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Gross Interst-Environment Games and Environment Crisis Theorems[1]

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Abstract: To study the interaction between the player's actions and environment in economic management game such as launching some projects with pollution in a certain area, we consider the relation between the player's interests and the environment. We introduce a so-called gross interest- environment game and the concept of common hazard degree based on binary number and n-person non-cooperative game theory. It is studied that properties of players' utility functions and common hazard degree. Basic on the concept of N-M stable set in set of Nash equilibria, we prove environment crisis theorems. Our main results are as follows: if it is an Nash equilibrium that every enterprise launching the project with pollution and it is not an Nash equilibrium that every not doing, then it is the most probable to realize Nash equilibrium with the greatest common hazard degree.

Key words: gross interest-environment game; common hazard degree; N-M stable set; environment crisis theorem

1. INTRODUCTION

In the traditional economic management game, one does not consider the interaction between the players' interests and the environment. The literature（Hardin G., 1968）is an example with which system economists are very familiar. The model shows that if a resource has no exclusive ownership, it will be excessively used (Zhang W.Y., 1996). This model is of great significance in environmental management science as well. For example, if fishermen should have unlimited fishing in high seas, the fish would be extinct. On the earth if enterprises should emit unlimitedly pollutants, the mankind survival environment would be increasingly worse.

The literatures (Jiang D.Y., etc., 2006; Jiang D.Y., *Computing, Information and Control*) studied the

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so-called condition games. The literature (Jiang D.Y., 2005) discussed applications of condition games to economic management science. The literatures (Jiang D.Y., 2005; Jiang D.Y., 2007) considered applications of them to environmental management science. These applications are be long to sustainable development issues in environmental and economic management sciences.

However there are other subjects as well. For example, we consider the case that some enterprises in a region want to start some projects with pollution. If these enterprises do not consider the interaction between their interest and the environment that enterprises damage to the environment and the environment affect interest of the enterprises and other objects that dependent on the environment, then the game is the traditional one. However, such environment interfere cannot be underestimated for the players' interests.

Our task will be to consider the environmental issues, a new game system that is gross interest – environment games. In the literature (Jiang D.Y., 2007), we introduced a so-called gross interest-environment game based on binary numbers and n-person non-cooperative game theory. It was studied that utility function of the game and conditions for Nash equilibria. In this paper, we shall study the concept of common hazard degree based on binary numbers and n-person non-cooperative game theory. It is studied that utility function of the game and properties of common hazard degree. Basic on the concept of N-M stable set in set of Nash equilibria (Jiang D.Y., 2007), we prove environment crisis theorems.

2. GROSS PROFIT-ENVIRONMENT GAMES

A system $\Gamma \equiv [N;(A_i);(P_i)]$ is called an n-person finite non-cooperative game, where $N = \{1, 2, \cdots, n\}$ is the finite set of all players, A_i is the finite set of player i's actions (or pure strategies), the Cartesian product $A = \prod_{i\in N} A_i$ of A_i is the set of situations of the game, and the real value function $P_i : A \to R$ is the player i's utility function, where $P_i(a_1 \cdots a_n)$ is the player i's utility under the situation $(a_1 \cdots a_n)$.

A situation $(a_1^* \cdots a_n^*) \in A$ is called Nash equilibrium if $P_i(a_1^* \cdots a_i^* \cdots a_n^*) \geq P_i(a_1^* \cdots a_{i-1}^* a_i a_{i+1}^* \cdots \ a_n^*)$ for any player i and any action $a_i \in A_i$.

In this paper, set of all Nash equilibria is denoted by NE.

Let $\Gamma \equiv [N;(A_i);(P_i)]$ be an n-person finite non-cooperative game. Every player i has exactly two actions 1 and 0. When i uses the action 1, he gets the gross profit g_i, and on the other hand, he destroys the environment which make each $j \in N$ of all players get an environmental negative utility $e_j^{(b_1 \cdots b_{i-1} 1 b_{i+1} \cdots b_n)}$, where $(b_1 \cdots b_{i-1} 1 b_{i+1} \cdots b_n)$ is the corresponding situation. When all players use his action 0, none of them can get either gross profit or environmental negative utility.

