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Unified Model of Relative Rotation, Non-linear Dynamics System and Chaos Movement Analysis[1]

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Abstract: The two end surface revolution axis relative rotation system and a kind of two qualities relative rotation system unified non-linear dynamics model are established, in the non-linear resiliency and the generalized damping force. And the chaos movement performance of unification system is studied, under weak cyclical strength condition as well.
Key words: Relative rotation; Non-linear dynamic system; Chaos movement

1. INTRODUCTION

Carmeli established the relativity theory of rotation mechanics in 1985， in the process of studying rotational motion and rotation mechanics (Carmeli . M, 1985; Carmeli . M, 1986). Luo shao Kai established the relativity analysis theory of rotation mechanics (LUO, 2008), in 1996. In reference to the relativity theory, Literature (DONG & LIU, 2002; DONG & WANG, 2004; WANG, 1005; ZHAO & LIU, 2006)established the constant coefficient linearity and the variable coefficient linear dynamics model of relative rotation between two random sections of circular cylinder and make qualitative and quantitative analysis to the system. Literature (WANG, 2005; SHI &LIU, 2007) established separately a kind of two end surface revolution axis system relative rotation non-linear dynamics model and a kind of two quality relative rotation nonlinear system dynamics model and discussed the system stability and the approximate solution. However, the above studies to rotation dynamics system have their limitations since they are confined to the condition that system recovery is linear resiliency (DONG & LIU, 2002; DONG & WANG, 2004; WANG, 2005; ZHAO & LIU, 2006; WANG, 2005; SHI &LIU, 2007).

This article illustrates the non-linear resiliency origin, then discusses the non-linear dynamics model unity for the above-mentioned two kinds of relative rotation dynamics system under the

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non-linear resiliency and finally applies chaos system analysis theory to reveal unified system chaos movement performance under the influence of weak cyclical disturbing force.

2. NON-LINEAR RESILIENCY ORIGIN

In the elastic system hooke's law expressed the elastic potential energy $U(x)$ and the displacement square is proportional, namely

$$U(x) = \frac{1}{2}Kx^2$$

in which K is the elasticity coefficient, then the system resiliency $g(x)$ and the displacement is proportional.

$$g(x) = \frac{dU}{dx} = Kx$$

by Newton second law system equation of motion

$$m\ddot{x} + Kx = 0$$

the above equation is linear, namely obeys the hooke's law the elastic system is linear.

Therefore, in fact many elastic systems (including project in each kind of component and so on) does not obey above simple rule (LIU & PENG, 2004), the general elastic potential energy takes the following form

$$U(x) = \frac{1}{2}kx^2 + \frac{1}{3}\lambda x^3 + \frac{1}{4}\mu x^4 + \cdots$$

then may result in the system the resiliency

$$g(x) = kx + \lambda x^2 + \mu x^3 + \cdots$$

the equation of motion obtain which by such U(x) and g(x) is naturally non-linear. Takes how many items in the potential energy multinomial only then appropriate or correct, should act according to the concrete question and the request determined.

3. THE UNITY OF TWO KIND OF RELATIVE ROTATION NONLINEAR SYSTEM DYNAMICS MODEL

If in literature (WANG, 2005), takes the damping force (damping moment) is

$$T_1^C = -f(\dot{\theta}_1 - \dot{\theta}_2), T_2^C = f(\dot{\theta}_1 - \dot{\theta}_2),$$

takes the non-linear elastic potential energy is

$$U(\theta_1 - \theta_2) = \frac{1}{2}k(\theta_1 - \theta_2)^2 + \frac{1}{3}\lambda(\theta_1 - \theta_2)^3 + \frac{1}{4}\mu(\theta_1 - \theta_2)^4 + \cdots$$

hen system resiliency

$$g(\theta_1-\theta_2)=k(\theta_1-\theta_2)+\lambda(\theta_1-\theta_2)^2+\mu(\theta_1-\theta_2)^3+\cdots$$

from this may result in two end surface revolution axis system relative rotation the non-linear dynamics model is

$$\begin{aligned}&\frac{1}{3}J\ddot{\theta}_1+\frac{1}{6}J\ddot{\theta}_2+f(\dot{\theta}_1-\dot{\theta}_2)+g(\theta_1-\theta_2)=T_1\\&\frac{1}{6}J\ddot{\theta}_1+\frac{1}{3}J\ddot{\theta}_2-f(\dot{\theta}_1-\dot{\theta}_2)-g(\theta_1-\theta_2)=T_2\end{aligned}\tag{1}$$

J is the circular cylinder during random two lateral section rotation inertia, θ_1 and θ_2 respectively is two lateral section corners, T_1 and T_2 respectively is two lateral section place sur- moments of force.

