

## Virtual Experiment Research of Armature-Controlled DC Motor

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### Abstract

In order to make students understand dc motor performances, we design a virtual experiment platform of the armature-controlled DC motor. The block diagram of an armature-controlled DC motor is established based on the research of control parameters. Through analyzing of the control characteristic of the armature-controlled dc motor, we set up block diagram of a speed-control and a position-control of armature-controlled DC motor. The influence factors of speed-control and position-control are analyzed respectively. Simulink model of open-loop speed, speed-control, and position-control is established with MATLAB and Simulink software. Finally we do the experiments using the designed Virtual experiment platform. It was a challenging task to develop platform that provides the students with practical appreciation and understanding of dc motors including modeling, system identification and controller design.

**Key words:** DC motor; Control; Virtual experiment

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### INTRODUCTION

To enhance their understanding of control systems, students must participate in labs and projects involving

practical problems. In this paper we have created a so-called virtual lab with problems mimicking actual practical system, a real system may exhibit properties that are not discussed in most theoretical textbooks but, affect the controller design.

In a realistic system, including an actuator (e.g., a DC motor) and mechanical (gears) and electrical components (amplifiers), issues such as saturation of the amplifier, friction in the motor or backlash in the gears will seriously affect the controller design. Students always become disenchanted after spending many hours designing an impractical controller than would ultimately not work in reality.

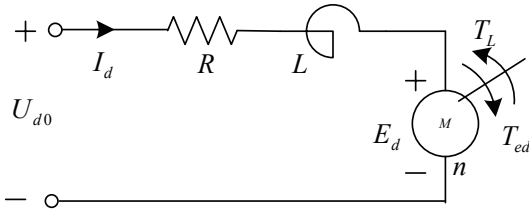
Through the use of MATLAB and Simulink, we managed to create a virtual lab representing some of the behaviors observed in an actual system. All the experiments presented here were compared to real systems in the lab environment, and their accuracy has been verified. These virtual labs include experiments on speed and position control of dc motors followed by a controller design project involving control of a simple robotic system. The focus on dc motors in these experiments was intentional, because of their simplicity and wide usage in numerous industrial applications.

## 1. THE MATHEMATICAL MODEL AND CLOSED-LOOP CONTROL SYSTEM OF DC MOTOR

### 1.1 The Mathematical Model of Dc Motor Armature Circuit in the Condition of Rated Excitation<sup>1</sup> (Samoylenko et al., 2007, 2008)

The mathematical model of dc motor armature circuit is shown as Figure 1.

<sup>1</sup> [http://www.mathworks.com/products/studentversion/sys\\_req.shtml](http://www.mathworks.com/products/studentversion/sys_req.shtml)



**Figure 1**  
Equivalent Circuit of Separately Excited DC Motor in the Condition of Rated Excitation

$$U_{d0} = RI_d + L \frac{dI_d}{dt} + E_d, \quad (1)$$

in the condition of zero initial, Laplace transform formula (1):

$$U_{d0} = RI_d(s) + LI_d(s) + E_d(s). \quad (2)$$

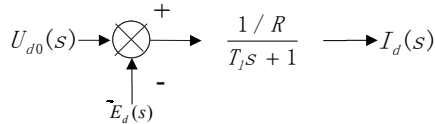
After the equivalent transformation, the Formula (3) can be concluded:

$$U_{d0}(s) - E_d(s) = (R + Ls)I_d(s) = R(1 + \frac{L}{R}s)I_d(s) = R(1 + T_l s)I_d(s) \quad (3)$$

By the Formula (3), the transfer function of voltage and current can be concluded

$$\frac{I_d(s)}{U_{d0}(s) - E_d(s)} = \frac{1/R}{1 + T_l s} \quad (4)$$

According to the Formula (4) the dynamic structure between the armature voltage and current of dc motor can be inferred, as shown in Figure 2.