Let B_n be the set of binary numbers with the word length n, for example,

$B_1 = \{0,1\}$, $B_2 = \{00, 01, 10, 11\}$ and $B_3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$.

We introduce the order relations on B_n as the following: (1) $b_1' \cdots b_n' \prec= b_1'' \cdots b_n''$ implies that $b_i\,'=1 \Rightarrow b_i\,''=1$, $i=1,2,\cdots,n$ and （2） $b_1\,'\cdots b_n\,' \prec b_1\,''\cdots b_n\,''$ implies that $b_1\,'\cdots b_n\,' \prec= b_1\,''\cdots b_n\,''$ and $b_1\,'\cdots b_n\,' \neq b_1\,''\cdots b_n\,''$.

It is obvious that $\prec=$ is a partial order relation and $\prec$ a quasi order relation.

Definition 1 An n-person finite non-cooperative game $\Gamma \equiv [N;(A_i);(P_i)]$ is called a gross profit-environment game, if $N=\{1,2,\cdots,n\}$, $A_i=\{0,1\}$ and

$$P_i(b_1\cdots b_n)=\begin{cases} g_i - e_i^{1(b_1\cdots b_n)}, & b_i=1 \\ -e_i^{0(b_1\cdots b_n)}, & b_i=0 \end{cases} ,\quad i=1,2,\cdots,n,$$

where $e_i^{0(b_1\cdots b_n)} \geq 0$ and the equal sign is holds if and only if $b_1\cdots b_n = 0\cdots 0$. Where g_i is the player i 'gross profit when he uses the action 1, $e_i^{1(b_1\cdots b_n)}$ is the player i 's negative utility when he uses the action 1 under the situation $(b_1\cdots b_{i-1}1b_{i+1}\cdots b_n)$, and $e_i^{0(b_1\cdots b_n)}$ is the player i 's negative utility when he uses the action 1 under the situation $(b_1\cdots b_{i-1}0b_{i+1}\cdots b_n)$.

Theorem 1 （**monotonicity of environmental negative utility**） For a gross profit-environment game $\Gamma \equiv [N;(A_i);(P_i)]$, let $(b_1\cdots b_n)\in A$ and

$$0<e_i^{(b_1\cdots b_n)}=\begin{cases} e_i^{0(b_1\cdots b_n)}, & b_i=0 \\ e_i^{1(b_1\cdots b_n)}, & b_i=1 \end{cases} ,\ i=1,2,\cdots,n.$$

Then

(1) $e_i^{(b_1'\cdots b_n')} = e_i^{(b_1''\cdots b_n'')}$ *if* $b_1\,'\cdots b_n\,' = b_1\,''\cdots b_n\,''$ *,* $i=1,2,\cdots,n$ *,*

(2) $e_i^{(b_1'\cdots b_n')} < e_i^{(b_1''\cdots b_n'')}$ *if* $b_1\,'\cdots b_n\,' \prec b_1\,''\cdots b_n\,''$ *,* $i=1,2,\cdots,n$ *, and*

(3) $e_i^{(b_1'\cdots b_n')} \leq e_i^{(b_1''\cdots b_n'')}$ *if* $b_1\,'\cdots b_n\,' \prec= b_1\,''\cdots b_n\,''$ *,* $i=1,2,\cdots,n$ *.*

Proof：We prove only the case $b_1\,'\cdots b_n\,' \neq 0\cdots 0$. The case $b_1\,'\cdots b_n\,' = 0\cdots 0$ is similar.

