a}''_i)^{(2)}$ and $(\mathrm{b}''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:

$$\mathrm{G}_i = \mathrm{G}^*_i + \mathbb{G}_i \quad , \mathrm{T}_i = \mathrm{T}^*_i + \mathbb{T}_i \tag{100}$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial \mathrm{T}_{17}}(\mathrm{T}^*_{17}) = (q_{17})^{(2)} ,$$

$$\frac{\partial(b''_i)^{(2)}}{\partial \mathrm{G}_j}\left((G_{19})^* \right) = s_{ij} \tag{101}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \tag{102}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \tag{103}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \tag{104}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{\mathrm{j}=16}^{18}\left(\mathrm{s}_{(16)(\mathrm{j})}\mathrm{T}^*_{16}\mathbb{G}_{\mathrm{j}}\right) \tag{105}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \tag{106}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \tag{107}$$

The characteristic equation of this system is

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right) \tag{108}$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right) (\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

More often than not, models begin with the assumption of 'steady state' and then proceed to trace out the path, which will be followed when the steady state is subjected to some kind of exogenous disturbance. Breathing pattern of terrestrial organisms is another parametric representation to be taken into consideration. It cannot be taken for granted that the sequence generated in this manner will tend to equilibrium i.e. a traverse from one steady state to another.

In our model, we have used the tools and techniques by Haimovici, Levin, Volttera, Lotka; have brought out implications of steady state, stability, asymptotic stability, behavioral aspects of the solution without any such assumptions, such as those mentioned in the fore going.

In the following, we give equations for the "dead organic matter-decomposer organism-terrestrial organism-oxygen consumption" system. Solutions and sine-qua-non theoretical aspects are dealt in the next paper (part II).

GOVERNING EQUATIONS

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13} \tag{1a}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14} \tag{2a}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15} \tag{3a}$$

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - (b'_{13})^{(1)}T_{13} \tag{4a}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - (b'_{14})^{(1)}T_{14} \tag{5a}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - (b'_{15})^{(1)}T_{15} \tag{6a}$$

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - (a'_{16})^{(2)}G_{16} \tag{7a}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - (a'_{17})^{(2)}G_{17} \tag{8a}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - (a'_{18})^{(2)}G_{18} \tag{9a}$$

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - (b'_{16})^{(2)}T_{16} \tag{10a}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - (b'_{17})^{(2)}T_{17} \tag{11a}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - (b'_{18})^{(2)}T_{18} \tag{12a}$$

Nutrients

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - (a'_{20})^{(3)}G_{20} \tag{13a}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - (a'_{21})^{(3)}G_{21} \tag{14a}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - (a'_{22})^{(3)}G_{22} \tag{15a}$$

Plants

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - (b'_{20})^{(3)}T_{20} \tag{16b}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - (b'_{21})^{(3)}T_{21} \tag{17b}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - (b'_{22})^{(3)}T_{22} \tag{18a}$$

GOVERNING EQUATIONS OF DUAL CONCATENATED SYSTEMS

Terrestrial Organisms - Oxygen Consumption System

$(-b_i'')^{(1)}(G_{13}, G_{14}, G_{15}, t) = -(b_i'')^{(1)}(G, t)$ i =13,14,15 the contribution of the consumption of oxygen due to cellular respiration to the dissipation coefficient of the terrestrial organisms.

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]\,G_{13} \tag{19a}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]\,G_{14} \tag{20a}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]\,G_{15} \tag{21a}$$

Where $+(a''_{13})^{(1)}(T_{14}, t)$, $+(a''_{14})^{(1)}(T_{14}, t)$, $+(a''_{15})^{(1)}(T_{14}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism.

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]\,T_{13} \tag{22a}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]\,T_{14} \tag{23a}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]\,T_{15} \tag{24a}$$

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

Dead Organic Matter - Decomposer Organism System

$(-b_i'')^{(2)}(G_{16}, G_{17}, G_{18}, t) = -(b_i'')^{(2)}(G_{19}, t)$, i=16,17,18 the contribution of the decomposer organism for the disintegration of dead organic matter.

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17}, t)}\right]G_{16} \tag{25a}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17}, t)}\right]G_{17} \tag{26a}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17}, t)}\right]G_{18} \tag{27a}$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19}, t)}\right]T_{16} \tag{28a}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19}, t)}\right]T_{17} \tag{29a}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19}, t)}\right]T_{18} \tag{30a}$$

Where $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter.