**Figure 2**  
The Dynamic Structure Between the Armature Voltage and Current of DC Motor

Dynamic equation of DC motor shaft:

$$T_{ed} - T_L = \frac{GD^2}{375} \frac{dn}{dt} \quad (5)$$

The relationship between the load torque and electromagnetic torque is shown in Formula (6), The relationship between the rotational speed and counter electromotive force is shown in Formula (7) and (8) in the condition of rated excitation.

$$T_L = C_m I_L, \quad (6)$$

$$T_{ed} = C_m I_d, \quad (7)$$

$$n = E_d / C_e. \quad (8)$$

The Formula (6) plug in the Formula (5), the Formula (9) can be deduced

$$T_{ed} - T_L = \frac{GD^2}{375 C_e} \frac{dE_d}{dt} \quad (9)$$

The Formula (7) plug in the Formula (8), the Formula (10) can be deduced

$$C_m(I_d - I_L) = \frac{GD^2}{375 C_e} \frac{dE_d}{dt} \quad (10)$$

In the condition of zero initial, Laplace transforms Formula (10). It can be pushed into the Formula (11)

$$C_m[I_d(s) - I_L(s)] = \frac{GD^2}{375 C_e} s E_d(s). \quad (11)$$

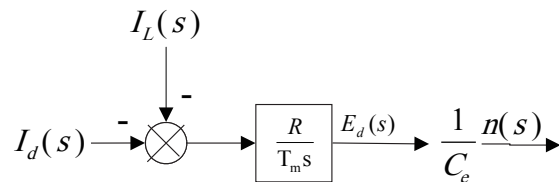
Respectively the numerator and denominator on the right side expression of Formula (11) multiply by R, and the Formula (12) can be deduced.

$$\begin{aligned} [I_d(s) - I_L(s)] &= \frac{GD^2}{375 C_e C_m} s E_d(s) \\ &= \frac{GD^2 R}{375 C_e C_m} \frac{s E_d(s)}{R} = \frac{T_m}{R} s E_d(s). \end{aligned} \quad (12)$$

According to the Formula (12), the transfer function of electrodynamic force and current can be concluded, as shown in Formula (13).

$$\begin{cases} \frac{I_d(s)}{U_{d0} - E_d(s)} = \frac{1/R}{1 + T_l s} \\ \frac{I_d(s)}{I_d(s) - I_L(s)} = \frac{R}{T_m s} \end{cases} \quad (13)$$

According to the Formula (13) the dynamic structure between the armature electrodynamic force and current of DC motor armature circuit can be inferred. As shown in Figure 3.

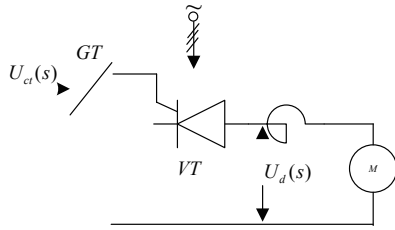


**Figure 3**  
The Dynamic Structure Between the Electrodynamic Force and Current of DC Motor Armature Circuit

## 1.2 A Dynamic Mathematical Model of Power Electronic Converter

### 1.2.1 Amplification Coefficient and Transfer Function of Thyristor Trigger GT and Rectifier VT (Eppinger & Weering, 2000)

Schematic diagram of thyristor-motor speed control system (V-M system) can be shown in Figure 4. In figure, VT is a thyristor controlled rectifier, GT is the trigger. In V-M system, the trigger for thyristor and a rectifier is regard as a link. When the analysis and design of closed-loop speed control system can be in progress, the amplification coefficient and transfer function of the link will be obtained.

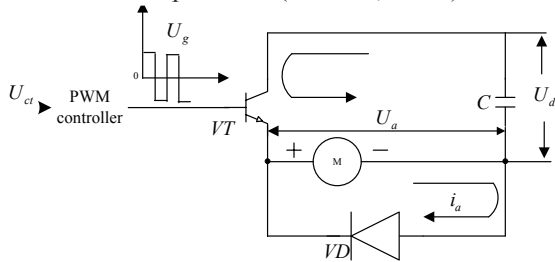


**Figure 4**  
**Schematic Diagram of Thyristor-Motor Speed Control System (V-M System)**

The input of this link is control voltage of flip flops  $U_{ct}$ , The output is the output voltage of the rectifier  $U_{d0}$ . Between the amplifying coefficients  $K_s$  of input and output can be estimated through measuring the characteristics or according to the device parameters.