In literature (SHI & LIU, 2007), takes also the non-linear elastic potential energy is $U(\theta_1-\theta_2)$, then a kind of two quality relative rotation nonlinear system dynamics model is

$$\begin{aligned}&I_1\ddot{\theta}_1+f(\dot{\theta}_1-\dot{\theta}_2)+g(\theta_1-\theta_2)=F_1\\&I_2\ddot{\theta}_2-f(\dot{\theta}_1-\dot{\theta}_2)-g(\theta_1-\theta_2)=F_2\end{aligned}\tag{2}$$

In which I_1, I_2 for the system centralism quality rotation inertia, θ_1 and θ_2 respectively is two lateral section corners, F_1 and F_2 respectively is two lateral section place sur- moments of force.

by the equation (1) and (2) obtains

$$\ddot{\theta}_1-\ddot{\theta}_2+\frac{12}{J}f(\dot{\theta}_1-\dot{\theta}_2)+\frac{12}{J}g(\theta_1-\theta_2)=\frac{6}{J}(T_1-T_2)\tag{3}$$

$$\ddot{\theta}_1-\ddot{\theta}_2+\frac{I_1+I_2}{I_1I_2}f(\dot{\theta}_1-\dot{\theta}_2)+\frac{I_1+I_2}{I_1I_2}g(\theta_1-\theta_2)=\frac{1}{I_1I_2}(I_2F_1-I_1F_2)\tag{4}$$

in the equation (3) and in (4), makes

$\varphi=\theta_1-\theta_2$, $a_1=\frac{12}{J}$ or $\frac{I_1+I_2}{I_1I_2}$, $a_2=\frac{12}{J}$ or $\frac{I_1+I_2}{I_1I_2}$,

$F(t)=\frac{6}{J}(T_1-T_2)$ or $\frac{1}{I_1I_2}(I_2F_1-I_1F_2)$

obtains two kind of systematic unified form relative rotation non-linear dynamics models is

$$\ddot{\varphi}+a_1f(\dot{\varphi})+a_2g(\varphi)=F(t)\tag{5}$$

4. THE HOMOCLINIC ORBIT AND CHAOS OF SYSTEM

In the equation (5), takes

$$f(\dot{\varphi}) = u_1\dot{\varphi} + u_3\dot{\varphi}^3,\ \ g(\varphi) = -\varphi^3 + \varphi^5,\ a_1 = \varepsilon,\ a_2 = 1,\ F(t) = \varepsilon K_e \sin(\Omega t)$$

then obtains

$$\ddot{\varphi} + \varepsilon(u_1\dot{\varphi} + u_3\dot{\varphi}^3) - \varphi^3 + \varphi^5 = \varepsilon K_e \sin\Omega t \tag{6}$$

Makes $x = \varphi, y = \dot{x}$, then by equation (6) obtains

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= x^3 - x^5 + \varepsilon(K_e \sin(\Omega t) - u_1 y - u_3 y^3) \end{aligned} \tag{7}$$

Supposes $X = (x, y)^T$, $f(X) = (y, x^3 - x^5)^T$, $g(X,t) = [0, K_e \sin(\Omega t) - u_1 y - u_3 y^3]^T$, results in matrix equation

$$\dot{X} = f(X) + \varepsilon g(X,t) \tag{8}$$

When $\varepsilon = 0$, the system (7) is non-disturbance system, namely

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= x^3 - x^5 \end{aligned} \tag{9}$$

Makes

$$\begin{aligned} \dot{x} &= y = 0 \\ \dot{y} &= x^3 - x^5 = 0 \end{aligned} \tag{10}$$

results in fixed point $(-1,0)$ $(1,0)$ and the tertiary fixed point $(0,0)$.

The system (9) secular equation is

$$\begin{vmatrix} 0-\lambda & 1 \\ 3x^2 - 5x^4 & 0-\lambda \end{vmatrix} = 0 \tag{11}$$

$$\lambda = \pm\sqrt{3x^2 - 5x^4} \tag{12}$$

When the fixed point is $(\pm 1,0)$, $\lambda = \pm\sqrt{2}i$, therefore $(\pm 1,0)$ is the center. When the fixed point is $(0,0)$, $\lambda = 0$, must discuss the fixed point 4 nature.