Nutrients–Plant System

$(-b_i'')^{(3)}(G_{20}, G_{21}, G_{22}, t) = -(b_i'')^{(3)}(G_{23}, t)$, i=20,21,22 the contribution of the

Nutrients

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21}, t)}\right]G_{20} \tag{31a}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21}, t)}\right]G_{21} \tag{32a}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21}, t)}\right]G_{22} \tag{33a}$$

Where $\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3 due to plants dissipating nutrients

Plants

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23}, t)}\right]T_{20} \tag{34a}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23}, t)}\right]T_{21} \tag{35a}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23}, t)}\right]T_{22} \tag{36a}$$

Where $\boxed{-(b''_{20})^{(1)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(1)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(1)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3 due to plants consuming nutrients

GOVERNING EQUATIONS OF CONCATENATED SYSTEM OF TWO CONCATENATED DUAL SYSTEMS

Terrestrial Organisms - Dead Organic Matter System

Dead Organic Matter Dissipates Terrestrial Organism

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) - (a''_{13})^{(1,1)}(T_{14}, t)]\ G_{16} \quad (37a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) - (a''_{14})^{(1,1)}(T_{14}, t)]\ G_{17} \quad (38a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) - (a''_{15})^{(1,1)}(T_{14}, t)]\ G_{18} \quad (39a)$$

Where $+ (a''_{16})^{(2)}(T_{17}, t)$, $+ (a''_{17})^{(2)}(T_{17}, t)$, $+ (a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism $- (a''_{13})^{(1,1)}(T_{14}, t)$, $- (a''_{14})^{(1,1)}(T_{14}, t)$, $- (a''_{15})^{(1,1)}(T_{14}, t)$ are second detrition coefficients for category 1, 2 and 3 due to terrestrial organisms.

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t) + (b''_{16})^{(2,2)}(G_{19}, t)]\ T_{13} \quad (40a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t) + (b''_{17})^{(2,2)}(G_{19}, t)]\ T_{14} \quad (41a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t) + (b''_{18})^{(2,2)}(G_{19}, t)]\ T_{15} \quad (42a)$$

Where $- (b''_{13})^{(1)}(G, t)$, $- (b''_{14})^{(1)}(G, t)$, $- (b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption $+ (b''_{16})^{(2,2)}(G_{19}, t)$, $+ (b''_{17})^{(2,2)}(G_{19}, t)$, $+ (b''_{18})^{(2,2)}(G_{19}, t)$ are second augmentation coefficients for category 1, 2 and 3 due to dead organic matter.

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)}\right] G_{13} \quad (43a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)}\right] G_{14} \quad (44a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)}\right] G_{15} \quad (45a)$$

Where $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism.

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)}\right] T_{16} \quad (46a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)}\right] T_{17} \quad (47a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)}\right] T_{18} \quad (48a)$$

Where $- (b''_{16})^{(2)}(G_{19}, t)$, $- (b''_{17})^{(2)}(G_{19}, t)$, $- (b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter.

Decomposer Organisms Dissipates Nutrients

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) + (a''_{16})^{(2,2)}(T_{17}, t)]\ G_{20} \quad (49a)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t) + (a''_{17})^{(2,2)}(T_{17}, t)]\ G_{21} \quad (50a)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t) + (a''_{17})^{(2,2)}(T_{17}, t)]\ G_{22} \quad (50a)$$

$+ (a''_{20})^{(3)}(T_{21}, t)$, $+ (a''_{21})^{(3)}(T_{21}, t)$, $+ (a''_{22})^{(3)}(T_{21}, t)$ are first augmentation coefficients for category 1, 2 and 3.

$+ (a''_{16})^{(2,2)}(T_{17}, t)$, $+ (a''_{17})^{(2,2)}(T_{17}, t)$, $+ (a''_{17})^{(2,2)}(T_{17}, t)$ are second augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) - (b''_{20})^{(3,3)}(G_{23}, t)]\ T_{16} \quad (52a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) - (b''_{21})^{(3,3)}(G_{23}, t)]\ T_{17} \quad (53a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{16} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) - (b''_{22})^{(3,3)}(G_{23}, t)]\ T_{18} \quad (54a)$$

$- (b''_{16})^{(2)}(G_{19}, t)$, $- (b''_{17})^{(2)}(G_{19}, t)$, $- (b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter. $- (b''_{20})^{(3,3)}(G_{23}, t)$, $- (b''_{21})^{(3,3)}(G_{23}, t)$, $- (b''_{22})^{(3,3)}(G_{23}, t)$ are second detrition coefficients for category 1, 2 and 3.

