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Hawking Radiation - An Augmentation Attrition Model

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Abstract

A system of Hawking radiation that dissipates the energy and mass of black hole is investigated. With the methodology reinforced with the explanations, we write the governing equations with the nomenclature for the system of matter antimatter and concatenate those equations with that of energy and mass of Black hole and Hawking radiation, such concatenation process, ipso facto fait accompli, following the same processual formalities for solvability.

Key words: Hawking radiation; Black body; Fundamental forces; Energy; Mass; Matter; Antimatter; Black hole; Governing equations

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INTRODUCTION

Hawking radiation is black body radiation that is predicted to be emitted by black holes, due to quantum effects near the event horizon. It has non-zero temperature and entropy. Alexei Starobinsky showed that according to the quantum mechanical uncertainty principle, rotating black holes should create and emit particles. Hawking radiation reduces the mass and the energy of the black hole. Shrink and ultimately vanish. Micro black holes (MBHs) are predicted to be larger net emitters of radiation than larger black holes and should shrink and dissipate faster. Black holes are sites of immense gravitational attraction. Classically, the gravitation is so powerful that nothing, not even electromagnetic radiation, can escape from the black hole. It is yet unknown how gravity can be incorporated into quantum mechanics, nevertheless, far from the black hole the gravitational effects can be weak enough for calculations to be reliably performed in the framework of quantum field theory in curved space-time. Hawking showed that quantum effects allow black holes to emit exact black body radiation, which is the average thermal radiation emitted by an idealized thermal source known as a black body. The electromagnetic radiation is as if it were emitted by a black body with a temperature that is inversely proportional to the black hole's mass. Physical insight into the process may be gained by imagining that particle-antiparticle radiation is emitted from just beyond the event horizon. This radiation does not come directly from the black hole itself, but rather is a result of virtual particles being "boosted" by the black hole's gravitation into becoming real particles. A slightly more precise, but still much simplified, view of the process is that vacuum fluctuations cause a particle-antiparticle pair to appear close to the event horizon of a black hole. One of the pair falls into the black hole whilst the other escapes. In order to preserve total energy, the particle that fell into the black hole must have had a negative energy (with respect to an observer far away from the black hole). By this process, the black hole loses mass, and, to an outside observer, it would appear that the black hole has just emitted a particle. In another model, the process is a quantum tunneling effect, whereby particle-antiparticle pairs will form from the vacuum, and one will tunnel outside the event horizon. Thus thermal radiation contains information about the Hawking radiation body that emitted it, while Hawking radiation seems to contain no such information, and depends only on the mass, angular momentum, and charge of the black hole (the no-hair theorem). This leads to the black hole information. However, according to the conjectured gauge-gravity duality (also, black holes in certain cases (and perhaps in general)) are equivalent to solutions of quantum field theory at a non-zero temperature. This means that no information loss is expected in black holes (since no such loss exists in the quantum field theory), and the radiation emitted by a black hole is probably the usual thermal radiation. If this is correct, then Hawking's original calculation should be corrected, though it is not known how.

A black hole of one solar mass has a temperature of only 60 nanokelvins; in fact, such a black hole would absorb far more cosmic microwave background radiation than it emits. A black hole of 4.5 × 1022 kg (about the mass of the Moon) would be in equilibrium at 2.7 Kelvin, absorbing as much radiation as it emits. Yet smaller primordial black holes would emit more than they absorb, and thereby lose mass.

TRANS-PLANCKIAN PROBLEM

The trans-Planckian problem is the observation that Hawking's original calculation requires talking about quantum particles in which the wavelength becomes shorter than the Planck length near the black hole's horizon. It is due to the peculiar behavior near a gravitational horizon where time stops as measured from far away. A particle emitted from a black hole with a finite frequency, if traced back to the horizon, must have had an infinite frequency there and a trans-Planckian wavelength. The Unruh effect and the Hawking effect both talk about field modes in the superficially stationary space-time that change frequency relative to other coordinates which are regular across the horizon. This is necessarily so, since to stay outside a horizon requires acceleration which constantly Doppler shifts the modes.

An outgoing Hawking radiated photon, if the mode is traced back in time, has a frequency which diverges from that which it has at great distance, as it gets closer to the horizon, which requires the wavelength of the photon to "scrunch up" infinitely at the horizon of the black hole. In a maximally extended external Schwarzschild solution, that photon's frequency only stays regular if the mode is extended back into the past region where no observer can go. That region doesn't seem to be observable and is physically suspect, so Hawking used a black hole solution without a past region which forms at a finite time in the past. In that case, the source of all the outgoing photons can be identified–it is a microscopic point right at the moment that the black hole first formed.

The quantum fluctuations at that tiny point, in Hawking's original calculation, contain all the outgoing radiation. The modes that eventually contain the outgoing radiation at long times are redshifted by such a huge amount by their long sojourn next to the event horizon, that they start off as modes with a wavelength much shorter than the Planck length.

The same effect occurs for regular matter falling onto a white hole solution. Matter which falls on the white hole accumulates on it, but has no future region into which it can go. Tracing the future of this matter, it is compressed onto the final singular endpoint of the white hole evolution, into a trans-Planckian region. The reason for these types of divergences is that modes which end at the horizon from the point of view of outside coordinates are singular in frequency there. The only way to determine what happens classically is to extend in some other coordinates that cross the horizon.

The key point is that similar trans-Planckian problems occur when the modes occupied with Unruh radiation are traced back in time. In the Unruh effect, the magnitude of the temperature can be calculated from ordinary Minkowski field theory, and is not controversial.

EMISSION PROCESS

Hawking radiation is required by the Unruh effect and the equivalence principle applied to black hole horizons. Close to the event horizon of a black hole, a local observer must accelerate to keep from falling in. An accelerating observer sees a thermal bath of particles that pop out of the local acceleration horizon, turn around, and free-fall back in. The condition of local thermal equilibrium implies that the consistent extension of this local thermal bath has a finite temperature at infinity, which implies that some of these particles emitted by the horizon are not reabsorbed and become outgoing Hawking radiation.

Following points are to borne in the mind:

(1) Schwarzschild black hole has a metric

(2) Black hole is the background space-time for a quantum field theory.

(3) The field theory is defined by a local path integral, so if the boundary conditions at the horizon are determined, the state of the field outside will be specified. The local metric describes a frame that is accelerating to keep from falling into the black hole. The horizon is not a special boundary, and objects can fall in. So the local observer should feel accelerated in ordinary Minkowski space by the principle of equivalence. The near-horizon observer must see the field excited at a local inverse temperature relation, so a black hole can only be in equilibrium with a gas of radiation at a finite temperature. Since radiation incident on the black hole is absorbed, the black hole must emit an equal amount to maintain detailed balance. The black hole acts as a perfect blackbody radiating at this temperature.

Assuming that a small black hole has zero entropy, the integration constant is zero. Forming a black hole is the most efficient way to compress mass into a region, and this entropy is also a bound on the information content of any sphere in space time. The form of the result strongly suggests that the physical description of a gravitating theory can be somehow encoded onto a bounding surface.

BLACK HOLE EVAPORATION

When particles escape, the black hole loses a small amount of its energy and therefore of its mass (mass and energy are related by Einstein's equation $E = mc^2$).

The power emitted by a black hole in the form of Hawking radiation can easily be estimated for the simplest case of a nonrotating, non-charged Schwarzschild black hole of mass. Combining the formulas for the Schwarzschild radius of the black hole, the Stefan–Boltzmann law of black-body radiation, the formula for the temperature of the radiation, and the formula for the surface area of a sphere (the black hole's event horizon), equation derivation is obtained.