（1）Let $b_1\,'\cdots b_n\,' = b_1\,''\cdots b_n\,''$. We have $b_i\,''=0$ if $b_i\,'=0$. So

$$e_i^{(b_1'\cdots b_n')} = e_i^{0(b_1'\cdots b_{i-1}'0b_{i+1}'\cdots b_n')} = e_i^{0(b_1''\cdots b_{i-1}''0b_{i+1}''\cdots b_n'')} = e_i^{(b_1''\cdots b_n'')}.$$

Similarly, we have $e_i^{(b_1'\cdots b_n')} = e_i^{(b_1''\cdots b_n'')}$ if $b_i\,'=1$.

（2）Let $b_1\,'\cdots b_n\,' \prec b_1\,''\cdots b_n\,''$. Let $b_j\,'=0$ and $b_j\,''=1$ for some $j(1\leq j\leq n)$. For any $i(1\leq i\leq n, i\neq j)$, we analysis the three subcases:

A． $b_i\,''=1$ if $b_i\,'=1$. It shows that the player i is harmed by himself action 1 and the player j 's action 1 under the situation $(b_1''\cdots b_n'')$. However the player i is not harmed by player j 's action 1 under the situation $(b_1'\cdots b_n')$. Therefore

$$e_i^{(b_1'\cdots b_n')} = e_i^{1(b_1'\cdots b_n')} < e_i^{1(b_1''\cdots b_n'')} = e_i^{(b_1''\cdots b_n'')}.$$

B · Let $b_i'=0$ and $b_i''=0$. Since $b_j''=1$, we have $i \neq j$. This shows that the player i uses the action 0 under the situations $(b_1''\cdots b_n'')$ and $(b_1'\cdots b_n')$. However he is harmed by the player j 's action 1 under first situation and not under the second one. Therefore

$$e_i^{(b_1'\cdots b_n')} = e_i^{0(b_1'\cdots b_n')} < e_i^{0(b_1''\cdots b_n'')} = e_i^{(b_1''\cdots b_n'')}.$$

C · Let $b_i'=0$ and $b_i''=1$. Similarly, we have

$$e_i^{(b_1'\cdots b_n')} = e_i^{0(b_1'\cdots b_n')} < e_i^{1(b_1''\cdots b_n'')} = e_i^{(b_1''\cdots b_n'')}.$$

To sum up, we obtain $e_i^{(b_1'\cdots b_n')} < e_i^{(b_1''\cdots b_n'')}$, $i=1,2,\cdots,n$.

（3）It is obvious from (1) and (2).

$e_i^{(b_1\cdots b_n)}$ is called by joint name environmental negative utility.

3. COMMON HAZARD DEGREE

Definition 3 $h(b_1\cdots b_n) = \sum_{i=1}^n e_i^{(b_1\cdots b_n)} \Big/ \sum_{i=1}^n e_i^{(1\cdots 1)}$ is called common hazard degree of the situation $(b_1\cdots b_n)$.

By monotonicity of environmental negative utility, we have

Theorem 2（**Monotonicity of Common hazard degree**） $h(b_1'\cdots b_n') < h(b_1''\cdots b_n'')$ *if* $b_1'\cdots b_n' \prec b_1''\cdots b_n''$.

It is explained as that the more the players who use their actions 1 are, the larger the common hazard degree is.

Theorem 3（Common Hazard Degree Inequality）

（1） $h(b_1\cdots b_n)=0$ *iff* $b_1\cdots b_n = 0\cdots 0$, *（2）* $h(b_1\cdots b_n)=1$ *iff* $b_1\cdots b_n = 1\cdots 1$, *and（3）* $0 \le h(b_1\cdots b_n) \le 1$, $\forall (b_1\cdots b_n) \in A$.

Proof: By theorem 1, we have $e_i^{(1\cdots 1)} > e_i^{(0\cdots 0)} = 0$, $i=1,2,\cdots,n$. So $\sum_{i=1}^n e_i^{(1\cdots 1)} > 0$.