PLANTS DISSIPATE OXYGEN CONSUMPTION

Oxygen Consumption

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) + (a''_{20})^{(3,3)}(T_{21}, t)]\ G_{13} \quad (55a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} -$$

$$[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) + (a''_{21})^{(3,3)}(T_{21}, t)]\ G_{14} \quad (56a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} -$$

$$[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) + (a''_{22})^{(3,3)}(T_{21}, t)]\ G_{15} \quad (57a)$$

Where $+ (a''_{13})^{(1)}(T_{14}, t)$, $+ (a''_{14})^{(1)}(T_{14}, t)$, $+ (a''_{15})^{(1)}(T_{14}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism. $+(a''_{20})^{(3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3)}(T_{21}, t)$ are second augmentation coefficients for category 1, 2 and 3.

Plants

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} -$$

$$\left[(b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23}, t)}\boxed{-(b''_{13})^{(1,1)}(G, t)}\right]T_{20} \quad (58a)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} -$$

$$\left[(b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23}, t)}\boxed{-(b''_{14})^{(1,1)}(G, t)}\right]T_{21} \quad (59a)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} -$$

$$\left[(b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23}, t)}\boxed{-(b''_{15})^{(1,1)}(G, t)}\right]T_{22} \quad (60a)$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3 due to

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1)}(G, t)}$ are second detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

Nutrients

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21}, t)}\right]G_{20} \quad (61a)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21}, t)}\right]G_{21} \quad (62a)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21}, t)}\right]G_{22} \quad (63a)$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3.

OXYGEN CONSUMPTION-DECOMPOSER ORGANISM SYSTEM

Decomposer Organism Dissipates Oxygen Consumption

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} -$$

$$\left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19}, t)}\boxed{-(b''_{13})^{(1,1)}(G, t)}\right]T_{16} \quad (64a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} -$$

$$\left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19}, t)}\boxed{-(b''_{14})^{(1,1)}(G, t)}\right]T_{17} \quad (65a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} -$$

$$\left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19}, t)}\boxed{-(b''_{15})^{(1,1)}(G, t)}\right]T_{18} \quad (66a)$$

Where $-(b''_{16})^{(2)}(G_{19}, t)$, $-(b''_{17})^{(2)}(G_{19}, t)$, $-(b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter. $-(b''_{13})^{(1,1)}(G, t)$, $-(b''_{14})^{(1,1)}(G, t)$, $-(b''_{15})^{(1,1)}(G, t)$ are second detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} -$$

$$\left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14}, t)}\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}\right]G_{13} \quad (67a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} -$$

$$\left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14}, t)}\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}\right]G_{14} \quad (68a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} -$$

$$\left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14}, t)}\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}\right]G_{15} \quad (69a)$$

Where $(a''_{13})^{(1)}(T_{14}, t)$, $(a''_{14})^{(1)}(T_{14}, t)$, $(a''_{15})^{(1)}(T_{14}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism. $+(a''_{16})^{(2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2)}(T_{17}, t)$, are second augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17}, t)}\right]G_{16} \quad (70a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17}, t)}\right]G_{17} \quad (71a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17}, t)}\right]G_{18} \quad (72a)$$

Where $+ (a''_{16})^{(2)}(T_{17}, t)$, $+ (a''_{17})^{(2)}(T_{17}, t)$, $+ (a''_{18})^{(2)}(T_{17}, t)$, are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G, t)}\right]T_{13} \quad (73a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G, t)}\right]T_{14} \quad (74a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G, t)}\right]T_{15} \quad (75a)$$

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$, are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

TERESTRIAL ORGANISMS DISSIPATES PLANTS

Terestrial Organisms

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{-(b''_{20})^{(3,3)}(T_{23},t)}\right] T_{13} \quad (76a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{-(b''_{21})^{(3,3)}(G_{23},t)}\right] T_{14} \quad (77a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{22})^{(3,3)}(G_{23},t)}\right] T_{15} \quad (78a)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$, are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption. $-(b''_{20})^{(3,3)}(T_{23},t)$, $-(b''_{21})^{(3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3)}(G_{23},t)$ are second detrition coefficients for category 1, 2 and 3 due to green plants.