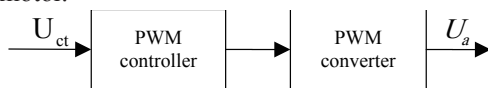
**1.2.2 The Mathematical Model of PWM Converter in Speed Control of DC Motor System**

The block diagram of irreversible PWM converter-DC motor system is shown in Figure 5. In the figure power switch device adopts IGBT (or IGCT, IEGT).



**Figure 5**  
**The Block Diagram of Irreversible PWM Converter-DC Motor System**

The block diagram of the PWM controller and the converter is shown in Figure 6, In the figure  $U_a$  is DC average voltage of output PWM converter in the steady state. The driving voltage  $U_g$  of the main circuit switching device is generated by the PWM controller.  $U_{ct}$  is the control voltage of the PWM controller.  $U_d$  is a DC power supply voltage. C is filter capacitor. VT is power switching device. VD is a free wheeling diode. MD is a DC motor.



**Figure 6**  
**Block Diagram of the PWM Controller and the Converter**

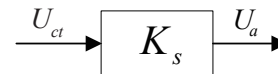
On the basis of the PWM converter working situation the average voltage of the output PWM converter varies linearly with the control voltage, Therefore, the amplification coefficient of PWM converter can be obtained, As shown in the Formula (14).

$$K_s = \frac{U_a}{U_{ct}} \quad (14)$$

When the switching frequency is 10 kHz,  $T=0.1$  ms Thus response delay can be ignored. This response delay is the output voltage of PWM converter to PWM control signal. PWM control and transform is considered real-time. Therefore, the mathematical model of the PWM converter can be written as a Formula (15)

$$W_s = \frac{U_a(s)}{U_{ct}(s)} = K_s \quad (15)$$

Formula (15) can be expressed as the structure diagram of PWM converter, as shown in Figure 7.

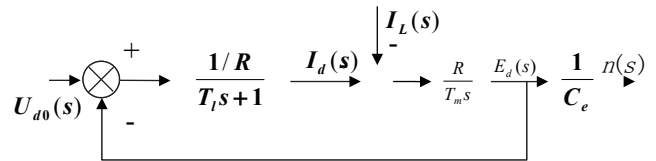


**Figure 7**  
**The Structure Diagram of PWM Converter**

**1.3 The Controlled Object Model of DC Speed Control System (Ishimoto & Takkakaw, 2009)**

**1.3.1 Dynamic Structure Diagram of DC Motor in Rated Excitation State**

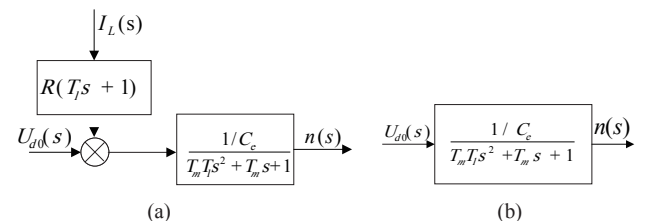
Dynamic structure diagram of dc motor in rated excitation state is shown in Figure 8.



**Figure 8**  
**Dynamic Structure Diagram of DC Motor in Rated Excitation State**

It can be known from Figure 8, that DC motor has two inputs, one is Ideal no-load voltage  $U_{d0}$  of the armature circuit, and the other is a load current  $I_L$ .

The former is the control input; the latter is the disturbance input. If you do not need to show a current  $I_d$  in the structure diagram, Comprehensive point of perturbation can be moved forward, Feedback point of electromotive force move backward. Then does the equivalent transformation. The dynamic structure diagram of is shown as Figure 9(a). When no-load  $I_L=0$ , the structure diagrams, it can be simplified into Figure 9(b).