Where there is the energy outflow, there is the reduced Planck constant, there is the speed of light, and there is the gravitational constant. It is worth mentioning that the formula for local metric has not yet been derived in the framework of semi classical gravity.

• Black hole evaporation produces a more consistent view of black hole thermodynamics, by showing how black holes interact thermally with the rest of the universe.

• Unlike most objects, a black hole's temperature increases as it radiates away mass. The rate of temperature increase is exponential, with the most likely endpoint being the dissolution of the black hole in a violent burst of gamma rays. A complete description of this dissolution requires a model of quantum gravity, however, as it occurs when the black hole approaches Planck mass and Planck radius.

• The simplest models of black hole evaporation lead to the black hole information paradox. The information content of a black hole appears to be lost when it dissipates; as under these models the Hawking radiation is random (it has no relation to the original information). A number of solutions to this problem have been proposed, including suggestions that Hawking radiation is perturbed to contain the missing information, that the Hawking evaporation leaves some form of remnant particle containing the missing information, and that information is allowed to be lost under these conditions.

Gravitation is extremely weak; it always wins over cosmological distances and therefore is the most important force for the understanding of the large scale structure and evolution of the Universe. Gravitational force in a local representation or referential frame is a electrostatic push in effect effects which encompass Electro Magnetic Force. Gravitational force is fundamentally of electromagnetic origin. Both follow inverse square laws EMF are present on a basic level across the universe, and its theory is similar to the theory of gravitation.

UNIFICATION OF THE FORCES OF NATURE

Although the fundamental forces in our present Universe are distinct and have very different characteristics, the current thinking in theoretical physics is that this was not always so. There is a rather strong belief (although it is yet to be confirmed experimentally) that in the very early Universe when temperatures were very high compared with today, the weak, electromagnetic, and strong forces were unified into a single force. Only when the temperature dropped did these forces separate from each other, with the strong force separating first and then at a still lower temperature the electromagnetic and weak forces separating to leave us with the 4 distinct forces that we see in our present Universe. The process of the forces separating from each other is called spontaneous symmetry breaking.

There is further speculation, which is even less firm than that above, that at even higher temperatures (the Planck Scale) all four forces were unified into a single force. Then, as the temperature dropped, gravitation separated first and then the other 3 forces separated as described above. The time and temperature scales for this proposed sequential loss of unification are illustrated in the following table.

Table 1
Loss of Unity in the Forces of Nature

Characterization	Forces unified	Time since beginning	Temperature (GeV)*
All 4 forces unified	Gravity, Strong, Electromagnetic, Weak	~0	~infinite
Gravity separates (Planck scale)	Strong, Electromagnetic, Weak	10-43 s	1019
Strong force separates (GUTs scale)	Electromagnetic, Weak	10-35 s	1014
Split of weak and electromagnetic forces	None	10-11 s	100
Present universe	None	1010 y	10-12

Theories that postulate the unification of the strong, weak, and electromagnetic forces are called Grand Unified Theories (often known by the acronym GUTs). Theories that add gravity to the mix and try to unify all four fundamental forces into a single force are called Super unified Theories. The theory that describes the unified electromagnetic and weak interactions is called the Standard Electroweak Theory, or sometimes just the Standard Model.

Grand Unified and Super unified Theories remain theoretical speculations that are as yet unproven, but there is strong experimental evidence for the unification of the electromagnetic and weak interactions in the Standard Electroweak Theory. Furthermore, although GUTs are not proven experimentally, there is strong circumstantial evidence to suggest that a theory at least like a Grand Unified Theory is required to make sense of the Universe. In general relativity, space time is assumed to be smooth and continuous and such an assumption is done in the mathematical sense in the theory of quantum mechanics.

There is an inherent discreteness incorporated in physics, in an attempt towards the reconciliation of these two theories. It is proposed that space time should be quantized at the very smallest scales. Current interest is on the locus and focus of nature of space-time at the Planck scale, Causal sets, Loop quantum gravity, string theory, and black hole thermodynamics. All predict a quantized space-time at the Planck scale. There are two kinds of dimensions spatial and temporal. Former is bidirectional and second is unidirectional. If we assume that and the temporal dimension by setting aside and marginalization of the compactified directions invoked by String theory, physical ramifications of such an assumption can be expatiated in no uncertain terms, the justificatory argument being anthropic in thematic and discursive form. Barrow, in fact attributed the manifestation of inverse square law in nature to the three dimensionality in space and time. Law of gravitation follows from the concept of flux and the proportional nature of flux density and the strength of the field. Ehrenfest showed that if N is even then the different part of a wave impulse travel at different speeds. For N=3, there shall be detrimental and pernicious implications of distortion. Weyl confirmed that Maxwell's Theory worked for N=3 and T=1. In the due course instability of electron orbits are proved for N greater than 3. They either collapse in to the nucleus or disperse. Behavior of the physical systems remains unpredicted if it is greater than 1.Protons and electrons would be unstable and could decay in to particles having greater mass themselves.

THE FUNDAMENTAL FORCES OF NATURE

The four forces of nature are considered to be the gravitational force, the electromagnetic force, which has residual effects, the weak nuclear force, and the strong nuclear force, which also has residual effects. Each of these forces reacts only on certain particles, and has its own range and force carrier, the particles that transmit the force, by traveling between the affected particles.

The range of any force is directly related to its force carrier. This is because force carriers must be emitted from one particle and reach another to create a force. However, the emitting particle can be considered at rest in its own reference frame. Emitting a force carrying particle violates conservation of mass-energy, since the force carrier contains some energy. However, this can be allowed by the uncertainty principle. If the force of electromagnetism were greater the atoms would not share electrons with other atoms. On the other hand, if the force of electromagnetism were weaker atoms would not hold on to electrons at all. Strong nuclear force is the degree to which protons and neutrons stick together. Besides, weak nuclear force governs radioactive decay. In the eventuality of the fact that weak nuclear force was stronger, matter would be converted in to heavy metals. If the weak nuclear force was much weaker, matter would remain in the form of the lightest element. Gravitational force determines how hot the stars burn. If the number of electrons and protons had not been equal, galaxies stars planets would never have formed. In other words electromagnetic force is far more than the gravitational force and the ratio must be less than 1. Strong nuclear force, on the other hand, is greater than the gravitational force and electromagnetic force and the ratio must be greater than 1. These are very important points that are used in building of the models and concatenation of the representative equations given at the end of the paper.

In fact, gravitational force is a residual force of force. At the quantum levels, gluons exhibit its effect beyond just a single proton and this is the reason as to why nuclear fusion can occur.

Goradia in his essay "Microscopic Implications of General Theory of Relativity" says that mass tells space how to curve and space tells how to move. It must also be noted that Newtonian gravity analysis describes the relationship between mass and space by having mass effect as its numerator and space effect as its denominator. It is also a fact that Einstein tried to explain nuclear forces in terms of gravity. Gravitational force is an effect of color force. Modified Newtonian gravity can explain residual strong nuclear interaction. Stephen Hawking in his "A Brief History of Time" says that in the eventuality of the fact that light is made of particles, then they must be effected in the same manner as gravitational force affects cannon balls, stars, or galaxies. In this connection, it must be noted that quantum mechanics can explain wave like characteristics and observations of light in terms of photons.