（1）$h(b_1\cdots b_n)=0$ if and only if $\sum_{i=1}^n e_i^{(b_1\cdots b_n)} = 0$ if and only if $e_i^{(b_1\cdots b_n)}=0$, $i=1,2,\cdots,n$ if and only if $b_i = 0$, $i = 1, 2, \cdots, n$ if and only if $b_1 \cdots b_n = 0 \cdots 0$.

（2） Let $h(b_1 \cdots b_n) = 1$. Then $\sum_{i=1}^{n} e_i^{(b_1 \cdots b_n)} = \sum_{i=1}^{n} e_i^{(1 \cdots 1)}$, i.e. $\sum_{i=1}^{n} (e_i^{(1 \cdots 1)} - e_i^{(b_1 \cdots b_n)}) = 0$. We have $e_i^{(1 \cdots 1)} - e_i^{(b_1 \cdots b_n)} \ge 0$, $i = 1, 2, \cdots, n$, so $e_i^{(1 \cdots 1)} = e_i^{(b_1 \cdots b_n)}$, $i = 1, 2, \cdots, n$. Obviously, $b_1 \cdots b_n \le 1 \cdots 1$. Assume that $b_1 \cdots b_n < 1 \cdots 1$. By theorem 1, we have that $e_i^{(1 \cdots 1)} > e_i^{(b_1 \cdots b_n)}$, $i = 1, 2, \cdots, n$, a contradiction. Therefore $b_1 \cdots b_n = 1 \cdots 1$. Conversely, let $b_1 \cdots b_n = 1 \cdots 1$. By theorem 1, we have $e_i^{(1 \cdots 1)} = e_i^{(b_1 \cdots b_n)}$, $i = 1, 2, \cdots, n$. Hence $h(b_1 \cdots b_n) = 1$.

（3） Obviously, $0 \cdots 0 \prec= b_1 \cdots b_n \prec= 1 \cdots 1$, $\forall (b_1 \cdots b_n) \in A$. By（1）and（2）, the result is obtained.

Theorem 3 shows that if all players use their actions 0, then the common hazard is smallest and vice versa; if all players use their actions 1, then the common hazard is largest and vice versa.

4. CONDITIONS FOR NASH EQILIBRIA

Theorem 4 $(0 \cdots 0)$ is an Nash equilibrium of the gross profit-environment game $\Gamma \equiv [N; (A_i); (P_i)]$ if and only if $g_i \le e_i^{1(0 \cdots 010 \cdots 0)}$, $i = 1, 2, \cdots, n$.

Proof: $(0 \cdots 0)$ is Nash equilibrium if and only if

$$0 = P_i(0 \cdots 0 \cdots 0) \ge P_i(0 \cdots 010 \cdots 0) = g_i - e_i^{1(0 \cdots 010 \cdots 0)}, \; i = 1, 2, \cdots, n$$

if and only if $g_i \le e_i^{1(0 \cdots 010 \cdots 0)}$, $i = 1, 2, \cdots, n$.

Corollary If $(0 \cdots 0)$ is Nash equilibrium of the gross profit-environment game $\Gamma \equiv [N; (A_i); (P_i)]$, then $g_i \le e_i^{1(b_1 \cdots b_{i-1} 1 b_{i+1} \cdots b_n)}$, $i = 1, 2, \cdots, n$, for any $(b_1 \cdots b_{i-1} 1 b_{i+1} \cdots b_n) \in A$.

Theorem 5 $(1 \cdots \underset{m}{1} 0 \cdots 0)$ is an Nash equilibrium of the game $\Gamma \equiv [N; (A_i); (P_i)]$ if

$$g_i \ge e_i^{1(1 \cdots \underset{i}{1} \cdots \underset{m}{1} 0 \cdots 0)}, \; g_j \le e_j^{1(1 \cdots \underset{m}{1} 0 \cdots \underset{j}{1} \cdots 0)} - e_j^{0(1 \cdots \underset{m}{1} 0 \cdots \underset{j}{0} \cdots 0)}, \; i = 1, 2, \cdots, m, \; j = m+1, \cdots, n.$$