Because in $x^3 - x^5$ does not contain factor y , therefore $(0,0)$ is the isolated singular points. Not harasses system (9) is possible to write

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= b_k x^3 - c x^5 \end{aligned} \tag{13}$$

among them, $b_k = c = 1$, $k = 3$, k is fixed point renumbering. According to the Briot-Bouquet lemma may know , $(0,0)$ nature is decided by k , because of $b_k = b_{2\times1+1} = 1 > 0$, therefore fixed point $(0,0)$ is a saddle point. The system (9) Hamilton quantity is

$$H(x, y) = \frac{1}{2}y^2 - \frac{1}{4}x^4 + \frac{1}{6}x^6 \tag{14}$$

When $H(0,0) = 0$, existing connection saddle point $(0,0)$ same sleeping orbital $\{q_{\pm}^0(t)\}$, the homoclinic orbit solution to be as follows

Makes,

$$\begin{aligned} &H(0,0) = \frac{1}{2}y^2 - \frac{1}{4}x^4 + \frac{1}{6}x^6 = 0 \\ &\dot{x} = y \end{aligned} \tag{15}$$

By equation (15) obtaining

$$y = \pm\frac{x^2}{\sqrt{2}}\sqrt{1 - \frac{2}{3}x^2} \tag{16}$$

Namely

$$\frac{dx}{x^2\sqrt{1 - \frac{2}{3}x^2}} = \pm\frac{dt}{\sqrt{2}} \tag{17}$$

Makes $x = \sqrt{\frac{2}{3}}\cot z$ and the integral results in to (17) type,

$$-\sqrt{\frac{2}{3}}\cot z = \pm\frac{t}{\sqrt{2}} + C \tag{18}$$

$\cot z = \dfrac{\sqrt{3 - 2x^2}}{\sqrt{2}x}$ substitutes (18) type, attention when $t = 0$, $x_0 = \pm\sqrt{\frac{3}{2}}$, $C = 0$ .after coordinating, obtains

$$x = \pm\sqrt{\frac{6}{3t^2 + 4}} \tag{19}$$

To x derivation obtains

$$y = \pm\frac{3\sqrt{3}t}{(3t^2 + 4)^{3/2}} \tag{20}$$

Therefore, the two homoclinic orbit parameter is

$$\begin{aligned} x_{\pm}^0(t) &= \pm\sqrt{\frac{6}{3t^2 + 4}} \\ y_{\pm}^0(t) &= \mp\frac{3\sqrt{6}t}{(3t^2 + 4)^{3/2}} \end{aligned} \tag{21}$$

According to the homoclinic theory of $Smale - Birkhoff$ ，when ε is enough small, the system (7) possibly has under the $Smale$ horseshoe chaos. The system (7) Melnikov function is

$$\begin{aligned} M_{\pm}(t_0) &= \int_{-\infty}^{+\infty} f(q_{\pm}^{0}(t)) \wedge g(q_{\pm}^{0}(t), t+t_0)dt = \\ &\int_{-\infty}^{+\infty} [K_e \sin\Omega(t+t_0) - u_1 y_{\pm}^{0}(t) - u_3 (y_{\pm}^{0}(t))^3] y_{\pm}^{0}(t)dt = \\ &K_e I \sin(\Omega t_0) - u_1 I_1 - u_3 I_3 \end{aligned} \tag{22}$$

In which

$$I_1 = \int_{-\infty}^{+\infty} (y_{\pm}^{0}(t))^2 dt = \int_{-\infty}^{+\infty} \frac{2t^2}{[t^2 + (\frac{2}{\sqrt{3}})^2]^3} dt = \frac{3\sqrt{3}}{64}\pi$$

$$I_3 = \int_{-\infty}^{+\infty} (y_{\pm}^{0}(t))^4 dt = \int_{-\infty}^{+\infty} \frac{4t^2}{[t^2 + (\frac{2}{\sqrt{3}})^2]^6} dt = \frac{4321}{8197}\sqrt{3}\pi$$

$$I = \int_{-\infty}^{+\infty} \frac{3\sqrt{6}t}{(3t^2+4)^{3/2}} \sin(\Omega t)dt = \frac{\Omega}{3}\int_{-\infty}^{+\infty} \frac{1}{(3t^2+4)^{1/2}} \cos(\Omega t)dt$$

By (22) type, may know when $|I| > \left|\frac{1}{K_e}(I_1 u_1 + I_3 u_3)\right|$, this time must have t_0' to cause $M_{\pm}(t_0') = 0$, but $M_{\pm}'(t_0') = \pm\Omega K_e I \cos(\Omega t_0') \neq 0$, therefore $M_{\pm}(t_0)$ has the single zero point, therefore produces the $Smale$ horse's hoof and the chaos.