Weak nuclear force is responsible for beta decay. EMF (Electro Magnetic Force) and WNF (Weak Nuclear Force) are two aspects of electro weak interaction. Gauge Bosons carry weak force. It is left-right asymmetric, violates CP symmetry but conserves CPT. In respect of Strong nuclear force Yukawa predicted that it was associated with a massive particle whose mass is 100 MeV. It may also be mentioned in the passing that merger of General Theory Of Relativity and Quantum Mechanics or Quantum Field Theory, has given rise to expectations that gravitation is mediated by a mass less spin particle(spin=2) called gravitons. Frictions, rainbows, lightning, are explained by EMF. It has been documented that the changes in gravitational force and EMF fields propagate at the speed of light. Matter, energy, electric charges move at the speed of light Gravitational force is very much related to the mass.

The amount of energy borrowed multiplied by the time it is borrowed for cannot exceed Planck's constant. Since the amount of energy in the borrowed particle is equal to mass (m) times speed of light (c) squared, the time of existence cannot exceed Planck's constant (h) divided by m times c squared. The maximum distance the force carrier can travel in time t is ct. This must be equal to h/mc. Since this is the maximum distance the force carrier can travel without violating the uncertainty principle, this range is the maximum range of the given force, based on two constants, h and c, and m, the mass of the force carrier.

The Electromagnetic Force

The electromagnetic force operates between particles, which contain electric charge. The force carrier for the electromagnetic force is the photon. Photons, which are commonly called light waves, and referred to as gamma rays, X-rays, visible light, radio waves, and other names depending on their energy. Photons have no mass, which means that, according to the previous calculation, there is no limit on the distance of effect of the electromagnetic force. Photons also have no electric charge, no color, no strangeness, charm, topness, or bottomness, but do possess a spin of 1. EMF can be regarded as smooth, continuous field propagated in the form of a wave and follows Planck's law.

The electromagnetic force has a strength proportional to the product of the electric charges of the particles, and inversely proportional to the square of the distance between the particles' centers of mass. The electromagnetic force is the second strongest force, behind the strong force by two orders of magnitude at the distances in a nucleus, but can be either attractive or repulsive. Like charges attract and unlike charges repel. Over large-scale measurements, the overall charge of an area is most often neutral, and the electromagnetic force has no overall effect. It does have residual attractive forces between electrically neutral atoms that constrain the atoms into molecules. These interactions between atoms are referred to by chemists as chemical bonds, dipole-dipole interactions, or other such terms.

The Gravitational Force

The gravitational force is an interaction between mass-energy, and is thus experienced by all particles to some degree. The gravitational force is proportional to the product of the total energies of the interacting particles, and inversely proportional to the square of the separation between the particles. However, this implies that the gravitational force has no distance limit. By the previously determined relationship, the force carrier of the gravitational force must have no mass for gravity to have no limit to its distance. This particle, known as the graviton, had not been discovered, and is only hypothesized. However, it must exist for the current understanding of forces to be correct.

An interesting fact about gravity is that, although the weakest force, 42 factors of magnitude weaker than the strong nuclear force, it has the greatest effect in large scales. This is because total energies can only be positive, and gravity can therefore only be attractive. Over large areas, the qualities that the other charges act on tend to cancel out, but the effect of gravity merely increases as more mass-energy is involved.

The Weak Nuclear Force

The weak nuclear force is a force of interactions between quarks and leptons, both of which are fermions with spin 1/2. The force only affects particles which are spinning counter-clockwise while going away. In other words, the weak nuclear interaction affects left-handed particles (and right-handed anti-particles). Leptons come in electron, muon, and tau flavors of charge -1, each with associated neutrinos of neutral charge. Quarks appear as the up and down, charm and strange, and top and bottom flavors. The flavors are conserved, and weak interactions transform leptons to other leptons and quarks to other quarks, while preserving this conservation.

The weak nuclear force has a limit in range of only 10 to the -18th meters. This means that the carrier particles must indeed have mass. The weak nuclear force is found to have three carrier particles, two W bosons, one charged -1 and one charged +1, and the electrically neutral Z boson. The W bosons have a mass of 80.22 GeV/(c squared), and the Z boson has a mass of 91.187 GeV(c squared). All cariers have a spin of 1, however. The weak force, as its name implies, is weaker than the electromagnetic or strong nuclear force, about five factors of magnitude smaller than the strong nuclear force distances in an atom's nucleus. However it is very important in beta decay and pair annihilation/production, as well as other interactions.

The Strong Nuclear Force

The strong nuclear force is an interaction between color, and particles that possess color. Quarks possess one of three colors, green, red, or blue, and the strong force is an attractive force between these and the mediating particle, gluons. Gluons have two colors, one normal color and one anti-color. The strong force has no theoretical limit to its range, as gluons have no mass. In addition, they have no electric charge, and a spin of 1. In reality, the strong force is so strong that all color-charged gluons and quarks are bound tightly together into color neutral hadrons, either the mesons which consist of a quark and antiquark with corresponding color and anticolor, or the baryons, which consist of three quarks of the three colors, which cancel to color-neutrality. Since color does not appear outside of any hadrons, the strong force only directly has effects inside a hadron, at distances around 10 to the negative 17th power.

The previous paragraph describes the direct effects of the strong force, usually referred to as the fundamental strong interaction. The strong force also has a residual effect. The color-neutral hadrons can interact with the strong force due to their color-charged constituents, similar to the electromagnetic interaction. The force carriers in this case are the mesons, and all hadrons are affected. The mesons, which include the pions, the kaons, the rhos, the Ds, the etas, and many others, have masses ranging from. 140 Gev/(c squared) to around 3 Gev/(c squared). This gives the residual effects of the strong force a maximum distance to interact of about 10 to the negative 15 meters.

Strong force interactions are important in quark-antiquark reactions, and in holding hadrons together. The fundamental strong interaction holds the constituent quarks of a hadron together, and the residual force holds hadrons together with each other, such as the proton and neutrons in a nucleus.

ENERGY AND MASS OF BLACK HOLE

Assumptions:

Energy and mass of the black hole is classified into three categories:

Category 1 representative of the Energy and mass of the black hole in the first interval vis-à-vis category 1 of Hawking radiation.

Category 2 (second interval) comprising of energy and mass of black hole corresponding to category 2 of Hawking radiation regimentation.

Category 3 constituting energy and mass of black hole belong to higher age than that of category 1 and category 2. This is concomitant to category 3 of Hawking radiation classification.

In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the energy and mass of black hole and corresponding Hawking radiation would be in the fitness of things. For category 3 “Over and above” nomenclature could be used. Similarly, a “less than” scale for category 1 can be used.

a) The speed of growth of energy and mass of a black hole (note that black holes also gobble up matter) under category 1 is proportional to the speed of growth of energy and mass of black hole under category 2. In essence the accentuation coefficient in the model is representative of the constant of proportionality between total energy and mass of black hole under category 1 and category 2 this assumptions is made to foreclose the necessity of addition of one more variable, that would render the systemic equations unsolvable.

b) The dissipation of energy and mass of black hole is concomitant with the Hawking radiation in all the three categories. They are attributable to the following two phenomenon:

1) Aging phenomenon: The aging process leads to transference of the black hole to the next category, no sooner than the age of the black hole (note that the concomitant energy and mass) which is aged crosses the boundary of demarcation.

2) Depletion phenomenon: Natural calamities leading to destruction of universe and galaxy dissipates the growth speed by an equivalent extent. It is assumed that with the destruction of a certain amount of space, corresponding black holes are also evaporated. Model makes allowance for new baby black holes that are formed.

Notations

G_{13}: Energy and mass of black holes in category 1.

G_{14}: Energy and mass of the black holes corresponding to the hawking radiation in category 2.