Proof: Since

$$g_i \ge e_i^{1(1 \cdots \underset{i}{1} \cdots \underset{m}{1} 0 \cdots 0)} > e_i^{1(1 \cdots \underset{i}{1} \cdots \underset{m}{1} 0 \cdots 0)} - e_i^{0(1 \cdots \underset{i}{0} \cdots \underset{m}{1} 0 \cdots 0)}, \; i = 1, 2, \cdots, m,$$

$$g_j \le e_j^{1(1 \cdots \underset{m}{1} 0 \cdots \underset{j}{1} \cdots 0)} - e_j^{0(1 \cdots \underset{m}{1} 0 \cdots \underset{j}{0} \cdots 0)}, \; j = m+1, \cdots, n,$$

We have that

$$P_i(1\cdots 1\underset{i}{0}1\cdots \underset{m}{1}0\cdots 0) = -e_i^{0(1\cdots 1\underset{i}{0}1\cdots \underset{m}{1}0\cdots 0)} < g_i - e_i^{1(1\cdots 1\underset{i}{1}1\cdots \underset{m}{1}0\cdots 0)} = P_i(1\cdots 1\underset{i}{1}1\cdots \underset{m}{1}0\cdots 0),$$

$$P_i(1\cdots \underset{m}{1}0\cdots \underset{j}{1}\cdots 0) = g_j - e_j^{1(1\cdots \underset{m}{1}0\cdots \underset{j}{1}\cdots 0)} \le -e_j^{0(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0)} = P_i(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0),$$

$i = 1, 2, \cdots, m$, $j = m+1, \cdots, n$.

Theorem 6 $(0\cdots 0)$ *must be realized if* $(0\cdots 0) \in NE$.

Proof: Let $(1\cdots \underset{m}{1}0\cdots 0)$ and $(0\cdots 0)$ are Nash equilibria. By Corollary of theorem 4, we have

$$P_i(1\cdots \underset{i}{1}\cdots \underset{m}{1}0\cdots 0) = g_i - e_i^{1(1\cdots \underset{i}{1}\cdots \underset{m}{1}0\cdots 0)} \le 0 = P_i(0\cdots \underset{i}{0}\cdots \underset{m}{0}\cdots 0\cdots 0),\ i = 1, 2, \cdots, m,$$

$$P_j(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0) = -e_i^{0(1\cdots \underset{m}{1}0\cdots \underset{j}{0}\cdots 0)} < 0 = P_i(0\cdots \underset{m}{0}\cdots 0\cdots \underset{j}{0}\cdots 0),\ j = m+1, \cdots, n.$$

Hence $(0\cdots 0)$ is better than $(1\cdots \underset{m}{1}0\cdots 0)$ for every player. So $(0\cdots 0)$ must be realized.

5. N-M STABLE SETS

Now we consider the case $(0\cdots 0) \notin NE$.

Definition 4 (Jiang D.Y., 2007; 2008) $V(\varnothing \ne V \subseteq NE \setminus \{(0\cdots 0)\})$ is called an N-M stable set of $[NE \setminus \{(0\cdots 0)\}, \prec]$ if it satisfies the conditions

（1） $\neg(a \prec b) \wedge \neg(b \prec a), \forall a, b \in V$ and (2) $\forall a \in NE \setminus (V \bigcup \{(0\cdots 0)\}), \exists b \in V, a \prec b$.

Theorem 7 (Jiang D.Y., 2007; 2008) There exists one and only one N-M stable set in $[NE \setminus \{(0\cdots 0)\}, \prec]$.

Theorem 8 Suppose V is an N-M stable set in $[NE \setminus \{(0\cdots 0)\}, \prec]$. Then for any $a \in V$, there exists none $b \in NE \setminus \{(0\cdots 0)\}$ such $a \prec b$.