Note: The precise value of integral I is very difficult to work out, but easy to obtain the approximate value of integral I applying numerical integration.

5. THE HETEROCLINIC ORBIT AND CHAOS OF SYSTEM

In the equation(5),takes $f(\dot{\varphi}) = u_1\dot{\varphi} + u_3\dot{\varphi}^3$ ， $g(\varphi) = \varphi - \varphi^3$ ， $a_1 = \varepsilon$ ， $a_2 = 1$ ， $F(t) = \varepsilon K_e \sin(\Omega t)$ ，

Then obtains

$$\ddot{\varphi} + \varepsilon(u_1\dot{\varphi} + u_3\dot{\varphi}^3) + \varphi - \varphi^3 = \varepsilon K_e \sin\Omega t \tag{23}$$

Makes $x = \varphi, y = \dot{x}$,then by equation (23) obtains

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x + x^3 + \varepsilon(K_e \sin(\Omega t) - u_1 y - u_3 y^3) \end{aligned} \tag{24}$$

Supposes $X = (x, y)^T$, $f(X) = (y, -x + x^3)^T$, $g(X,t) = [0, K_e \sin(\Omega t) - u_1 y - u_3 y^3]^T$ results in matrix equation

$$\dot{X} = f(X) + \varepsilon g(X,t) \qquad (25)$$

When $\varepsilon = 0$,equation (25) is *Hamilton* system , obtains the heteroclinic orbit of system (LIU & CHEN, 2003)

$$q_{\pm}^{0}(t) = [x_{\pm}^{0}(t), y_{\pm}^{0}(t)]^{T} = [\pm th(\frac{\sqrt{2}}{2}t), \pm\frac{\sqrt{2}}{2}\sec h^{2}(\frac{\sqrt{2}}{2}t)]^{T} \qquad (26)$$

By (12)type , obtains

$$f(q_{\pm}^{0}(t)) = [y_{\pm}^{0}(t), -x_{\pm}^{0}(t) + (x_{\pm}^{0}(t))^{3}]^{T}$$

$$g(q_{\pm}^{0}(t), t+t_{0}) = [0, K_{e}\sin\Omega(t+t_{0}) - u_{1}y_{\pm}^{0}(t) - u_{3}(y_{\pm}^{0}(t))^{3}]^{T}$$

$$f(q_{i}^{0}(t)) \wedge g(q_{i}^{0}(t), t+t_{0}) = [K_{e}\sin\Omega(t+t_{0}) - u_{1}y_{\pm}^{0}(t) - u_{3}(y_{\pm}^{0}(t))^{3}]y_{\pm}^{0}(t)$$

therefore，equation (25) Melnikov function is

$$M_{\pm}^{0}(t_{0}) = \int_{-\infty}^{+\infty} f(q_{i}^{0}(t)) \wedge g(q_{i}^{0}(t), t+t_{0})dt$$

$$= \int_{-\infty}^{+\infty}[K_{e}\sin\Omega(t+t_{0}) - u_{1}y_{\pm}^{0}(t) - u_{3}(y_{\pm}^{0}(t))^{3}]y_{\pm}^{0}(t)dt \qquad (27)$$

$$= K_{e}I_{1} - u_{1}I_{2} - u_{3}I_{3} - hI_{4} - K_{5}I_{5}$$

Because

$$I_{1} = \int_{-\infty}^{+\infty} y_{\pm}^{0}(t)\sin\Omega(t+t_{0})dt = \int_{-\infty}^{+\infty}(\sin(\Omega t)\cos(\Omega t_{0}) + \cos(\Omega t)\sin(\Omega t_{0}))y_{\pm}^{0}(t)dt$$

$$= \sin(\Omega t_{0})\int_{-\infty}^{+\infty}\cos(\Omega t)y_{\pm}^{0}(t)dt = \pm\sqrt{2}\pi\csc h(\frac{\sqrt{2}}{2}\Omega\pi)\sin(\Omega t_{0})$$