Plants

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{20} - [(b'_{20})^{(1)} - (b''_{20})^{(3)}(G_{23},t) - (b''_{14})^{(1,1)}(G,t)]\, T_{20} \quad (79a)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(1)} - (b''_{21})^{(3)}(G_{23},t) - (b''_{14})^{(1,1)}(G,t)]\, T_{21} \quad (80a)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{20} - [(b'_{22})^{(1)} - (b''_{22})^{(3)}(G_{23},t) - (b''_{14})^{(1,1)}(G,t)]\, T_{22} \quad (81a)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ are first detrition coefficients for category 1, 2 and 3 due to green plants. $-(b''_{13})^{(1,1)}(G,t)$, $-(b''_{14})^{(1,1)}(G,t)$, $-(b''_{15})^{(1,1)}(G,t)$, are second detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

DECOMPOSER ORGANISM DISSIPATES OXYGEN CONSUMPTION

Terrestrial Organisms Dissipates Dead Organic Matter

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}\right] G_{16} \quad (82a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}\right] G_{17} \quad (83a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}\right] G_{18} \quad (84a)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$, are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism. And $+(a''_{13})^{(1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1)}(T_{14},t)$, are second augmentation coefficient for category 1, 2 and 3 due to terrestrial organisms.

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\right] T_{13} \quad (85a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\right] T_{14} \quad (86a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\right] T_{15} \quad (87a)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ are second detrition coefficient for category 1, 2 and 3 due to dead organic matter.

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\right] G_{13} \quad (88a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\right] G_{14} \quad (89a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\right] G_{15} \quad (90a)$$

Where $+\,(a''_{13})^{(1)}(T_{14},\, t)$, $+\,(a''_{14})^{(1)}(T_{14},\, t)$, $+\,(a''_{15})^{(1)}(T_{14},\, t)$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism. $+\,(a''_{16})^{(2,2,2)}(T_{17},\, t)$, $+\,(a''_{17})^{(2,2,2)}(T_{17},\, t)$, $+\,(a''_{18})^{(2,2,2)}(T_{17},\, t)$, are second augmentation coefficient for category 1, 2 and 3 due to decomposer organism.

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \boxed{-(b''_{13})^{(1,1,1)}(G,t)} \right] T_{16} \tag{91a}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \boxed{-(b''_{14})^{(1,1,1)}(G,t)} \right] T_{17} \tag{92a}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \boxed{-(b''_{15})^{(1,1,1)}(G,t)} \right] T_{18} \tag{93a}$$

where $-\,(b''_{16})^{(2)}(G_{19},\, t)$, $-\,(b''_{17})^{(2)}(G_{19},\, t)$, $-\,(b''_{18})^{(2)}(G_{19},\, t)$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter. $-\,(b''_{13})^{(1,1,1)}(G,\, t)$, $-\,(b''_{14})^{(1,1,1)}(G,\, t)$, $-\,(b''_{15})^{(1,1,1)}(G,\, t)$ are second detrition coefficients for category 1,2 and 3 due to oxygen consumption.

PLANTS DISSIPATE OXYGEN CONSUMPTION

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \right] G_{13} \tag{94a}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \right] G_{14} \tag{95a}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \right] G_{15} \tag{96a}$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism.

$+\,(a''_{16})^{(2,2,2)}(T_{17},\, t)$, $+\,(a''_{17})^{(2,2,2)}(T_{17},\, t)$, $+\,(a''_{18})^{(2,2,2)}(T_{17},\, t)$, are second augmentation coefficient for category 1, 2 and 3 due to decomposer organism.

$+\,(a''_{20})^{(3,3,3)}(T_{21},\, t)$, $+\,(a''_{21})^{(3,3,3)}(T_{21},\, t)$, $+\,(a''_{22})^{(3,3,3)}(T_{21},\, t)$, are third augmentation coefficient for category 1, 2 and 3.

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - \left[(b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23},t)} \boxed{-(b''_{13})^{(1,1)}(G,t)} \boxed{-(b''_{13})^{(1,1,1)}(G,t)} \right] T_{20} \tag{97a}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - \left[(b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23},t)} \boxed{-(b''_{14})^{(1,1)}(G,t)} \boxed{-(b''_{14})^{(1,1,1)}(G,t)} \right] T_{21} \tag{98a}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - \left[(b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23},t)} \boxed{-(b''_{15})^{(1,1)}(G,t)} \boxed{-(b''_{15})^{(1,1,1)}(G,t)} \right] T_{22} \tag{99a}$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ are first detrition coefficients for category 1, 2 and 3.

$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3.

$\boxed{-(b''_{13})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G,t)}$ are third detrition coefficients for category 1, 2 and 3.

TERESTRIAL ORGANISMS DISSIPATES NUTRIENTS

Terestrial Organisms

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3)}(G_{23},t)} \right] T_{13} \tag{100a}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3)}(G_{23},t)} \right] T_{14} \tag{101a}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3)}(G_{23},t)} \right] T_{15} \tag{102a}$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3 due to nutrients.

