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Option Pricing Model With Continuous Dividends

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Abstract

This paper discusses the problem of pricing on European options in jump-diffusion model by martingale method. We assuming jump process are more commen then Possion process a kind of nonexplosive counting process. Supposing that the dividend for each share of the security is paid continuously in time at a rate equal to a fixed fraction of the price of the security. By changing the basic assumption of R.C.Merton option pricing model to the assumption. It is established that the behavior model of the stock pricing process is jump-diffusion process. With risk-neutral martingale measure, pricing formula and put-call parity for European options with continuous dividends are obtained by stochastic analysis method. The results of Margrabe are generalized.

Key words: Continuous dividend; Jump-diffusion; Option pricing; Count process

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INTRODUCTION

Option pricing theory is always one of the kernel problems on financial mathematics. Together with the capital asset pricing theory ,the portfolio selection theory, the effective theory of market and acting issue, it is regarded as one of the five theory modules in modern finance. Many scholars have done a great deal of researches on option pricing theory and obtained many results which are instructive in financial practice. However, the appearance of important information will cause the stock price to a kind of not continual jumps. A mass of finance practicial has indicated that there is a serious warp between the hypothesis of Black-Scholes model (Black & Scholes, 1973) for the underlying asset price and the realistic markets. Therefore, many scholars put forward many new kinds of option pricing models (Ball & Roma, 1993; Harold & Kushner, 2000; Gill & Wong, 2011; Cai & Mao, 2002; Kou, 2001) by relaxing some assuming conditions of Black-Scholes model. Option pricing theory with jump-diffusion is one of them. But we show that real data cannot always be fit by a geometric Brownian motion model, and that more general models may need to be considered. The appearance of important information will cause the stock price to a kind of no continual jumps (Yang, Zhang, & Xia, 2013; Rieger, 2012; Yang & Hao, 2013; Angelo, 2009). When markets are complete, the existence of optimal strategies can be found Merton (1971), Jeanblanc and Pontier (1990). Follmer and Leukert (2000) discussed semimartingale model, Pham (2000) discussed continuous markets model, Nakano (2004) discussed jump-diffusion mode. Merton (1976) established famous jump-diffusion model when jump process is Possion process and discussed the impact of the dividend on the option, the Black-Scholes formula was extended. Roll and Geske also put forward the pricing model of the American call option with dividend. The Black-Scholes partial differential equation was modified after they consider paying a dividend and the equation was solved. In a continuous setup where the evolution of a single stock is modelled by geometric Brownian motion, Black and Scholes derived a closed-form solution for the value of European-style call and put options by presenting a strategy that duplicates its payoff through continuous trading in the stock and the bond. One of the drawbacks of using geometric Brownian motion as a model for a security's price over time is that it does not allow for the possibility of a discontinuous price jump in either the up or down direction.because such jumps do occur in practice, it is advantageous to consider a model for price evolution that superimposes random jumps on a geometric Brownian motion.

Options are examples of exchange-traded derivative securities—that is,securities whose value depends on the prices of other more basic securities such as stocks or bonds. The option price is the only variable that changes with the market supply and demand, which directly affect the profit and loss of the buyers and sellers, that is the core issue of the options trading.

In this paper, We consider that price of underlying asset price obeys jump-diffusion process, , because of the reality the stock price jumps do not necessarily obey the Poisson process, jump process generalized conforms to the actual situation of stock price movement. We Establish the option-pricing model with continuous dividends. Pricing formula of European option is also given. The results of existing are generalized.

1. CONTINUOUS DIVIDENTS MODELS

For instance,if the stock's price is presently, then in the next dt time units the dividend payment per share of stock owned will be approximately $\rho S\mathrm{d}t$ when is dt small.To begin,we need a model for the evolution of the price of the security over time. One way to obtain a reasonable model is to suppose that all dividends are reinvested in the purchase of additional shares of the stock.Thus,we would be continuously adding additional shares at the rate ρ times the number of shares we presently own. Consequently,our number of shares in growing by a continuously compounded rate ρ.Therefor,if we purchased a single share at time 0, at time t we would have $e^{\rho t}$ shares with a total market value of $e^{\rho t}S$. It seems reasonable to suppose that $e^{\rho t}S$ follows a geometric brownian motion.