G_{15}: Energy and mass corresponding to hawking radiation in category 3.

$(a_{13})^{(1)},(a_{14})^{(1)},(a_{15})^{(1)}$: Accentuation coefficients.

$(a'_{13})^{(1)},(a'_{14})^{(1)},(a'_{15})^{(1)}$: Dissipation coefficients.

Formulation of the System

In the light of the assumptions stated in the foregoing, we infer the following:

(a) The growth speed in category 1 is the sum of a accentuation term $(a_{13})^{(1)} G_{14}$ and a dissipation term $-(a'_{13})^{(1)} G_{13}$, the amount of dissipation taken to be proportional to the concomitant category of energy and mass of black holes in the universe which has been classified depending upon the age.

(b) The growth speed in category 2 is the sum of two parts $(a_{14})^{(1)} G_{13}$ and $-(a'_{14})^{(1)} G_{14}$ the inflow from the category 1.

(c) The growth speed in category 3 is equivalent to $(a_{15})^{(1)} G_{14}$ and $-(a'_{15})^{(1)} G_{15}$ the dissipation, or the slowing down of the pace of hawking radiation due to galactic or natural calamities with distorts and has disastrous consequences, it may also be due to transformation of one type of energy in to another. Which accentuates the “loss” or “gain” depending upon the creation or destruction of matter, which is to be noted, is taking place simultaneously.

Governing Equations

The differential equations governing the above system can be written in the following form

$$\frac{(dG_{13})}{dt} = (a_{13})^{(1)} G_{14} - (a'_{13})^{(1)} G_{13} \quad (1)$$

$$\frac{(dG_{14})}{dt} = (a_{14})^{(1)} G_{13} - (a'_{14})^{(1)} G_{14} \quad (2)$$

$$\frac{(dG_{15})}{dt} = (a_{15})^{(1)} G_{14} - (a'_{15})^{(1)} G_{15} \quad (3)$$

$$(a_i)^{(1)} > 0 \ , \quad i = 13,14,15 \quad (4)$$

$$(a'_i)^{(1)} > 0 \ , \quad i = 13,14,15 \quad (5)$$

$$(a_{14})^{(1)} < (a'_{13})^{(1)} \quad (6)$$

$$(a_{15})^{(1)} < (a'_{14})^{(1)} \quad (7)$$

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{13}}{(a_{13})^{(1)} G_{14} - (a'_{13})^{(1)} G_{13}} = dt \quad (8)$$

$$\frac{dG_{14}}{(a_{14})^{(1)} G_{13} - (a'_{14})^{(1)} G_{14}} = dt \quad (9)$$

Or we write a single equation as

$$\frac{dG_{13}}{(a_{13})^{(1)} G_{14} - (a'_{13})^{(1)} G_{13}} = \frac{dG_{14}}{(a_{14})^{(1)} G_{13} - (a'_{14})^{(1)} G_{14}} = \frac{dG_{15}}{(a_{15})^{(1)} G_{14} - (a'_{15})^{(1)} G_{15}} = dt \quad (10)$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α ,β ,γ all positive we can write equation (10) as

$$\frac{\alpha dG_{13}}{\alpha\left((a_{13})^{(1)} G_{14} - (a'_{13})^{(1)} G_{13}\right)} = \frac{\beta dG_{14}}{\beta\left((a_{14})^{(1)} G_{13} - (a'_{14})^{(1)} G_{14}\right)} = \frac{\gamma dG_{15}}{\gamma\left((a_{15})^{(1)} G_{14} - (a'_{15})^{(1)} G_{15}\right)} = dt \quad (11)$$

$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t}$ where i=13,14,15 and C_{13}, C_{14}, C_{15} are arbitrary constant coefficients.

Stability Analysis

Supposing $G_i(0) = G_i^0(0) > 0$, and denoting by λ_i the characteristic roots of the system, it easily results that

1. If $(a'_{13})^{(1)} (a'_{14})^{(1)} - (a_{13})^{(1)} (a_{14})^{(1)} > 0$ all the components of the solution, i.e all the three parts in the expanding universe tend to zero, and the solution is stable with respect to the initial data.

2. If $(a'_{13})^{(1)} (a'_{14})^{(1)} - (a_{13})^{(1)} (a_{14})^{(1)} < 0$ and $(\lambda_{14} + (a'_{13})^{(1)} G_{13}^0 - (a_{13})^{(1)} G_{14}^0 \neq 0$, $(\lambda_{14} < 0)$, the first two components of the solution tend to infinity as t→∞, and $G_{15} \to 0$, ie. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 tend to zero.

3. If $(a'_{13})^{(1)} (a'_{14})^{(1)} - (a_{13})^{(1)} (a_{14})^{(1)} < 0$ and $(\lambda_{14} + (a'_{13})^{(1)} G_{13}^0 - (a_{13})^{(1)} G_{14}^0 = 0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i, the corresponding solution tends to infinity.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system of energy and mass of the black hole in the expanding universe converges to equilibrium.

2. The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point.

3. Conditions 1 and 2 are independent of the size and direction of initial disturbance.

4. The actual shape of the time path of energy and mass of the black hole in the expanding universe is determined by efficiency parameter , the strength of the response of the portfolio in question, and the initial disturbance.

5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold.

6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources (black holes need to gormandize something be it matter in the form of asteroids or something else) that are available in question.

7. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry. Mention may be made that the magnetic flux produced is the one responsible for the Hawking radiation (News paper report).

HAWKING RADIATION

Formulation of the System

a) The growth speed in category 1 is the sum of two parts:

1) A term $+(b_{13})^{(1)}T_{14}$ proportional to the balance of the radiation attributable to the energy and mass in the category 2.
2) A term $-(b'_{13})^{(1)}T_{13}$ representing the quantum of balance dissipated from category 1. This comprises of black hole, which have grown old, qualified to be classified under category 2 (and apparently the energy and mass thereof).

b) The growth speed in category 2 is the sum of two parts: It is to be noted that creation and destruction of black holes, their energy and mass and corresponding evaporation and concomitant Hawking radiation is taking place in the world continuously.

1) A term $+(b_{14})^{(1)}T_{13}$ constitutive of the amount of inflow from the category 1.
2) A term $-(b'_{14})^{(1)}T_{14}$ the dissipation factor arising due to radiation vis-à-vis black hole in the corresponding category getting obliterated or "evaporated".

c) The growth speed under category 3 is attributable to inflow from category 2.

Governing Equations

Following are the differential equations that govern the growth in the energy and mass of black holes and concomitant Hawking radiation thereof, dissipation attributable to "evaporation", and accentuation of the energy and mass ascribable to the "formation" of new baby black holes.

$$\frac{(dT_{13})}{dt}=(b_{13})^{(1)}T_{14}-(b'_{13})^{(1)}T_{13} \quad (12)$$

$$\frac{(dT_{14})}{dt}=(b_{14})^{(1)}T_{13}-(b'_{14})^{(1)}T_{14} \quad (13)$$

$$\frac{(dT_{15})}{dt}=(b_{15})^{(1)}T_{14}-(b'_{15})^{(1)}T_{15} \quad (14)$$

$$(b_i)^{(1)}>0, \quad i=13,14,15 \quad (15)$$

$$(b'_i)^{(1)}>0, \quad i=13,14,15 \quad (16)$$

$$(b_{14})^{(1)}<(b'_{13})^{(1)} \quad (17)$$

$$(b_{15})^{(1)}<(b'_{14})^{(1)} \quad (18)$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is $\alpha'_i T_i+\beta'_i T_i+\gamma'_i T_i=C'_i e_i^{\lambda'_i t}$, $i=13,14,15$ where C'_{13},C'_{14},C'_{15} are arbitrary constant coefficients and $\alpha'_{13},\alpha'_{14},\alpha'_{15},\gamma'_{13},\gamma'_{14},\gamma'_{15}$ corresponding multipliers to the characteristic roots of the system.