$\boxed{-(b''_{20})^{(3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3)}(G_{23},t)}$ are second detrition coefficients for category 1, 2 and 3 due to green plants.

Nutrients

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - \left[(a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21},t)} \right.$$

$$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\Big]G_{20} \tag{103a}$$

$$\frac{dG_{21}}{dt}=(a_{21})^{(3)}G_{20}-\Big[(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}$$

$$\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\Big]G_{21} \tag{104a}$$

$$\frac{dG_{22}}{dt}=(a_{22})^{(3)}G_{21}-\Big[(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}$$

$$\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\Big]G_{22} \tag{105a}$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to nutrients.

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3 due to decomposer organism. $+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3 due to terrestrial organisms.

PLANTS DISSIPATE DEAD ORGANIC MATTER

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\Big[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}$$

$$\boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}\Big]G_{16} \tag{106a}$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\Big[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}$$

$$\boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}\Big]G_{17} \tag{107a}$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\Big[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}$$

$$\boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}\Big]G_{18} \tag{108a}$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

And $\boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3 due to terrestrial organisms.

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3 due to nutrients.

$$\frac{dT_{20}}{dt}=(b_{20})^{(3)}T_{21}-\Big[(b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}$$

$$\boxed{-(b''_{13})^{(1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}\Big]T_{20} \tag{109a}$$

$$\frac{dT_{21}}{dt}=(b_{21})^{(3)}T_{20}-\Big[(b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}$$

$$\boxed{-(b''_{14})^{(1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}\Big]T_{21} \tag{110a}$$

$$\frac{dT_{22}}{dt}=(b_{22})^{(3)}T_{21}-\Big[(b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}$$

$$\boxed{-(b''_{15})^{(1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}\Big]T_{22} \tag{111a}$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ are first detrition coefficients for category 1, 2 and 3 due to green plants.

$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3 due to terrestrial organisms. $-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$, are third detrition coefficients for category 1, 2 and 3 due to decomposer organisms.

DECOMPOSER ORGANISMS DISSIPATES NUTRIENTS

$$\frac{dG_{20}}{dt}=(a_{20})^{(3)}G_{21}-\Big[(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}$$

$$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\Big]G_{20} \tag{112a}$$

$$\frac{dG_{21}}{dt}=(a_{21})^{(3)}G_{20}-\Big[(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}$$

$$\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\Big]G_{21} \tag{113a}$$

$$\frac{dG_{22}}{dt}=(a_{22})^{(3)}G_{21}-\Big[(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}$$

$$\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\Big]G_{22} \tag{114a}$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$

are first augmentation coefficients for category 1, 2 and 3 due to dead organic matter.

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$

are second augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{16})^{(2,2,2,2)}(T_{18},t)$, are second augmentation coefficients for category 1, 2 and 3 due to dead organic matter.

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\Big[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}$$

$$\boxed{-(b''_{13})^{(1,1,1)}(G,t)}\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\Big]T_{16} \quad (115a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \Big[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}$$

$$\boxed{-(b''_{14})^{(1,1,1)}(G,t)}\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\Big]T_{17} \quad (116a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \Big[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}$$

$$\boxed{-(b''_{15})^{(1,1,1)}(G,t)}\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}\Big]T_{18} \quad (117a)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$, are first detrition coefficients for category 1, 2 and 3 due to dead organic matter.

$\boxed{-(b''_{13})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G,t)}$

are second detrition coefficients for category 1,2 and 3 due to oxygen consumption. $\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$,

$\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3 due to green plants.

TERESTRIAL ORGANISMS DISSIPATE OXYGEN CONSUMPTION

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \Big[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}$$

$$\boxed{+(a''_{20})^{(3,3)}(T_{21},t)}\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\Big]G_{13} \quad (118a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \Big[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}$$

$$\boxed{+(a''_{21})^{(3,3)}(T_{21},t)}\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\Big]G_{14} \quad (119a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \Big[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}$$

$$\boxed{+(a''_{22})^{(3,3)}(T_{21},t)}\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\Big]G_{15} \quad (120a)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism

$\boxed{+(a''_{20})^{(3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3)}(T_{21},t)}$

are second augmentation coefficients for category 1, 2 and 3 due to green plants. $\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$,

$\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3 due to terrestrial organisms.

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \Big[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}$$

$$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}\Big]T_{13} \quad (121a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \Big[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}$$

$$\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}\Big]T_{14} \quad (122a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \Big[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}$$

$$\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}\Big]T_{15} \quad (123a)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$

are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$

are second detrition coefficients for category 1, 2 and 3 due to decomposer organism.

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are third detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

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