It is usual to suppose that,at the moment the dividend is paid, the price of a share instantaneously decreases by the amount of the dividend.If one assumes that the price never drops by at least the amount of the dividend,then buying immediately before and selling immediately after the payment of the dividend would result in an arbitrage; hence, there must be some possibility of a drop in price of at least the amount of the dividend, and the usual assumption-which is roughly in agreement with actual data-is that the price decreases by exactly the dividend paid.

Let $(\Omega,F,P^*,(F_t)_{0\le t\le T})$ be a probability space and $\{W^*(t),0\le t\le T\}$ be a standard Wiener process given on a probability space $(\Omega,F,P^*,(F_t)_{0\le t\le T})$ The market is built with a bond $B(t)$ and a risky asset $S(t)$. We suppose that $B(t)$ is the solution of the equation

$$\mathrm{d}B(t) = r(t)B(t)\mathrm{d}t,\ B(0) = 1. \tag{1}$$

$S(t)$satisfies the stochastic differential equation

$$\mathrm{d}S(t) = S(t\text{-})[(\mu(t))\mathrm{d}t + \sigma(t)\mathrm{d}W^*(t) + (U\text{-}1)\mathrm{d}M(t)],\ S(0) = s_0, \tag{2}$$

where $r(t)$ is risk-free interest rate, $\mu(t)$ is expected stock returns, $\sigma(t)$ is volatility. $M(t) = N(t)\text{-}\int_0^t\lambda(s)ds,\ T\ge t\ge 0$ is the compensated martingale of nonexplosive counting process $\{N_t,\ 0\le t\le T\}$ with intensity parameter $\lambda(t)$.We assume that the filtration $(F_t,\ 0\le t\le T)$ is generated by the $\{W^*(t),\ 0\le t\le T\}$ and martingale $\{M(t),\ 0\le t\le T\}$.

Let us consider the case when the dividend yield,rath than the dividend payoffs,is assumed to be known.More specifically, we assume that the stock continuously pays dividends at some fixed rate.Following the classic paper by Samuelson and Merton (1969), we assume that the effective dividend rate is proportional to the level of the stock price. Although this is rather impractical as a realistic dividend policy associated with a particular stock, Samuelson and Merton's model fits the case of a stock index opyion resonably well.The dividend payments should be used in full,either to purchase additional shares of stock, or to invest in risk free bonds. Consequently,a trading strategy is said to be self-financing when its wealth process satisfies definition 1.

Definition1 (Yuan, 2008) A strategy $\{a(t),b(t)\}$is called self-financing if wealth process

$V(t) = a(t)S(t) + b(t)B(t)$ is satisfied

$$\mathrm{d}V(t) = a(t)\mathrm{d}S(t) + b(t)\mathrm{d}B(t) + \rho(t)a(t)S(t)\mathrm{d}t. \tag{3}$$

The continuous dividend rate is $\rho(t)$, it follows from (1),(2),(3) that

$$\mathrm{d}V(t) = \{a(t)S(t)(u(t) + \rho(t)) + b(t)r(t)B(t)\}\mathrm{d}t + a(t)S(t)\sigma(t)\mathrm{d}W^*(t) + a(t)S(t)(U\text{-}1)\mathrm{d}M(t). \tag{4}$$

Let $V^*(t) = \mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}V(t)$, $S^*(t) = \mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}S(t)$,we can prove the following proposition.

Proposition 1 The following are equivalent

(1)A strategy $\{a(t),\ b(t)\}$is called self-financing.