ENERGY AND MASS OF BLACK HOLES AND CONCOMITANT HAWKING RADIATIONS-THE DUAL SYSTEM PROBLEM

We will denote

1) By $T_i(t)$, $i=13,14,15$, the three parts of the radiation analogously to the G_i of the energy and mass in black hole portfolio.
2) By $(a''_i)^{(1)}(T_{14},t)$ $(T_{14}\geq 0, t\geq 0)$, the contribution of the radiation to the dissipation coefficient of the energy and mass of black holes.
3) By $(-b''_i)^{(1)}(G_{13},G_{14},G_{15},t)=-(b''_i)^{(1)}(G,t)$, the contribution of the energy and mass of black holes (when they evaporate or destroyed) to the dissipation coefficient of Hawking radiation.

Governing Equations

$$\frac{(dG_{13})}{dt}=(a_{13})^{(1)}G_{14}-[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14},t)]G_{13} \quad (19)$$

$$\frac{(dG_{14})}{dt}=(a_{14})^{(1)}G_{13}-[(a'_{14})^{(1)}+(a''_{14})^{(1)}(T_{14},t)]G_{14} \quad (20)$$

$$\frac{(dG_{15})}{dt}=(a_{15})^{(1)}G_{14}-[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14},t)]G_{15} \quad (21)$$

$$\frac{(dT_{13})}{dt}=(b_{13})^{(1)}T_{14}-[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G,t)]T_{13} \quad (22)$$

$$\frac{(dT_{14})}{dt}=(b_{13})^{(1)}T_{13}-[(b'_{14})^{(1)}-(b''_{14})^{(1)}(G,t)]T_{14} \quad (23)$$

$$\frac{(dT_{15})}{dt}=(b_{15})^{(1)}T_{14}-[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G,t)]T_{15} \quad (24)$$

$+(a''_{13})^{(1)}(T_{14},t)=$ First augmentation factor attributable to HR (Hawking Radiation) to the dissipation of EMOBH (energy and mass of black holes)

$-(b''_{13})^{(1)}(G,t)=$ First detrition factor contributed by HR dissipating EMOBH

Where we suppose

(A) $(a_i)^{(1)},(a'_i)^{(1)},(a''_i)^{(1)},(b_i)^{(1)},(b'_i)^{(1)},(b''_i)^{(1)}>0, i,j=13,14,15$

(B) The functions $(a''_i)^{(1)},(b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)},(r_i)^{(1)}$: (25)

$$(a''_i)^{(1)}(T_{14},t)\leq(p_i)^{(1)}\leq(\hat{A}_{13})^{(1)} \quad (26)$$

$$(b''_i)^{(1)}(G,t)\leq(r_i)^{(1)}\leq(b'_i)^{(1)}\leq(\hat{B}_{13})^{(1)}$$

(C) $$\lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t)=(p_i)^{(1)} \quad (27)$$

$$\text{im}_{G\to\infty}(b''_i)^{(1)}(G,t)=(r_i)^{(1)} \quad (28)$$

Definition of $(\hat{A}_{13})^{(1)},(\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}$,$(\hat{B}_{13})^{(1)}$, $(p_i)^{(1)}$, $(r_i)^{(1)}$ are positive constants and (i=13,14,15). They satisfy Lipschitz condition:

$$|(a_i'')^{(1)}(T_{14}',t)-(a_i'')^{(1)}(T_{14},t)|\leq(\hat{k}_{13})^{(1)}|T_{14}-T_{14}'|e^{-(\hat{M}_{13})^{(1)}t} \quad (29)$$

$$|(b_i'')^{(1)}(G',t)-(b_i'')^{(1)}(G,t)|\leq(\hat{k}_{13})^{(1)}||G-G'||e^{-(\hat{M}_{13})^{(1)}t} \quad (30)$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(k_{13})^{(1)},(\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)}=1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to hawking radiation, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}$, $(\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}$, $(\hat{k}_{13})^{(1)}$, are positive constants (31)

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \ \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} <1$$

Definition of $(\hat{P}_{13})^{(1)}$, $(\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}$, $(\hat{k}_{13})^{(1)}$, $(\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}$, $(a_i')^{(1)}$, $(b_i)^{(1)}$, $(b_i')^{(1)}$, $(p_i)^{(1)}$, $(r_i)^{(1)}$, i = 13,14,15, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)}+(a_i')^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}]<1 \quad (32)$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)}+(b_i')^{(1)}+(\hat{B}_{13})^{(1)}+(\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}]<1 \quad (33)$$

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t)\leq(\hat{P}_{13})^{(1)}e^{(\hat{m}_{13})^{(1)}t}$, $\quad (G_i(0)=G_i^0>0)$

$T_i(t)\leq(\hat{Q}_{13})^{(1)}e^{(\hat{m}_{13})^{(1)}t}$, $\quad (T_i(0)=G_i^0>0)$

Proof:

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, T_i: $\mathbb{R}_+\to\mathbb{R}_+$ which satisfy

$$G_i(0)=G_i^0,\ T_i(0)=T_i^0,\ G_i^0\leq(\hat{P}_{13})^{(1)},\ T_i^0\leq(\hat{Q}_{13})^{(1)}, \quad (34)$$

$$0\leq G_i(t)-G_i^0\leq(\hat{P}_{13})^{(1)}e^{(\hat{m}_{13})^{(1)}t} \quad (35)$$

$$0\leq T_i(t)-T_i^0\leq(\hat{Q}_{13})^{(1)}e^{(\hat{m}_{13})^{(1)}t} \quad (36)$$

By

$$\bar{G}_{13}(t)=G_{13}^0+\int_0^t\left[(a_{13})^{(1)}G_{14}(s_{(13)})-\left((a_{13}')^{(1)}+a_{13}'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\right)G_{13}(s_{(13)})\right]ds_{(13)} \quad (37)$$

$$\bar{G}_{14}(t)=G_{14}^0+\int_0^t\left[(a_{14})^{(1)}G_{13}(s_{(13)})-\left((a_{14}')^{(1)}+(a_{14}'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\right)G_{14}(s_{(13)})\right]ds_{(13)} \quad (38)$$

$$\bar{G}_{15}(t)=G_{15}^0+\int_0^t\left[(a_{15})^{(1)}G_{14}(s_{(13)})-\left((a_{15}')^{(1)}+(a_{15}'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\right)G_{15}(s_{(13)})\right]ds_{(13)} \quad (39)$$

$$\bar{T}_{13}(t)=T_{13}^0+\int_0^t\left[(b_{13})^{(1)}T_{14}(s_{(13)})-\left((b_{13}')^{(1)}-(b_{13}'')^{(1)}(G(s_{(13)}),s_{(13)})\right)T_{13}(s_{(13)})\right]ds_{(13)} \quad (40)$$

$$\bar{T}_{14}(t)=T_{14}^0+\int_0^t\left[(b_{14})^{(1)}T_{13}(s_{(13)})-\left((b_{14}')^{(1)}-(b_{14}'')^{(1)}(G(s_{(13)}),s_{(13)})\right)T_{14}(s_{(13)})\right]ds_{(13)} \quad (41)$$

$$\bar{T}_{15}(t)=T_{15}^0+\int_0^t\left[(b_{15})^{(1)}T_{14}(s_{(13)})-\left((b_{15}')^{(1)}-(b_{15}'')^{(1)}(G(s_{(13)}),s_{(13)})\right)T_{15}(s_{(13)})\right]ds_{(13)} \quad (42)$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying 34,35,36 into itself. Indeed it is obvious that

$$G_{13}(t)\leq G_{13}^0+\int_0^t\left[(a_{13})^{(1)}\left(G_{14}^0+(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\right)\right]ds_{(13)}=$$

$$\left(1+(a_{13})^{(1)}t\right)G_{14}^0+\frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)}t}-1\right) \quad (43)$$