$$I_{2} = \int_{-\infty}^{+\infty}(y_{\pm}^{0}(t))^{2}dt = \frac{1}{2}\int_{-\infty}^{+\infty}\sec h^{4}\frac{\sqrt{2}}{2}tdt = \frac{\sqrt{2}}{2}\int_{-\infty}^{+\infty}\sec h^{4}udu =$$

$$\frac{\sqrt{2}}{2}\int_{-\infty}^{+\infty}(1 - th^{2}u)dthu = \frac{2}{3}\sqrt{2}$$

$$I_{3} = \int_{-\infty}^{+\infty}(y_{\pm}^{0}(t))^{4}dt = \frac{1}{4}\int_{-\infty}^{+\infty}\sec h^{8}\frac{\sqrt{2}}{2}tdt = \frac{\sqrt{2}}{4}\int_{-\infty}^{+\infty}\sec h^{6}udu =$$

$$\frac{\sqrt{2}}{4}\int_{-\infty}^{+\infty}(1 - 3thu + 3th^{2}u - th^{3}u)dthu = \frac{\sqrt{2}}{6}$$

Therefore, obtains

$$M_{\pm}^{0}(t_{0}) = \pm\sqrt{2}\pi K_{e}\csc h(\frac{\sqrt{2}}{2}\Omega\pi)\sin(\Omega t_{0}) - \frac{2}{3}\sqrt{2}u_{1} - \frac{\sqrt{2}}{6}u_{3} \qquad (28)$$

By (28) type , may know when $|K_{e}| < \frac{1}{\pi}\left|(\frac{2}{3}u_{1} + \frac{1}{6}u_{3})sh(\frac{\sqrt{2}}{2}\Omega\pi)\right|$, this time must t_{0}' to cause $M_{\pm}(t_{0}') = 0$ ，but $M_{\pm}'(t_{0}') = \pm\sqrt{2}\pi\Omega\csc h(\frac{\sqrt{2}}{2}\Omega\pi)\cos(\Omega t_{0}') \neq 0$, therefore $M_{\pm}(t_{0})$ has the single zero point , therefore *Smale* horseshoe and the chaos.

6. CONCLUSION

This article elaborates that non-linear dynamics system of relative rotation of two end surface revolution axis has the unified dynamics model as one class two quality relative rotation non-linear dynamics system. Under the disturbance of weak cyclical force, the system is proved to have the homoclinic orbit, paving way for conditions that the system will engender *Smale* horseshoe chaos movement.

REFERENCES

Carmeli . M.. (1985). The dynamics of rotating bodies. *Found. Phys*, 889-903.

Carmeli . M.. (1986). Rotational relativity theory. *Int. J. Theor Phys* ., 89-94 .

DONG . Q . L & Liu, B..(2002). A similarity dynamic equation of an elastic rotation shaft with twin sides under the Gelileo coordinate conversion. *Acta Phys. Sin*. Beijing: Chinese Physical Society, 2191-2196.

DONG, Q . L, Wang, K & Liu. (2004). B.. An integral solution for the relative-rotation dynamic equation of a cylinder. *Acta Phys. Sin*. Beijing: Chinese Physical Society, 337-342.

LIU, B . Z & PENG, J . H.. (2004). *Nonlinear Dynamics*. Beijing: High education press.

LIU, Y . Z & Chen, L . Q.. (2003). *Nonlinear Vibrations*. Beijing: High education press.

LUO, S . K ,. (2008)The theory of relativistic analytical mechanics of the rotational systems. *Appl. Math. and Mech.* Chongqing: Chinese Applied Mathematics and Mechanics Society, 43-53 .

WANG, K. (2005). Steadiness of relatively rotation dynamics equation and it's solution for a kind stickiness flexibility. *Acta Phys. Sin*. Beijing: Chinese Physical Society, 3987-3991.

WANG, K. (2005) Steadiness and approximative solution of nonlinear dynamics system of two endsurfaces rotation spindle's relative rotation. *Acta Phys. Sin*. Beijing: Chinese Physical Society, , 5530-5533.

SHI, P . M & Liu, B. (2007). Stability of nonlinear dynamical system of relative rotation and approximate solution under forced excitation. *Acta Phys. Sin*. Beijing: Chinese Physical Society, 3678-3682.

ZHAO, W & Liu, B.. (2006). Analysis of stability of the equilibrium state of periodic motion in a nonlinear relative-rotation system. *Acta Phys. Sin*. Beijing: Chinese Physical Society, 3852-3857.