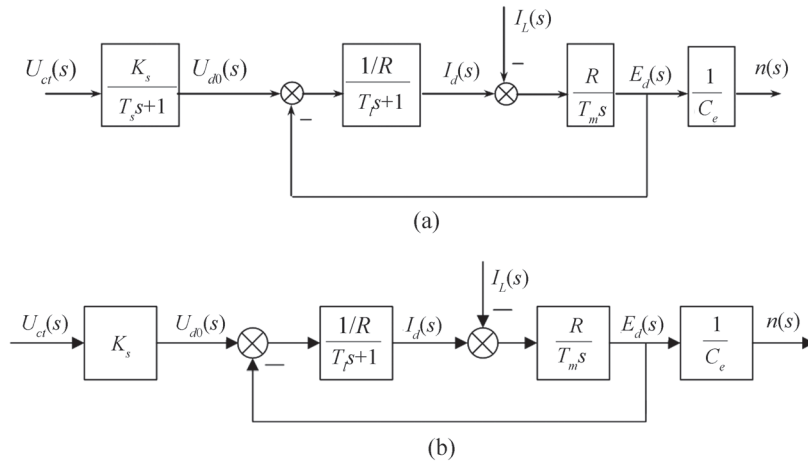
**Figure 9**  
**The Change and Simplification of DC Motor Dynamic Structure Diagram. (a)  $I_L \neq 0$ , (b)  $I_L = 0$**

In rated excitation the DC motor is a two order linear link, its characteristic equation is shown as Formula (16)

$$T_m T_i s^2 + T_m s + 1 = 0. \tag{16}$$

Time constants  $T_m$  is electrical inertia, time constants  $T_i$  are and electromagnetic inertia. If  $T_m > 4T_i$ , the two roots of the characteristic equation are the two negative real numbers. At this time, the transfer function between

$U_{d0}$  and  $n$  can be decomposed into two linear link, when Instantaneously increasing the input, speed will change linearly. If  $T_m > 4T_i$ , the character equation has a pair of conjugate solution that has negative real part. Therefore, the DC motor is a two order oscillation systems; this suggests that the dc motor has character of oscillation in the running.



**Figure 10**  
**Dynamic Structure Diagram of the Rotating Armature System in Rated Excitation Condition. (a) In Rated Excitation State, The Dynamic Structure Diagram of Thyristor- DC Motor Speed Control System, (b) In rated Excitation State, the Dynamic Structure Diagram of PWM- DC Motor Speed Control System**

**1.3.2 Dynamic Structure Diagram of the Rotating Armature System in Rated Excitation Condition**

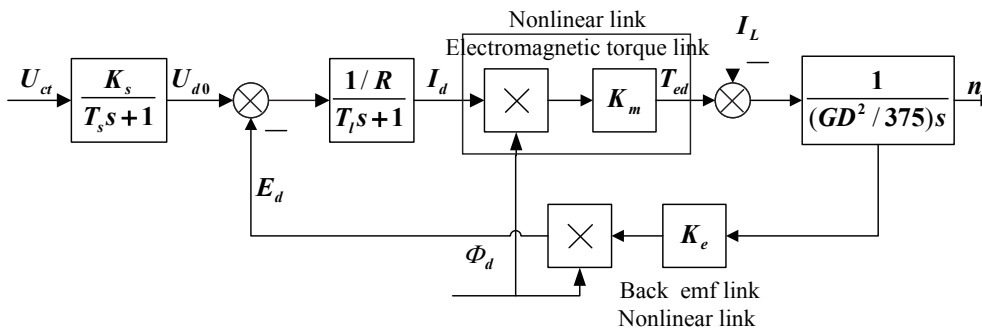
When the flux  $\Phi_d$  is a variable, Parameter  $C_e = K_e \Phi_d$ ,  $C_m = K_m \Phi_d$  is no longer constant. In order to analyze the problems,  $\Phi_d$  should be reflected in the counter electromotive force equation and the electromagnetic torque equation, Equation can be expressed as Formula

(17) and (18):

$$E_d = C_e n = K_e \Phi_d n, \tag{17}$$

$$T_{ed} = C_m I_d = K_m \Phi_d I_d. \tag{18}$$

According to the structure diagram 10 and the Formula (17), Formula (18) structure diagram can be obtained in the condition of weak magnetic, as shown in Figure 11.