From which it follows that
$(G_{13}(t) - G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t} \leq$

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)} + G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)} + (\hat{P}_{13})^{(1)}\right] \quad (44)$$

(G_i^0) is as defined in the statement of theorem 1.
Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

It is now sufficient to take, $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + ((\hat{P}_{13})^{(1)} + G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad (45)$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad (46)$$

In order that the operator $\mathcal{A}^{(1)}$transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself
The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},$$

$$\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\} \quad (47)$$

Indeed if we denote
Definition of $\tilde{G}, \tilde{T}$:
$(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$ (48)

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)} - G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)} +$$
$$\int_0^t\{(a_{13}^{'})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}} +$$
$$(a_{13}^{''})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}} +$$
$$G_{13}^{(2)}\left|(a_{13}^{''})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a_{13}^{''})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)} \quad (49)$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$
From the hypotheses on 25,26,27,28 and 29 it follows

$$|G^{(1)} - G^{(2)}|e^{-(\hat{M}_{13})^{(1)}t} \leq \frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right) d\left((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)})\right) \quad (50)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis (34, 35, 36) the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ and $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$, i=13,14,15 depend only on T_{14} and respectively on G (*and not on t*) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$. (51)

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\}ds_{(13)}\right]} \geq 0$$
$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for } t > 0 \quad (52)$$

Definition of
$\left((\hat{M}_{13})^{(1)}\right)_1, \left((\hat{M}_{13})^{(1)}\right)_2$ *and* $\left((\hat{M}_{13})^{(1)}\right)_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15}. indeed if
$G_{13} < (\hat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\hat{M}_{13})^{(1)}\right)_1 - (a_{14}^{'})^{(1)}G_{14}$

and by integrating

$$G_{14} \leq \left((\hat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\hat{M}_{13})^{(1)}\right)_1/(a_{14}^{'})^{(1)} \quad (53)$$

In the same way , one can obtain

$$G_{15} \leq \left((\hat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\hat{M}_{13})^{(1)}\right)_2/(a_{15}^{'})^{(1)} \quad (54)$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13}, G_{15} and G_{13}, G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} .The proof is analogous with the preceding one. An analogous property is true if $G_{14}$is bounded from below.

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(1)}(G(t), t)) = (b_{14}^{'})^{(1)}$ then $T_{14} \to \infty$. (55)

Definition of $(m)^{(1)}$ and ε_1 :
Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i^{''})^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t),t) = (b'_{15})^{(1)}$ (56)

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42.

BEHAVIOR OF THE SOLUTIONS OF EQUATION 37 TO 42

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:four constants satisfying

$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$ (57)

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$ (58)

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: (59)

(b) By $(\nu_1)^{(1)} > 0, (\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations (60)

$(a_{14})^{(1)}(\nu^{(1)})^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ (61)

and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ and

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: (62)

By $(\bar{\nu}_1)^{(1)} > 0, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the

roots of the equations $(a_{14})^{(1)}(\nu^{(1)})^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ (63)

$(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ (64)

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ (65)

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by (66)

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if \ (\nu_0)^{(1)} < (\nu_1)^{(1)}$ (67)

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if \ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$ (68)

and $(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, if \ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$ (69)

and analogously

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, if \ (u_0)^{(1)} < (u_1)^{(1)}$ (70)

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if \ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$ (71)

and $(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ (72)

are defined by 59 and 61 respectively
Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{13}^0 e^{\left((S_1)^{(1)}-(p_{13})^{(1)}\right)t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t} \quad (73)$$

where$(p_i)^{(1)}$ is defined by equation 25

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{\left((S_1)^{(1)}-(p_{13})^{(1)}\right)t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t} \quad (74)$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}\left((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)}\right)}\left[e^{\left((S_1)^{(1)}-(p_{13})^{(1)}\right)t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t)\right.$$

$$\left.\le \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}\left((S_1)^{(1)} - (a_{15}')^{(1)}\right)}\left[e^{(S_1)^{(1)}t} - e^{-(a_{15}')^{(1)}t}\right] + G_{15}^0 e^{-(a_{15}')^{(1)}t}\right) \quad (75)$$

$$T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{\left((R_1)^{(1)}+(r_{13})^{(1)}\right)t} \quad (76)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{\left((R_1)^{(1)}+(r_{13})^{(1)}\right)t} \quad (77)$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}\left((R_1)^{(1)} - (b_{15}')^{(1)}\right)}\left[e^{(R_1)^{(1)}t} - e^{-(b_{15}')^{(1)}t}\right] + T_{15}^0 e^{-(b_{15}')^{(1)}t} \le T_{15}(t) \le$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}\left((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)}\right)}\left[e^{\left((R_1)^{(1)}+(r_{13})^{(1)}\right)t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t} \quad (78)$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a_{13}')^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b_{13}')^{(1)}$ (79)

$(R_2)^{(1)} = (b_{15}')^{(1)} - (r_{15})^{(1)}$

Proof : From 19,20,21,22,23,24 we obtain

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a_{13}')^{(1)} - (a_{14}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t)\right) - (a_{14}'')^{(1)}(T_{14}, t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)} \quad (80)$$

Definition of $\nu^{(1)}$ $\nu^{(1)} = \frac{G_{13}}{G_{14}}$

It follows $-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \le \frac{d\nu^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$ (81)

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner, we get

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})t\right]}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}} \tag{82}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$ (83)

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, (84)

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}((\nu_1)^{(1)} - (\nu_2)^{(1)})t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}((\nu_1)^{(1)} - (\nu_2)^{(1)})t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)} \tag{85}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}} \tag{86}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}} \tag{87}$$

Now, using this result and replacing it in 19, 20, 21, 22, 23, and 24 we get easily the result stated in the theorem.

Particular case:

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case.

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then $(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Theorem 3: If $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ are independent on t, and the conditions (with the notations 25, 26, 27, 28).

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0 \tag{88}$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}$, $(r_{14})^{(1)}$ as defined by equation 25 are satisfied, then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \qquad (89)$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \qquad (90)$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \qquad (91)$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \qquad (92)$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \qquad (93)$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \qquad (94)$$

has a unique positive solution , which is an equilibrium solution for the system (19 to 24).

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{13},G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0 \qquad (95)$$

Definition and uniqueness of T_{14}*:

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \;,$$

$$G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]} \qquad (96)$$

(b) By the same argument, the equations 92,93 admit solutions G_{13},G_{14} if

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0 \qquad (97)$$

Where in $G(G_{13},G_{14},G_{15}),G_{13},G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*)=0$

Finally we obtain the unique solution of 89 to 94

G_{14}^* given by $\varphi(G^*)=0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G^a_{13} = \frac{(1_{31})^{(G)}1^a_4]}{a(1^1_{3\prime})^{(+)}a(1'^1_{3\prime})^{(()}T1^a_{4*})]}$$

$$G^a_{15} = \frac{(a_{15})^{(1)}G^a_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^a_{14})]} \qquad (98)$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]}$$

$$T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]} \qquad (99)$$

Obviously, these values represent an equilibrium solution of 19, 20, 21, 22, 23, 24.