(2) Wealth process satisfied

$\mathrm{d}V^*(t) = a(t)S^*(t)(\sigma(t)\mathrm{d}W(t) + (U\text{-}1)\mathrm{d}M(t))$, where

$$W(t) = W^*(t) + \int_0^t \theta(s)\mathrm{d}s,\ \theta(t) = \frac{u(t)+\rho(t)-r(t)}{\sigma(t)}, 0\le t\le T.$$

Proof Applying Ito's lemma yield (Yang & Luo, 2006), we have

$$\mathrm{d}V^*(t) = S^*(t)a(t)\Big[\big(u(t)+\rho(t)-r(t)\big)\mathrm{d}t + \sigma(t)\mathrm{d}W^*(t) + (U-1)\mathrm{d}M(t)\Big], \tag{5}$$

let, $W_t = W_t^* + \int_0^t \theta(s)\mathrm{d}s,\ \theta(t) = \dfrac{u(t)+\rho(t)-r(t)}{\sigma(t)}$, $0\le t\le T(5)$ equivalently

$$\mathrm{d}V^*(t) = a(t)S^*(t)\big(\sigma(t)\mathrm{d}W(t) + (U-1)\mathrm{d}M(t)\big).$$

We find it convenient to introduce an auxiliary process $S^{**}(t) = \mathrm{e}^{\int_0^t \rho(s)\mathrm{d}s}S^*(t)$, whose dynamics are given by the stochastic differential equation.

Proposition 2 Let $S^{**}(t)=\mathrm{e}^{\int_0^t \rho(s)\mathrm{d}s}S^*(t)$. We have $\{S^{**}(t)\}$ is satisfied

$$\mathrm{d}S^{**}(t)=S^{**}(t)\big(\sigma(t)\mathrm{d}W(t)+(U-1)\mathrm{d}M(t)\big). \tag{6}$$

Proof For $S^{**}(t)=\mathrm{e}^{\int_0^t \rho(s)\mathrm{d}s}S^*(t)$ and $S^*(t)=\mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}S(t)$,

using Ito's lemma yield, we have

$$\mathrm{d}S^{**}(t)=\mathrm{e}^{\int_0^t \rho(s)\mathrm{d}s}\mathrm{d}S^*(t)+\rho(t)\mathrm{e}^{\int_0^t \rho(s)\mathrm{d}s}S^*(t)\mathrm{d}t, \tag{7}$$

$$\mathrm{d}S^*(t)=\mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}\mathrm{d}S(t)-r(t)\mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}S(t)\mathrm{d}t \cdot \tag{8}$$

Together with (2), (7) and (8), we have

$$\mathrm{d}S^{**}(t)=S^{**}(t)\Big[\big(u(t)+\rho(t)-r(t)\big)\mathrm{d}t+\sigma(t)\mathrm{d}W^*(t)+(U-1)\mathrm{d}M(t)\Big], \tag{9}$$

let, $W_t=W_t^*+\int_0^t\theta(s)\mathrm{d}s$, $\theta(t)=\dfrac{u(t)+\rho(t)-r(t)}{\sigma(t)}$, $0\le t\le T$(9) equivalently

$$\mathrm{d}S^{**}(t)=S^{**}(t)\big(\sigma(t)\mathrm{d}W(t)+(U-1)\mathrm{d}M(t)\big).$$

Applying Dolease-Dade exponential formula (Duffie, 1996), stochastic differential equation (6) equivalently

$$S^{**}(t)=S_0^{**}\prod_{i=1}^{N(t)}U_i\exp\{-\frac{1}{2}\int_0^t\sigma^2(s)\mathrm{d}s+(1-E(U))\int_0^t\lambda(s)\mathrm{d}s+\int_0^t\sigma(s)\mathrm{d}W(s)\}. \tag{10}$$

so $S(t)=s_0\prod_{i=1}^{N(t)}U_i\exp\{\int_0^t[r(s)-\rho(s)-\frac{1}{2}\sigma^2(s)]\mathrm{d}s+(1-E(U))\int_0^t\lambda(s)\mathrm{d}s+\int_0^t\sigma(s)\mathrm{d}W(s)\}$.

Proposition 3 Let $\dfrac{\mathrm{d}P}{\mathrm{d}P^*}=\exp\left\{-\int_0^T\theta(s)\mathrm{d}W^*(s)-\frac{1}{2}\int_0^T\theta^2(s)\mathrm{d}s\right\}$ and $E(U)$ be bounded,then self-financing wealth process is $V_0=E_P\left(e^{-\int_0^T r(s)\mathrm{d}s}V_T\,|F_0\right)$ P is risk-neutral martingale measure.