Proof: Assume there exist $a \in V$ and $b \in NE \setminus \{(0\cdots 0)\}$ such that $a \prec b$. By the condition 1, we have $b \in NE \setminus (V \bigcup \{(0\cdots 0)\})$. By the condition 2, we have $b \prec a$. It contradicts to $a \prec b$.

Definition 5 An element $b \in NE \setminus \{(0\cdots 0)\}$ is called a greatest element in $[NE \setminus \{(0\cdots 0)\}, \prec]$ if $a \prec b, \forall a \in NE \setminus \{(0\cdots 0), b\}$.

Theorem 9 $V=\{b\}$ is N-M stable set of $[NE\setminus\{(0\cdots 0)\},\prec]$ if and only if b is the greatest element in $NE\setminus\{(0\cdots 0)\}$.

6. ENVIROMENT CRISIS THEOREMS

Definition 6 A player $j(m+1\le j\le n)$ is called a sufferer about the situation $(1\cdots \underset{m}{1}0\cdots 0)$.

Theorem 10 There exist elements in N-M stable set V whose common hazard degree is greater than that of any one in $NE\setminus(V\bigcup\{(0\cdots 0)\})$ and probability that they are realized is greater than that of any one in $NE\setminus(V\bigcup\{(0\cdots 0)\})$.

Proof: We have $(b_1\cdots b_n)\prec(b_1^*\cdots b_n^*)$ for any $(b_1\cdots b_n)\in NE\setminus(V\bigcup\{(0\cdots 0)\})$. By theorem 3，we have that $h(b_1\cdots b_n)<h(b_1^*\cdots b_n^*)$. And for any $(b_1\cdots b_n)\in NE\setminus(V\bigcup\{(0\cdots 0)\})$, there exists $(b_1^*\cdots b_n^*)\in V$ such that $(b_1\cdots b_n)\prec(b_1^*\cdots b_n^*)$. Sufferers about $(b_1\cdots b_n)$ are more than ones about $(b_1^*\cdots b_n^*)$. Therefore players to prefer $(b_1^*\cdots b_n^*)$ are more than players to do $(b_1\cdots b_n)$. Thus probability that $(b_1^*\cdots b_n^*)$ is realized is greater than one that $(b_1\cdots b_n)$ is.

Theorem 11 It is the most probable event to realize Nash equilibrium $(1\cdots 1)$ with the greatest common hazard degree if $(1\cdots 1)\in NE$ and $(0\cdots 0)\notin NE$.

REFERENCES

Hardin G.. (1968). The tragedy of he Commons. *Science*, *162:*1243-1248

Zhang W.Y. . (1996). *Game Theory and Information Economics.* Shanghai: Shanghai People Press.

Jiang D.Y., etc. . (2006). Realizblity of Expected Nash Equilibria of N-Person Condition Games Under Strong Common Knowledge System. *International Journal of Innovative Computing, Information and Control,* 2(4): 761-770

Jiang D.Y. Equivalent Representations of Bi-matrix Games. *International Journal of Innovative Computing. Information and Control*, (to appear)

Jiang D.Y. . (2005). etc., Nash Equilibrium in Expectation of n-person Non-cooperative Condition Games under Greatest Entropy Criterion. *Systems Engineering, 23(11)*: 108-111

Jiang D.Y. . (2005). etc., Applications of n-person non cooperative condition game to environmental management. *Journal of Danlian Maritime University, 31(4)*: 49-52

Jiang D.Y. . (2007). A Sustainable Development Game in Management Science. *Proceeding of International Conference of Modelling and Simulation*. World Academic Press, pp.79-82

Jiang D.Y. . (2007). Gross Interest-Environment Games in Economic Management Science. *Management Science and Engineering, 1(1)*:25-32.

Jiang D.Y. . (2007). Neumann-Morgenstern Stable Set of a Finite Static Strategy Game. *Journal of Mathematical Control Science and Applications, 1(2)*:251-260

Jiang D.Y. . (2008). *Theory of game with entropy and its applications.* Beijing: Science Press