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i$, $\mathbb{T}_i$

$$G_i = G_i^* + \mathbb{G}_i \,,\; T_i = T_i^* + \mathbb{T}_i \qquad (100)$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T^*_{14}) = (q_{14})^{(1)} \quad , \quad \frac{\partial(b''_i)^{(1)}}{\partial G_j}(\,G^*\,) = s_{ij} \qquad (101)$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \qquad (102)$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \qquad (103)$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \qquad (104)$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \qquad (105)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \qquad (106)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \qquad (107)$$

The characteristic equation of this system is

$$\begin{aligned}
&\left((\lambda)^{(1)}+(b'_{15})^{(1)}-(r_{15})^{(1)}\right)\{\left((\lambda)^{(1)}+(a'_{15})^{(1)}+(p_{15})^{(1)}\right)\\
&\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\right]\\
&\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(14)}T^{*}_{14}\right)\\
&+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)\\
&\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)\\
&\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)\\
&\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)\\
&+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}\\
&+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)\\
&\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0
\end{aligned} \tag{108}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

COMBINATRIONICS MECHANICS

Governing Equations

Energy and Mass of Black Holes

$$\frac{dG_{13}}{dt}=(a_{13})^{(1)}G_{14}-(a'_{13})^{(1)}G_{13} \tag{1a}$$

$$\frac{dG_{14}}{dt}=(a_{14})^{(1)}G_{13}-(a'_{14})^{(1)}G_{14} \tag{2a}$$

$$\frac{dG_{15}}{dt}=(a_{15})^{(1)}G_{14}-\left(a'_{15}\right)^{(1)}G_{15} \tag{3a}$$

Hawking Radiation

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-(b'_{13})^{(1)}T_{13} \tag{4a}$$

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-(b'_{14})^{(1)}T_{14} \tag{5a}$$

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-\left(b'_{15}\right)^{(1)}T_{15} \tag{6a}$$

Matter

Force of gravity was due to the presence of matter, specifically its mass. In fact the existence of black holes was postulated by Karl Schwarzschild who in 1916 derived an equation for the Schwarzschild Radius of a black hole (Rs = GM/c^2, where Rs is the Schwarzschild radius, G is Newton's gravitational constant, M is the mass of the black hole and c the speed of light).

To form a black hole matter **collapses** under its own gravitational field, such as in the **death of** a large star. If the matter in question is massive enough then its gravitational attraction will be so great that it will **overcome** all of the other forces trying to resist the collapse and the matter will continue to **shrink** until it becomes no more than a point, known as a singularity. This point will have an infinite density and will be infinitely small. The **effect** on space time will be such that it is **distorted** to the point where light can no longer escape from the black hole, hence the name black. At singularities the known laws of physics break down which is why so much time and effort is spent examining these strange features of our universe.

The Schwarzschild radius describes a property of black holes known as the event horizon. This is the point between space where light can escape from the black hole's gravitational field and the space where it cannot. Although the singularity inside the black hole is infinitely small, the black hole would appear to be the size of its event horizon and to all effects is.

When matter falls into the event horizon it becomes isolated from the rest of space and time and has, effectively, disappeared from the universe that we exist in. Once inside the black hole the matter will be torn apart into its smallest subatomic components, which will be stretched and squeezed until they to become part of the singularity and increase the radius of the black hole accordingly

Interestingly enough it has now been shown, by Stephen Hawking that the matter inside a black hole is

not completely isolated from the rest of the universe and that given a sufficient length of time black holes will **gradually dissolve** by radiating away the energy of the matter that they **contain**.

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - (a'_{16})^{(2)}G_{16} \tag{7a}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - (a'_{17})^{(2)}G_{17} \tag{8a}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - (a'_{18})^{(2)}G_{18} \tag{9a}$$

Antimatter

Black Holes and Antimatter Cloud

According to Paul Gilster, gamma rays coming out of galactic center, **flagging** the presence of an antimatter cloud of enormous extent, have spawned few explanations more exotic than the one we consider today: Black holes. Primordial black holes, in fact, **produced** in their trillions at the time of the Big Bang and left evaporating through so-called 'Hawking radiation' ever since. That's the theory of Cosimo Bambi (Wayne State University) and colleagues, who are studying the same antimatter cloud. Hawking offers a mechanism for small black holes **to lose** mass over time. But since the phenomenon has never been observed, the upcoming launch of the GLAST (Gamma-ray Large Area Space Telescope) satellite again looms large in significance.

But assuming that black holes do evaporate, the trick is to figure out how fast, and that rate depends upon mass, with more massive black holes **producing** fewer evaporated particles. What Bambi's team argues that a mass of about 10^{16} grams, roughly that of a fairly common asteroid, will **produce** the right amount of antimatter to explain the detections. Theoretically, the signature radiation from black holes of this particular size should be observable given the right equipment, Paul Gilster opines that his team have considered evaporating primordial BHs [black holes], as a possible **source** of positrons to generate the observed photon 511 keV line from the Galactic Bulge. The analysis of the accompanying continuous photon background produced, in particular, by the same evaporating BHs, **allows to** fix the mass of the evaporating BHs near 10^{16} g. It is interesting that the necessary amount of BHs could be of the **same order** of magnitude as the amount of dark matter in the Galactic Bulge. This **opens a possibility** that such primordial BHs may form all cosmological dark matter. The background MeV **photons created** by these primordial BHs can be registered in the near future, according to Gilster, while the neutrino flux may be still beyond observation. The significance of this model would be difficult to overestimate, because these BHs would present a **unique link connecting** early universe and particle physics.

It thus bears ample testimony and infallible observatory and impeccable demonstration by Gilster and his team that Primordial black holes **produce** explanation for dark matter itself. But bear in mind that along with the x-ray binaries so recently considered in **relation** to galactic antimatter, other explanations are still in play, including type Ia supernovae and a host of far more exotic possibilities. Gilster is of the opinion that GLAST should help, but the apprehensions about the qualitative gradient of structural gradient and diffuse solidarity abstraction **grows that** the antimatter cloud at galactic center may remain enigmatic for some time to come.

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - (b'_{16})^{(2)}T_{16} \tag{10a}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - (b'_{17})^{(2)}T_{17} \tag{11a}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - (b'_{18})^{(2)}T_{18} \tag{12a}$$

Governing Equations of Dual Concatenated Systems

Energy and Mass of Black Holes

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\right]G_{13} \tag{13a}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\right]G_{14} \tag{14a}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\right]G_{15} \tag{15a}$$

Where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to hawking radiation dissipating energy and mass of the black holes classified based on age thus augmenting the dissipation coefficient of energy and mass of black holes and reducing the dissipation coefficient of hawking radiation due to evaporation of black holes

Hawking Radiation Corresponding to the Black Hole Classification

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\right]T_{13} \tag{16a}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\right]T_{14} \tag{17a}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\right]T_{15} \tag{18a}$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3.