**Figure 11**  
**The Dynamic Structure Diagram of Generalized Controlled Object That Is DC Speed Control System in Weak Magnetic State**

We can get the conclusions from the above, in the weak magnetic state Electromagnetic torque form link ( $T_{ed} = C_m I_d = K_m \Phi_d I_d$ ) At the same time Counter electromotive force form link ( $E_d = C_e n = K_e \Phi_d n$ ). Therefore, the mathematical model of DC motor contain two nonlinear links. the mechanical and electrical time

constant is as shown in Formula (19).

$$T_m = \frac{GD^2 R}{375 K_e K_m \Phi_d^2}. \tag{19}$$

When  $\Phi_d$  reduced into time-varying parameter, In the weak magnetic process, the math model of the controlled

object of DC speed control system has nonlinear characteristics. The dynamic structure diagram shown in Figure 12, consist linear and nonlinear link. Linear link can be expressed by transfer function, the input and output nonlinear link can only be expressed by time domain. The connection of linear link and nonlinear link is viewed as a kind of connection structure. This is must pay attention to the problems existing in the application.

## 2. THE VIRTUAL EXPERIMENT

The experiments that you will perform are intended to give you hands-on experience in and analyzing the system components and experimenting with various feedback control schemes. The setup components are as follows:

- (a) A dc motor with a speed sensor (normally a tachometer) or a position sensor (usually an encoder with incremental rotation measurement)
- (b) A power supply and amplifier to power the motor
- (c) Interface cards to monitor the sensor and provide a command voltage to the amplifier input and a PC running MATLAB and Simlink to control the system and to record the response (alternatively, the controller may be composed of an analog circuit system)

The components are described in the next section 2.1, 2.2, 2.3.

### 2.1 DC Motor

The motor is used a permanent magnet dc motor, the parameters are shown in Table 1:

**Table 1**  
**Parameters of Dc Motor**

Parameters	Signification	The values
$k_m$	Motor constant	0.10 Nm/A
$k_b$	Speed	0.10 V/rad/sec
$R_a$	Armature resistance	0.35 ohm
$L_A$	Armature inductance	0.56 mH
$J_m$	Armature moment of inertia	$0.0019 \times 10^{-4}$ kg/m <sup>2</sup>
$\tau_m$	Motor mechanical time constant	$2.3172 \times 10^{-5}$ sec

A reduction gear head may be attached to the output disk to the motor shaft. If the motor shaft angular rotation is considered as the output, the gear head will scale the inertia of the load by  $1/n^2$  in the system model, where n is the gear ration.

### 2.2 Position Sensor or Speed Sensor

For position-control application, an incremental encoder or a potentiometer may be attached directly to connect a tachometer to the motor shaft to measure the rotation of

the armature. To speed control, it is customary to connect a tachometer to the motor shaft. Normally sensor-shaft inertia and damping is too small to be included in the system model. The output of each sensor is proportional to the variable it is measuring, we will assume a proportionality gain of 1, that is,  $K_v=1$ (speed), and  $K_s=1$ (position)

### 2.3 Power Amplifier

The purpose of the amplifier is to increase the current capacity of the voltage signal from the analog output interface card. The output current from the interface should normally be limited, whereas the motor can draw many times the current. The details of the amplifier design are somewhat complex and will not be discussed here, but we should note two important points regarding the amplifier:

(a) The maximum voltage that can be output by the amplifier is effectively limited to 20 V.

(b) The maximum current that the amplifier can provide to the motor is limited to 8A, Therefore,

- ① Amp gain 2 V/V
- ② Amplifier input saturation limits  $\pm 10V$
- ③ Current saturation limits  $\pm 4V$

### 2.4 Virtual Experiment

#### 2.4.1 Open-Loop Speed

The first step is to model the motor. Using the parameter values in section 3.1 and simulating the open-loop velocity response of the motor to a step voltage applied to the armature. You should run the following tests using SIMLab:

(a) Apply step inputs of +5V, +15V, and -10V. Note that the steady-state speed should be approximately the applied armature voltage divided by  $k_b$  as in Formula (5) (try dc motor alone with no gear head or load applied in this case).