Proof Applying Girsanov theorem, $W(t)=W^*(t)+\int_0^t\theta(s)\mathrm{d}s$ is standard Wiener process under the martingale P,$M(t)(T\ge t\ge 0)$ is P martingale.

For, because $\sigma(t)$ is integrable function,and $E(U)$ is bounded,we have $V^*(t)=\mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}V(t)$ is P martingale.It follows that $V^*(0)=E_P(V^*(T)|F_0)$.Finally,

$$V_0=E_P\left(e^{-\int_0^T r(s)\mathrm{d}s}V_T\,|F_0\right).$$

2. OPTION PRICING FORMULA WITH CONTINUOUS DIVIDENT

Proposition 4 Assume that the dynamics of a bond $B(t)$ and a risky asset $S(t)$ are given by (1), (2), maturity date T, exercise price K. Then the price of European call option satisfy

$$C(0,S_T)=\sum_{n=0}^{\infty}P_n(T)E[s_0\Phi(d_1)\exp\{-\int_0^T[\rho(s)+(E(U)-1)\lambda(s)]\mathrm{d}s\}\prod_{i=0}^{n}U_i-Ke^{-rT}\Phi(d_2)]$$

where $d_1=\dfrac{\ln\dfrac{s_0\prod_{i=1}^{n}U_i}{K}+(1-E(U))\int_0^T\lambda(s)\mathrm{d}s+\int_0^T\left[r(s)-\rho(s)+\frac{1}{2}\sigma^2(s)\right]\mathrm{d}s}{\sqrt{\int_0^T\sigma^2(s)\mathrm{d}s}}$, $d_2=d_1-\sqrt{\int_0^T\sigma^2(s)\mathrm{d}s}$.

Proof Since $V^*(t)=\mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}V(t)$ is the martingale under risk-neutral martingale measure, then

$$\begin{aligned}C(0,S_T)&=E[\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s}(S(T)-K)^+\,|\mathbf{F}_0]\\&=E\left[\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s}S_T I_{\{S_T\ge K\}}\,|\mathbf{F}_0\right]-KE\left[\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s}I_{\{S_T\ge K\}}\,|\mathbf{F}_0\right]\end{aligned} \tag{11}$$

Let $S_T^n = s_0 \prod_{i=1}^{n} U_i \exp\{\int_0^T [r(s)-\rho(s)-\frac{1}{2}\sigma^2(s)]\mathrm{d}s + (1-E(U))\int_0^T \lambda(s)\mathrm{d}s + \int_0^T \sigma(s)\mathrm{d}W(s)\}$.

$$E\left[\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} S_T I_{\{S_T \geq K\}} \left|\mathbf{F}_0\right.\right] = \sum_{n=0}^{\infty} P_n(T) E(\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} S_T^n I_{\{S_T^n \geq K\}})$$

$$= \sum_{n=0}^{\infty} P_n(T) E(s_0 \prod_{i=1}^{n} U_i \exp\{-\int_0^T [\rho(s)+(E(U)-1)\lambda(s)]\mathrm{d}s\}.$$

$$\int_{-\infty}^{d} \exp\{-\frac{1}{2}\int_0^T \sigma^2(s)\mathrm{d}s - y - \frac{y^2}{2\int_0^T \sigma^2(s)\mathrm{d}s}\} \frac{1}{\sqrt{2\pi\int_0^T \sigma^2(s)\mathrm{d}s}} dy]$$

$$= \sum_{n=0}^{\infty} P_n(T) E(s_0 \prod_{i=1}^{n} U_i \exp\{-\int_0^T [\rho(s)+(E(U)-1)\lambda(s)]\mathrm{d}s\}.$$

$$\int_{-\infty}^{d} \exp\{-\frac{(y+\int_0^T \sigma^2(s)\mathrm{d}s)^2}{2\int_0^T \sigma^2(s)\mathrm{d}s}\} \frac{1}{\sqrt{2\pi\int_0^T \sigma^2(s)\mathrm{d}s}} dy]$$