Matter and Antimatter System: Corresponding Concatenated Equations

Matter

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)}\right] G_{16} \quad (19a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)}\right] G_{17} \quad (20a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)}\right] G_{18} \quad (21a)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to dissipation ofmatter by antimatter on the same lines as that of blackhole-radiation system

Antimatter

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)}\right] T_{16} \quad (22a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)}\right] T_{17} \quad (23a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)}\right] T_{18} \quad (24a)$$

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

GOVERNING EQUATIONS OF CONCATENATED SYSTEM OF TWO CONCATENATED DUAL SYSTEMS

Black Holes-Radiation-Matter-Antimatter System:a Cross Cultural Combination Governing Equations

Matter

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{+(a''_{13})^{(1,1)}(T_{14},t)}\right] G_{16} \quad (25a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1)}(T_{14},t)}\right] G_{17} \quad (26a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1)}(T_{14},t)}\right] G_{18} \quad (27a)$$

Hawking Radiation

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{-(b''_{16})^{(2,2)}(G_{19},t)}\right] T_{13} \quad (28a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{-(b''_{17})^{(2,2)}(G_{19},t)}\right] T_{14} \quad (29a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{18})^{(2,2)}(G_{19},t)}\right] T_{15} \quad (30a)$$

Energy and Mass of Black Holes

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)}\right] G_{13} \quad (31a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)}\right] G_{14} \quad (32a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)}\right] G_{15} \quad (33a)$$

Where $+(a''_{13})^{(1)}(T_{14},t)$, $+(a''_{14})^{(1)}(T_{14},t)$, $+(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3 due to hr dissipating energy and mass of black holes.

Antimatter

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)}\right] T_{16} \quad (34a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)}\right] T_{17} \quad (35a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)}\right] T_{18} \quad (36a)$$

Where $-(b''_{16})^{(1)}(G_{19},t)$, $-(b''_{17})^{(1)}(G_{19},t)$, $-(b''_{18})^{(1)}(G_{19},t)$, are first detrition coefficients for category 1, 2 and 3.

Governing Equations of Hawking Radiation-Energy and Mass of Black Holes-Matter-Antimatter System:system of Holistic Equations

Hawking Radiation

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19},t) - (b''_{13})^{(1,1)}(G,t)]T_{16} \quad (37a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} -$$

$[(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) - (b''_{14})^{(1,1)}(G, t)]T_{17}$ (38a)

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} -$
$[(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) - (b''_{15})^{(1,1)}(G, t)]T_{18}$ (39a)

Where $-(b''_{16})^{(2)}(G_{19}, t)$, $-(b''_{17})^{(2)}(G_{19}, t)$, $-(b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 3. $-(b''_{13})^{(1,1)}(G, t)$, $-(b''_{14})^{(1,1)}(G, t)$, $-(b''_{15})^{(1,1)}(G, t)$ are second detrition coefficients for category 1, 2 and 3.

Energy and Mass of Black Holes

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} -$
$[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) + (a''_{16})^{(2,2)}(T_{17}, t)]G_{13}$ (40a)

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} -$
$[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) + (a''_{17})^{(2,2)}(T_{17}, t)]G_{14}$ (41a)

$\frac{dG_{15}}{dt}\ (a_{15})^{(1)}G_{14} -$
$[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) + (a''_{18})^{(2,2)}(T_{17}, t)]G_{15}$ (42a)

Where $(a''_{13})^{(1)}(T_{14}, t)$, $(a''_{14})^{(1)}(T_{14}, t)$, $(a''_{15})^{(1)}(T_{14}, t)$, are first augmentation coefficients for category 1, 2 and 3. $+(a''_{16})^{(2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2)}(T_{17}, t)$ are second augmentation coefficients for category 1, 2 and 3.

Matter

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)}\right] G_{16}$ (43a)

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)}\right] G_{17}$ (44a)

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)}\right] G_{18}$ (45a)

Where $+(a''_{16})^{(2)}(T_{17}, t)$, $+(a''_{17})^{(2)}(T_{17}, t)$, $+(a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3.

Hawking Radiation

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)}\right] T_{13}$ (46a)

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)}\right] T_{14}$ (47a)

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)}\right] T_{15}$ (48a)

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3.

Governing Equations of the System

Matter

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} -$

$\left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}\right] G_{16}$ (49a)

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} -$

$\left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}\right] G_{17}$ (50a)

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} -$

$\left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}\right] G_{18}$ (51a)

Where $+(a''_{16})^{(2)}(T_{17}, t)$, $+(a''_{17})^{(2)}(T_{17}, t)$, $+(a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to anti matter dissipating matter in the universe and $+(a''_{13})^{(1,1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1)}(T_{14}, t)$ are second augmentation coefficient for category 1, 2 and 3 due to hawking radiation dissipating the matter namely the energy and mass of black holes.

Hawking Radiation

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} -$

$\left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}\right] T_{13}$ (52a)

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} -$
$\left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}\right] T_{14}$ (53a)

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} -$

$\left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}\right] T_{15}$ (54a)

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3. $-(b''_{16})^{(2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2)}(G_{19}, t)$, are second detrition coefficient for category 1, 2 and 3.

Energy and Mass Black Holes

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} -$
$[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) + (a''_{16})^{(2,2,2)}(T_{17}, t)]G_{13}$ (55a)

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} -$
$[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) + (a''_{17})^{(2,2,2)}(T_{17}, t)]G_{14}$ (56a)

$\frac{dG_{15}}{dt}\ (a_{15})^{(1)}G_{14} -$
$[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) + (a''_{18})^{(2,2,2)}(T_{17}, t)]G_{15}$ (57a)

Where $+(a''_{13})^{(1)}(T_{14}, t)$, $+(a''_{14})^{(1)}(T_{14}, t)$, $+(a''_{15})^{(1)}(T_{14}, t)$ are first augmentation coefficients for category 1, 2 and 3 due HR $+(a''_{17})^{(2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2)}(T_{17}, t)$ are second augmentation coefficient for category 1, 2 and 3 due to antimatter.

Antimatter

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} -$
$[(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) - (b''_{13})^{(1,1,1)}(G, t)]T_{16}$ (58a)

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} -$
$[(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) - (b''_{14})^{(1,1,1)}(G, t)]T_{17}$ (59a)

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) - (b''_{15})^{(1,1,1)}(G, t)] T_{18} \tag{60a}$$

where $-(b''_{16})^{(2)}(G_{19}, t)$, $-(b''_{17})^{(2)}(G_{19}, t)$, $-(b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 due to matter dissipating antimatter *HR* $-(b''_{13})^{(1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1)}(G, t)$ are second detrition coefficients for category 1, 2 and 3 due toemobh dissipating antimatter (please refer introduction).

VERY IMPORTANT EPILOGUE

In the above equations, we have explored all the possibilities if energy and mass of black holes,hawking radiation,matter and antimatter interacting in various permutations and combinations. The equations can be solved with the application of the processual formalities and procedural regularities of the paper which has been elucidated in detail. In the foregoing. Nevertheless such possibilities and probabilities would be discussed both with reference to structure orientation and process orientation in future papers. Notwithstanding, it can be said in unmistakable terms that with the same conditional ties and functionalities consummated we shall obtain the results as has been obtained in the above paper in the consolidated and concretised fashion.

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The introduction is a collection of information from, articles, abstracts of the articles, paper reports, home pages of the authors, textbooks, research papers, and various other sources including the internet including Wikipedia. We acknowledge all authors who have contributed to the same. Should there be any act of omission or commission on the part of the authors in not referring to the author, it is authors' sincere entreat, earnest beseech, and fervent appeal to pardon such lapses as has been done or purported to have been done in the foregoing. With great deal of compunction and contrition, the authors beg the pardon of the respective sources. References list is only illustrative and not exhaustive. We have put all concerted efforts and sustained endeavors to incorporate the names of all the sources from which information has been extracted. It is because of such eminent, erudite, and esteemed people allowing us to piggy ride on their backs, we have attempted to see little forward, or so we think.

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