(b) Study the effect of viscous friction on the steady-state motor speed. First set  $B=0$  in the simlink motor parameter window. Then gradually increase its value and check the speed response.

(c) Repeat step 2 and connect the gear head with a gear ratio of 5.2:1, using additional load inertia at the output shaft of the gear head, of 0.05 kg/m<sup>2</sup> (requires modification of J in the Simulink motor parameters).

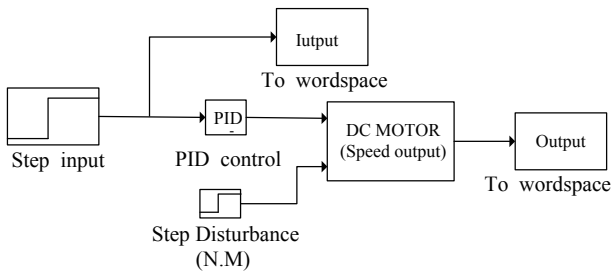
(d) Determine the viscous friction required for output shaft to reduce the motor speed by 50 percent from the speed it would rotate as if there were no viscous friction.

(e) Introduce a step-disturbance input  $T_L$  and study its effect on the system in Step 3.

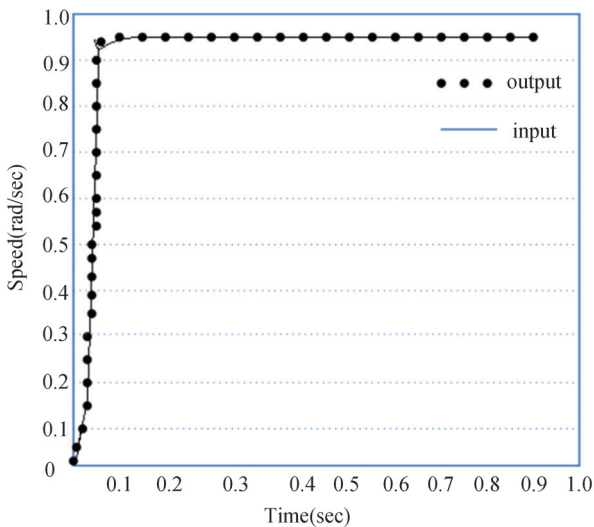
(f) Assuming that you do not know the overall inertia J for the system in Step 3, can you see the speed-response plot to estimate its value? If so, confirm the values of motor and load inertia. How about the viscous-damping coefficient? Can you use the time response to find other system parameters?



In this experiment, we use the open-loop model represented. The Simlink system model is shown in Figure 12, representing a simple open-loop model with a motor speed output. A typical open-loop speed response is shown in Figure 13.



**Figure 12**  
Simlab Open-Loop Speed Response of DC Motor Experiment



**Figure 13**  
Speed Response of the Open-Loop System

### 2.4.2 Speed Control

Having simulated the motor characteristics in previous sections, we can now extend the model to include velocity feedback from the motor and use a proportional controller. Assume that the motor velocity is measured using a sensor that provides 1V/rad/sec. The block diagram that you should be modeling is shown in Figure 4. For proportional gains of 1, 10, and 100, perform the following tests using SIMLab:

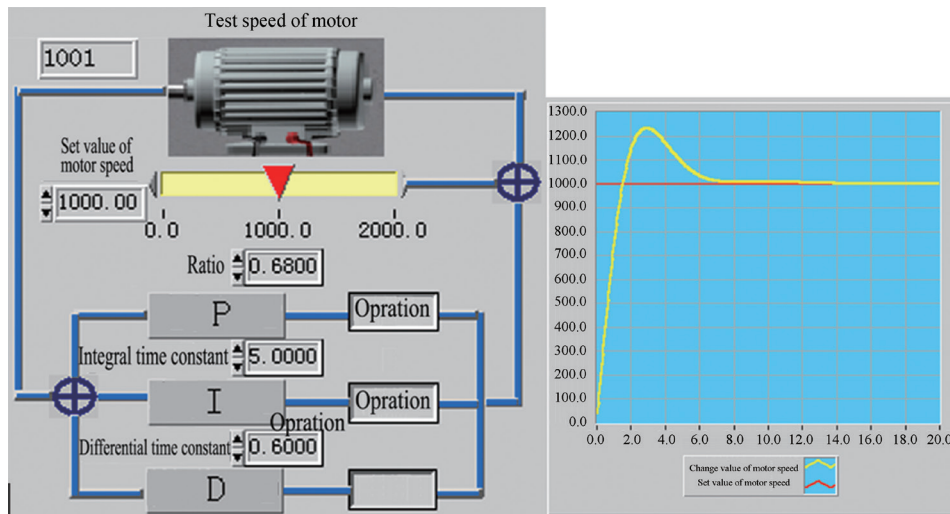
(a) Apply step input of +5V, +15V, and -10V (try dc motor alone; no gear head or load applied in this case).