$$= \sum_{n=0}^{\infty} P_n(T) E(s_0 \prod_{i=1}^{n} U_i \exp\{-\int_0^T [\rho(s)+(E(U)-1)\lambda(s)]\mathrm{d}s\}\Phi(\frac{d+\int_0^T \sigma^2(s)\mathrm{d}s}{\sqrt{\int_0^T \sigma^2(s)\mathrm{d}s}}), \qquad (12)$$

where $d = \ln\dfrac{S_0 \prod_{i=1}^{n} V_i}{K} + (1-E(V))\int_0^T \lambda(s)\mathrm{d}s + \int_0^T \left[r(s)-\rho(s)-\frac{1}{2}\sigma^2(s)\right]\mathrm{d}s$

$$KE\left[\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} I_{\{S_T \geq K\}} \left|\mathbf{F}_0\right.\right] = K\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} \sum_{n=0}^{\infty} P_n(T) E[I_{\{S_T^n \geq K\}}]$$

$$= K\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} \sum_{n=0}^{\infty} P_n(T) E[P(S_T^n \geq K)]$$

$$= K\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} \sum_{n=0}^{\infty} P_n(T) E[P(-\int_0^T \sigma(s)\mathrm{d}W(s) \leq d)]$$

$$= K\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} \sum_{n=0}^{\infty} P_n(T) E[\Phi(\frac{d}{\sqrt{\int_0^T \sigma^2(s)\mathrm{d}s}})] \quad . \qquad (13)$$

let $d_1 = \dfrac{d+\int_0^T \sigma^2(s)\mathrm{d}s}{\sqrt{\int_0^T \sigma^2(s)\mathrm{d}s}} = \dfrac{\ln\dfrac{s_0 \prod_{i=1}^{n} U_i}{K} + (1-E(U))\int_0^T \lambda(s)\mathrm{d}s + \int_0^T \left[r(s)-\rho(s)+\frac{1}{2}\sigma^2(s)\right]\mathrm{d}s}{\sqrt{\int_0^T \sigma^2(s)\mathrm{d}s}}$,

$$d_2 = \frac{d}{\sqrt{\int_0^T \sigma^2(s)\mathrm{d}s}} = d_1 - \sqrt{\int_0^T \sigma^2(s)\mathrm{d}s}.$$

Together with (11), (12) and (13), we have

$$C(0,S_T) = \sum_{n=0}^{\infty} P_n(T) E_n[s_0\Phi(d_1)\exp\{-\int_0^T [\rho(s)+(E(U)-1)\lambda(s)]\mathrm{d}s\}\prod_{i=0}^{n} U_i - Ke^{-rT}\Phi(d_2)].$$

Proposition 5 （put-call parity relation）Assume that the dynamics of a bond $B(t)$ and a risky asset $S(t)$ are given by (1), (2), maturity date T,exercise price K. Then the put-call parity relation may be rewritten as

$$C(t,S_T) - P(t,S_T) = e^{-\int_t^T \rho(s)ds} S_t - Ke^{-\int_t^T r(s)ds}.$$

Proof since $\{S^{**}(t), 0 \leq t \leq T\}$ is P martingale, we have

$$V^*(t) = \mathrm{e}^{-\int_0^t r(s)\mathrm{d}s} V(t),\ S^*(t) = \mathrm{e}^{-\int_0^t r(s)\mathrm{d}s} S(t)$$

We find that put-call parity relation is not affected by the jump process of stock price, but it is affected by continuous dividends. We can useput-call parity to find the price of a European put option on a stock with the same parameters as earlier.When the continuous dividend rate $\rho(t) = 0$ and nonexplosive counting process $\{N_t, 0 \le t \le T\}$ is Possion process with intensity parameter $\lambda(t) = \lambda$, The results of this article are Merton R C's. (1976) conclusion.

In this paper, we assume that the dividends that will be paid to the shareholders during an option's lifetime can be predicted with certainty. We discuss arbitrage pricing within the option pricing model under the assumption that the stock upon which an option is written pays dividends during option's lifetime. Because jumps do occur in practice, it is advantageous to consider a model for price evolution that superimposes random jumps on a geometric Brownian motion. Assumption that jump process is a count process that more general than Poisson process. It is established that the behavior model of the stock pricing process is jump-diffusion process. Pricing formula of European option and put-call parity relation are also given.

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