(b) Repeat step 1 using additional load inertia at the output shaft of the gear head (gear ratio 5.2:1) of 0.05 kg/m<sup>2</sup> (requires adjustment of the J value in SIMLab)

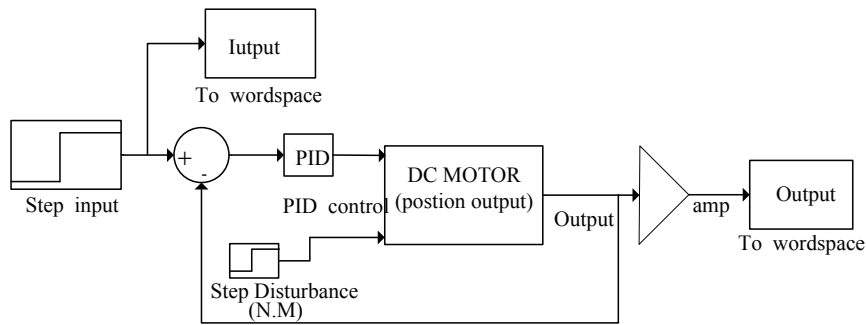
(c) Apply the same viscous friction to the output shaft as obtained, and observe the effect of closed-loop control. By how much does the speed change?

Open up Experiment: speed control of the SIMLab menu window. A screen similar to Figure 14 will be displayed, next, select the Enter Model Parameters to get the system Simulink model, as shown in Figure 15. This figure is a simple PID speed-control model. Double-clicking on the PID block displays the editable PID values. The values of the step-input and disturbance-torque blocks may also be adjusted. The disturbance-torque default value is set to zero. To change an input value, simply change its final value.

Increasing the proportional gain in the PID block will decrease the rise time. For an unsaturated model, the SIMLab version of this experiment could exhibit extremely fast rise times at very high proportional gains, because the dc motor can utilize unlimited voltage and select the Look under Mask to obtain the motor model similar. Double-click both the voltage and current blocks and adjust their values to very large (or delete

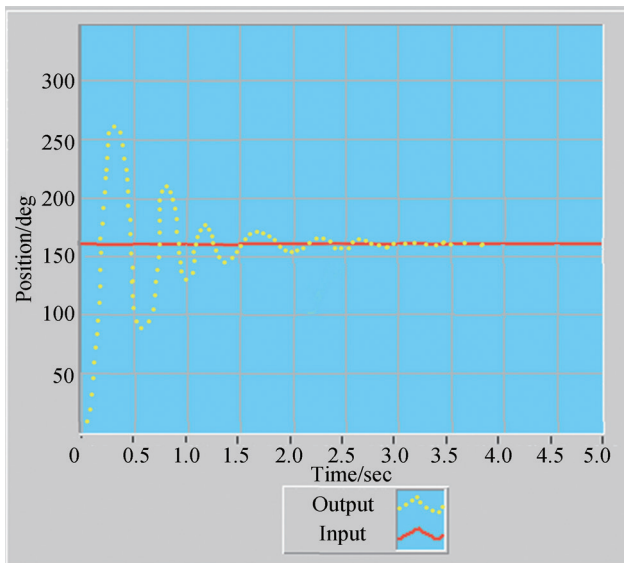


**Figure 14**  
Speed-Control Response in the Simlab Control Window



**Figure 15**  
**Position Control Simulink Model**

their blocks). Recall from Section 1.2 that the default saturation limits are  $\pm 10V$  and  $\pm 4V$ , respectively. Figure 16 displays a typical speed response from the SIMLab.



**Figure 16**  
**Position-Control Response in the SIMLab Control Window**

For a given input to change the proportional gain values, enter in the following sets of PID values and print all three plots on the same figure (use the print to button

- ①  $P=1 \quad I=0 \quad D=0$ ,
- ②  $P=10 \quad I=0 \quad D=0$ ,
- ③  $P=100 \quad I=0 \quad D=0$ .

The input units used in these simulations are specified in volt, while the feedback units at the motor are in radians per second. This was done intentionally to illustrate the scaling (or conversion) that is performed by the sensor. Had the velocity been specified in effect of the radians per second, a different response would have been obtained. To check the effect of the velocity feedback scaling, repeat the preceding experiments using a proportional gain of 10, but assume that the velocity feedback signal is a voltage

generated by a sensor with a conversion factor of  $0.2V/\text{rad/sec}$ . (Note: In commonly used industry standards the tachometer gain is in volt per PPM)

Next, for the Virtual Lab test the following:

Apply step inputs of  $\pm 5V$ ,  $\pm 15V$ , and  $-10V$ . How different are the results from the SIMLab.

Select the enter model parameters to get the system simulink model, as shown in Figure 14. This figure is simple PID speed-control model, Figure 14 displays a typical speed response from the SIMLab.

### 2.4.3 Position Control

Investigate the closed-loop position; modify your model to position feedback. For proportional gains of 1, 10, and 100 (requires modification of PID block parameters), perform the following tests using SIMLab:

- (a) for the motor alone, apply a  $160^\circ$  step input;
- (b) Apply a step disturbance torque ( $-0.1$ ) and repeat 1;
- (c) Examine the effect of integral control in step 2 by modifying the Simulink PID block;
- (d) Repeat Step 1 using additional load inertia at the output shaft of  $0.05 \text{ kg/m}^2$  and the gear ratio 5.2:1 (requires modification of  $J_m$  and B in the motor parameters);
- (e) Set  $B=0$  and repeat Step 4 ;
- (f) Examine the effect of voltage and current saturation blocks (requires modification of the saturation blocks in the motor model);
- (g) In all above cases, comment on the validity of Formula (13).

Open up Experiment-position control from the SIMLab menu window. A screen similar to Figure 8 will be displayed. Next select Enter Model Parameters to get the system Simulink model, as shown in Figure 8. This figure is a simple PID position-control model. Double-clicking on PID block displays the editable PID values. The Deg to Rad and Rad to Deg gain blocks convert the input and output such that the user enters inputs and receives outputs in degrees only. The values of the step-input and the disturbance-torque blocks may also adjust. The disturbance-torque default value is set to zero. To change an input values, simply change its final value. Figure 9 displays a typical position response from the SIMLab.

The position time response can be animated. The Animate Position Response button initiates the animation, and the Stop Animation button halts it. This is a very useful tool that gives the user a physical sense of how a real motor turns. The time input-angle, and out-angle values are displayed on the animation field.

The nonlinearities due to voltage and current limits cause the time response to saturate at a high enough proportional gain. In the SIMLab version, the saturation values may be adjusted as discussed. Try to examine the saturation effect for different proportional gain values. For proportional gain of 1, 10, and 100 (requires modification of PID block parameters), repeat step 1 and 3 tests using Virtual lab)

Select the enter model parameters to get the system simulink model, as shown in Figure 15. This figure is simple PID position-control model, Figure 16 displays a typical speed response from the SIMlab.

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## CONCLUSION

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According to the needs of practice teaching, we described the SIMLab and Virtual Lab software to improve students understanding of control and to provide a better practical appreciation of the subject. We discussed that in a

realistic system including an actuator and mechanical and electrical components, issues such as saturation of the amplifier friction in the motor, or backlash in gears will seriously affect the controller design, we focused on problems involving dc motors including modeling and virtual experiment. We presented experiments on speed and position control of dc motors. The focus on dc motors in these experiments was intentional, because of their simplicity and wide use in industrial applications.

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